A treatise on the use of magic squares

JACQUES SESIANO
seziano@bk.ru

ABSTRACT: The (anonymous) Arabic treatise studied below, a 17th-century copy of a late mediaeval work, deals with the application of magic squares as amulets. With the association of numerical values to Arabic letters, to each word or to each sentence may be attributed a certain sum. Placing this word or sentence in some row of an empty square, the task will be to complete the square so that it would display this quantity as a magic sum. In the first part, the author presents numerical magic squares filled with the first consecutive natural numbers (size 3×3 to 10×10), the particularity of which is to ease subsequent placing of the required word or sentence in some row and then complete the square. Examples of such constructions are presented in the second part. The third part is devoted to the applications: once the desired magic square is constructed, the reader is taught on which material and at which time it must best be drawn in order to ensure successful use.

KEY WORDS: Arabic magic squares, amulets.

Un tractat sobre l'ús de quadrats màgics

RESUM: El tractat àrab anònim estudiat, una còpia del s. XVII d'una obra tardomedieval, tracta de l'aplicació de quadrats màgics com a amulets. Una certa suma s'associa a paraules o frases senceres a partir dels valors numèrics a lletres àrabs; es col·loca aquesta paraula o frase en una fila del quadrat buit, i llavors es completa un quadrat màgic, de manera que la suma sigui la suma màgica. A la primera part l'autor del tractat presenta quadrats màgics numèrics compostos amb els primers nombres naturals (de mida 3×3 a 10×10), amb l'objectiu de simplificar l'operació de col·locar la paraula o frase requerida en un quadrat per completar-lo. A la segona part es donen exemples. La tercera part es dedica a les aplicacions: un cop es construeix el quadrat màgic desitjat, es prescriu en quin material i en quin moment cal dibuixar el quadrat per assegurar-ne un ús exitós.

PARAULES CLAU: Quadrats màgics àrabs, amulets.

Sesiano, Jacques. (2022). «A treatise on the use of magic squares». *Suhayl* 19, pp. 89–150. ISSN: 1576-9372.
DOI: 10.134/SUHAYL2022.19.3.

Introduction and historical remarks

A *magic square* is a square divided into a square number of cells in which natural numbers, all different, are arranged in such a way that the same («magic») sum is found in each horizontal row, each vertical row, and each of the two main diagonals. For squares of given *orders*, that is, with a given number of cells on each side, the smallest possible magic sums are those obtained when the squares are filled with the first consecutive natural numbers. We shall call them basic magic sums. Their values are given in Fig. 1 for the orders 3 to 10, the only ones occurring in the treatise studied here, together with the formula for calculating them for any order n. Note that a magic square of order 2 is not possible, whereas those of all larger orders are.

3	4	5	6	7	8	9	10	...	n
15	34	65	111	175	260	369	505	...	$\frac{n\,(n^2+1)}{2}$

Figure 1

Besides these general, *ordinary* magic squares, there is the category of *bordered* magic squares, which we shall also meet in our text. Their characteristic is that when the borders are successively removed, what remains is still a magic square, just with a smaller magic sum. In this case pairs of opposite cells (horizontally, vertically, and diagonally for the corners) all make the same sum, namely n^2+1 for a square of order n, which explains their special feature: each removal of a border reduces the magic sum by $n^2 + 1$. Note that a bordered square of odd order may be progressively reduced to the smallest possible square, that of order 3, whereas an even-order square will end with a square of order 4 (since, as said above, there is no magic square of order 2).

Still another type of magic squares is that of *composite* squares, briefly alluded to in our text. Not only is the whole square magic, but so are the square parts inside it. Such a square must have at least the order 9, with inner squares of order 3; generally, the possibility for larger composite squares will depend on the divisibility of the main square's order.

There exist for the construction of magic squares generally applicable methods, but they are at best applicable to just one of three categories of orders: odd orders, thus equal to an odd number (from 3 on); evenly-even orders, thus equal to an even number divisible by 4 (from 4 on); evenly-odd orders, thus equal to an even number divisible by 2 but not by 4, the smallest of which is therefore the square of order 6.

From the earliest times of their existence magic squares have attracted two very different kinds of people. First, mathematicians, who initiated the search for general methods of construction; indeed, trial-and-error constructions served, at best, to obtain a few squares of small orders. The other kind were those who, admiring the characteristic and not understanding its background, saw it as a «magic» property.

The history of magic squares begins in antiquity. A square of order 3 is said to have appeared first in China, at an unknown time and surrounded by legends. That was apparently an isolated occurrence. Another isolated occurrence is found in Greece, but of quite a different nature, with an anonymous treatise, surviving in an Arabic translation, displaying methodical ways to construct bordered and composite squares (not ordinary ones, simply mentioned in the introduction).[1] These simple methods are used, but without explanation of their background, and only the more difficult applications are mathematically justified. Obviously, this is a high-level treatise, not intended for beginners. It must be the only surviving result of earlier studies, a situation comparable to that of Diophantus' *Arithmetica*.

This Arabic translation gave rise to the first properly Arabic treatise, that by Abū'l-Wafā' Būzjānī (940-997/8). He attempted to give a mathematical background to the science of magic squares, and tried in particular to find the origin of the general constructions found in the Greek treatise. But his efforts, despite his competence as a mathematician, were only partly successful: he first noticed the general method for bordered squares of odd orders, since it appeared from the examples given (order 3 to 11), and was able to explain it; but failed in the case of even orders, a main reason being his overlooking the difference between the two kinds of even orders. Indeed, he treated separately the squares of orders 4, 6, 8 in various ways, but did not find common features in their arrangements which could be applied to higher orders. He also examined the construction of ordinary squares, but here too his attempts remained limited to the first orders.[2]

1. See *An ancient Greek treatise on magic squares*.

2. See *Le traité d'Abū'l-Wafā'* (full text and translation) or *Magic squares in the tenth century* (extracts and comparative study). The latter includes the edition of another 10th-century treatise, that by Anṭākī (d. 987); his work is here rather irrelevant since he merely copied the translation of the Greek treatise (among other works translated from the Greek).

The eleventh century witnessed the success of the mathematical approach, this time not based on isolated examples; the result was the discovery of general methods for constructing ordinary squares (remember that, for bordered squares, the Greek treatise transmitted the way to obtain them). The basis was the study of the properties of the *natural squares* for each of the three categories of order, that is, the squares having the same order as the ones to be constructed but containing the first numbers in their natural order. It was noted that, first, all their diagonals, main and broken, always contain the magic sum while, second, pairs of symmetrically placed lines and columns differ from the magic sum by the same amount but with a different sign. The first property gave the clue to one general construction for odd orders and the second was applied to even orders.[3] Arabic studies were, in the following centuries, to produce various other methods and improvements in construction techniques, but often to the detriment of the mathematical explanations: most readers merely wished to have a quick and easy way to obtain magic squares.

The discovery of easy construction methods thus increased the interest in magic squares, in particular among non-mathematicians. There was further enthusiasm on account of another «magic» aspect, which relied primarily on the association of Arabic letters with numerals. This association has its origin in the Greek numerical system, an adaptation of which was in use in early Islamic times (Fig. 2), and remained employed after the adoption of Indian numerals. Since to each word or sentence corresponds a numerical quantity, namely the sum of its letters, the aim would be to construct a magic square displaying this numerical quantity in any row (horizontal, vertical, main diagonal). A further refinement consisted in having the values of the individual letters of the word, or of the individual words in a sentence, appear in a row of a square with the corresponding order, which was then to be completed. This gave rise to a new kind of squares, those filled with non-consecutive numbers, a mathematically interesting problem since the construction may be subject to various restrictions. For practical use, such squares were of the first seven orders, thus from 3 to 9: they were commonly associated with the seven planets then known (including Moon and Sun), of which they embodied the respective, good or evil, qualities. The reader was then taught on what material and when to draw each of these squares, since both the nature of the material and the astrologically predetermined time for doing so were said to increase the square's efficacy.

3. On this turning point, see *Les carrés magiques*, pp. 25–27, 49–55, 89–91 (Russian edition pp. 33–35, 58–65, 100–102), or *Magic squares, their history* (...), pp. 25–28, 51–56, 93–95.

α̅ ا 1	β̅ ب 2	γ̅ ج 3	δ̅ د 4	ε̅ ه 5	ϛ̅ و 6	ζ̅ ز 7	η̅ ح 8	ϑ̅ ط 9
ι̅ ى 10	κ̅ ك 20	λ̅ ل 30	μ̅ م 40	ν̅ ن 50	ξ̅ س 60	ο̅ ع 70	π̅ ف 80	ϙ̅ ص 90
ρ̅ ق 100	σ̅ ر 200	τ̅ ش 300	υ̅ ت 400	φ̅ ث 500	χ̅ خ 600	ψ̅ ذ 700	ω̅ ض 800	ϡ̅ ظ 900
͵α̅ غ 1000								

FIGURE 2

But the task of completing the square was still too demanding for certain readers, who wanted a ready-made product. Thus many shorter texts merely give the figures of the first seven squares filled with the first consecutive numbers and describe their magical use. Of such a kind were the first Arabic texts translated into Latin in 14th-century Spain;[4] whence the denomination of *magici*, sometimes *planetarii*, which came to replace the original and more mathematical one of «Harmonious arrangement of numbers», *wafq al-aʿdād*, perhaps a translation of ἁρμονία τῶν ἀριθμῶν.

THE MANUSCRIPT AND ITS CONTENTS

The text studied here is preserved on fol. 43^{v}, 1 – 50^{r}, 5 of the manuscript Haci Mahmut Efendi 5726 (now in the Süleymaniye Library), copied in 1085h (AD 1675), with mostly 23 lines to a page. The original text might go back to the 13th or 14th century. It had a certain success: early readers left marginal annotations which were then incorporated into the text, by our copyist or a previous one. The present copyist thus did not really follow the text; still, he can be said to have been very careful, for practically all the diacritical points are extant. The figures (in numerals) sometimes contain errors, maybe due to our copyist since three re-

4. See our *Magic squares for daily life*.

peated figures (bordered seven by seven, ordinary and bordered six by six) do not display the same mistakes.

The author's intention is not to present a theory of magic squares; thus, he does not explain the basis of his constructions. His purpose is the application of magic squares as amulets, thus containing a magic sum equal to the numerical value of some word or sentence. His text therefore belongs to the first kind of magic ones mentioned above, namely those requiring from the reader some mathematical comprehension to complete such squares.

The treatise consists of three parts. In the first, ordinary magic squares filled with the first consecutive numbers are described, but in the crudest way: we are told in which cell each number must be put. At most we learn that a short sequence of numbers will be placed by the knight's move, but only as long as it remains within the figure; thus the author does not consider, or mention, that a (say) knight's move may be prolonged across the other side, horizontally and vertically, as if the same square were reproduced there virtually. Therefore there are no means of memorizing the construction. These tedious descriptions are for the orders 3 to 10 (the presence of order 10 raises some questions). The same applies to the bordered squares, with once again no attempt to justify the construction, let alone facilitate the memorization. The squares are a tool, not a goal.

The author does, however, digress to count the number of figures obtainable from the four types of 4×4 squares he presents. This is not the first time such an attempt has been made: Abū'l-Wafā' Būzjānī considered the number of possible borders in a square of order 5 and (disregarding changes in the inner part of order 3) obtained a result of 108 configurations;[5] all these squares will display the basic magic sum. Our author's attempt is less clear; for many of his 4×4 squares thus constructed will not be magic since the condition for the two main diagonals remains unfulfilled.

The second and third parts are devoted to applications proper, first theoretical then practical. In the second part we are taught, using the previous figures, how to construct squares containing a predetermined magic sum, be it as a whole or with individual parts, letters or words (that is, their numerical values), in one of the rows (here the first row); this placing requires some attention, but here the

5. Nine possibilities for the upper corners (instead of ten) multiplied by six plus six (instead of six by six) permutations within two opposite rows. See the edition of his treatise, pp. 153 & 212 (result missing), and MS. Delhi Arabic 110 (online: Qatar Digital Library), fol. 68^{r} (above result).

text provides the necessary explanations, illustrated by examples. The third part teaches how to prepare the amulets: choosing the material, making the appropriate fumigation and doing that at the most favourable time. This is also clearly explained. Thus, and once again, it is evident that the main purpose of this treatise is magical applications, but still aimed at readers of a certain level.

The ordinary magic squares described in the first part of our text display a very particular feature: for a square of order n, thus filled with n series of consecutive numbers, *each series has one, and only one, of its terms in each row* (horizontal, vertical, main diagonals). This is intentional: it will ease the construction of squares with either a given magic sum or given letters or words in one of their rows.

[1]	21	34	47	11	24	37
25	38	2	[15]	35	48	12
49	13	26	39	3	16	[29]
17	30	[43]	14	27	40	4
41	5	18	31	44	[8]	28
9	[22]	42	6	19	32	45
33	46	10	23	[36]	7	20

FIGURE 3

Consider, for instance, the ordinary square of order 7 filled with the first 49 natural numbers. Since 49 is the product of 7 by itself, it will contain seven series of seven consecutive numbers, namely the series 1,…,7; 8,…,14; 15,…,21; 22,…,28; 29,…,35; 36,…,42; 43,…,49. With our author's arrangement, there is, as said, one, and only one, term of each series in each of the rows displaying the magic sum 175 (Fig. 3). We are now to see how the same square may contain another, larger sum.

First case: Considering the sum of the letters

Evidently, if we add the same quantity to each term of the *last* series, the magic sum will be increased by this quantity. Adding the same quantity to the last two series, the magic sum will be increased by twice this quantity (which is perhaps more elegant than adding it twice to the last series, for the quantity of missing numbers will be smaller; see below, *iii*). This can be continued, taking the series from the last, and we shall thus obtain a magic square with numbers all different, and consecutive except where the increment first took place, the quantity of missing numbers then being equal to this increment. Thus, by transforming squares in this way, we may obtain any magic sum.

Suppose then the required sum to be S and the basic magic sum for the order considered, M. Thus each row of the basic square must be increased by $S - M$, to be distributed among some or all n cells of each row.

(*i*) The simplest case is if the quantity $S - M$ when divided by the order n gives a quotient without remainder, say s; for then s will be added to each cell of the basic magic square. In that case, each row will now contain M increased by n times s, thus the required sum S, and the square will contain a continuous number sequence beginning with $s + 1$.

Consider the above example of the ordinary square of order 7, with the magic sum 175. In order to obtain the magic sum 210, which differs from the basic sum by $35 = 5 \cdot 7$, we shall increase all numbers of the basic square by 5. The number sequence in the square will be 6 to 54. But observe that since the difference equals five times seven, we have the possibility to add 7 to each of the last five series, that is, add 7 to the first term of the third series and proceed continuously; thus the series beginning originally with 15 will now begin with 22. The numbers placed will begin with 1, and there will be a gap from 15 to 21, when the first increment took place. See Fig. 4. See also (*iii*) below, where the whole increment is put in the last series.

(*ii*) But if $S - M$ is not exactly divisible by n, there will be an integral part, say again s, and a remainder, necessarily smaller than n, say n_2. So we shall add s to the n series and, in addition, a unit to the last n_2 series. Then each row will contain M increased by n times s plus n_2 units, thus the required sum S. The square will display two continuous sequences of numbers, from $s + 1$ to $s + n\,(n - n_2)$ and from $s + n\,(n - n_2) + 2$ to $s + n^2 + 1$, thus separated by one missing term.

[1]	28	41	54	11	31	44
32	45	2	[22]	42	55	12
56	13	33	46	3	23	[36]
24	37	[50]	14	34	47	4
48	5	25	38	51	[8]	35
9	[29]	49	6	26	39	52
40	53	10	30	[43]	7	27

FIGURE 4

[11]	32	45	58	21	35	48
36	49	12	[26]	46	59	22
60	23	37	50	13	27	[40]
28	41	[54]	24	38	51	14
52	15	29	42	55	[18]	39
19	[33]	53	16	30	43	56
44	57	20	34	[47]	17	31

FIGURE 5

Suppose that we wish the 7×7 square to have the magic sum 250. Subtracting 175 from it leaves 75, which, divided by 7 gives 10 and the remainder 5. We shall increase all numbers by 10 and share the remaining units among the last five series, that is, increase them by 11 instead of 10. The first two series, 1 to 7 and 8 to 14, will thus become 11 to 17 and 18 to 24, while the third series will begin with 26. The square will then contain the numbers from 11 to 60, with 25 missing (at the place of the new increment). See Fig. 5.

(*iii*) There is a simpler possibility, as hinted at by our author, which is applicable to both cases. We place the $n - 1$ first series as for the ordinary square and add the quantity $S - M$ to the n terms of the last series. The square will then contain the two continuous sequences of numbers from 1 to $n^2 - n$ and from $n^2 - n + 1 + S - M$ to $n^2 + S - M$.

In our two examples, we would keep the first six series unchanged and add $210 - 175 = 35$, respectively $250 - 175 = 75$, to the terms of the last series, which will thus begin, instead of 43, with 78 and 118, respectively.

Second case: Individual values in a given line (say the first)

s_1	s_2	s_3
$r+s_3-s_1$	r	$r+s_1-s_3$
$2r-s_3$	$2r-s_2$	$2r-s_1$

FIGURE 6

As seen from Fig. 6, where s_1, s_2, s_3 are the numerical values of the three letters or the three words, with the sum $S = 3r$, the square of order 3 is a particular case. Indeed, since a third of the sum *must* be put in the central cell, the numbers to be placed in the other cells are all determined. Our text states clearly that the given sum must be divisible by three (Section II, first kind, beginning; Arabic, line 253), but overlooks the possibility of some of the remaining numbers becoming negative.

Consider now the six other squares connected with planets, thus with the orders 4 to 9 (Fig. 7-12). The n given letters s_i, with the sum S, being placed in the first line, the remainder of the square will be filled as in the figures we have indicated below. Indeed, this corresponds exactly to how the series are placed in the basic squares of orders 4 to 9 filled by the author with the first consecutive natural numbers. Thus the series of s_i will comprise $n - 1$ further terms, distributed continuously on one or both sides of s_i. A problem of repetition might arise if the numerical values of several of the given letters correspond to one of the units or 10.

s_1	s_2	s_3	s_4
$s_3{+}1$	$s_4{-}1$	$s_1{+}1$	$s_2{-}1$
$s_4{-}2$	$s_3{-}2$	$s_2{+}2$	$s_1{+}2$
$s_2{+}1$	$s_1{+}3$	$s_4{-}3$	$s_3{-}1$

FIGURE 7

s_1	s_2	s_3	s_4	s_5
$s_3{+}3$	$s_4{-}2$	$s_5{-}2$	$s_1{+}3$	$s_2{-}2$
$s_5{+}1$	$s_1{+}1$	$s_2{+}1$	$s_3{+}1$	$s_4{-}4$
$s_2{-}1$	$s_3{-}1$	$s_4{-}1$	$s_5{-}1$	$s_1{+}4$
$s_4{-}3$	$s_5{+}2$	$s_1{+}2$	$s_2{-}3$	$s_3{+}2$

FIGURE 8

s_1	s_2	s_3	s_4	s_5	s_6
$s_3{+}2$	$s_6{-}2$	$s_4{-}2$	$s_1{+}1$	$s_2{-}1$	$s_5{+}2$
$s_6{+}2$	$s_1{+}5$	$s_5{+}1$	$s_3{-}2$	$s_4{-}3$	$s_2{-}3$
$s_4{-}5$	$s_5{+}4$	$s_1{+}2$	$s_2{+}1$	$s_6{-}1$	$s_3{-}1$
$s_2{-}4$	$s_4{-}4$	$s_6{+}1$	$s_5{+}3$	$s_3{+}1$	$s_1{+}3$
$s_5{+}5$	$s_3{-}3$	$s_2{-}2$	$s_6{-}3$	$s_1{+}4$	$s_4{-}1$

FIGURE 9

s_1	s_2	s_3	s_4	s_5	s_6	s_7
s_6+1	s_7+1	s_1+1	s_2-6	s_3+1	s_4+1	s_5+1
s_4+2	s_5+2	s_6+2	s_7+2	s_1+2	s_2-5	s_3-5
s_2-4	s_3-4	s_4-4	s_5+3	s_6+3	s_7+3	s_1+3
s_7+4	s_1+4	s_2-3	s_3-3	s_4-3	s_5-3	s_6+4
s_5-2	s_6-2	s_7+5	s_1+5	s_2-2	s_3-2	s_4-2
s_3-1	s_4-1	s_5-1	s_6-1	s_7-1	s_1+6	s_2-1

FIGURE 10

s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
s_3+1	s_4-1	s_5+1	s_6-1	s_7+1	s_8-1	s_1+1	s_2-1
s_5+2	s_6-2	s_7+2	s_8-2	s_1+2	s_2-2	s_3+2	s_4-2
s_7+3	s_8-3	s_1+3	s_2-3	s_3+3	s_4-3	s_5+3	s_6-3
s_2-4	s_1+4	s_8+4	s_7+4	s_6+4	s_5-4	s_4-4	s_3-4
s_8+3	s_7+5	s_6+3	s_5-3	s_4-5	s_3-3	s_2-5	s_1+5
s_6+2	s_5-2	s_4-6	s_3-2	s_2-6	s_1+6	s_8+2	s_7+6
s_4-7	s_3-1	s_2-7	s_1+7	s_8+1	s_7+7	s_6+1	s_5-1

FIGURE 11

s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
s_7-4	s_8+5	s_9-4	s_1-4	s_2-4	s_3+5	s_4+5	s_5+5	s_6-4
s_4+1	s_5+1	s_6+1	s_7+1	s_8+1	s_9+1	s_1-8	s_2+1	s_3+1
s_2-3	s_3+6	s_1-3	s_5+6	s_6-3	s_4-3	s_8+6	s_9-3	s_7-3
s_8+2	s_9-7	s_7+2	s_2+2	s_3+2	s_1-7	s_5+2	s_6+2	s_4+2
s_5+7	s_6-2	s_4-2	s_8+7	s_9-2	s_7-2	s_2-2	s_3-2	s_1-2
s_3+3	s_1-6	s_2+3	s_6-6	s_4+3	s_5+3	s_9-6	s_7+3	s_8+3
s_9-1	s_7-1	s_8-1	s_3-1	s_1-1	s_2-1	s_6-1	s_4-1	s_5+8
s_6-5	s_4+4	s_5+4	s_9-5	s_7+4	s_8+4	s_3+4	s_1-5	s_2-5

FIGURE 12

Translation[6]

In the name of God the merciful, the compassionate.
There are three classes of magic square: ordinary, bordered, literal.[7]

Section 1: Numerical magic squares

The purpose in a *numerical magic square* is that the signification be a number, whether the denotative element is a letter or a numeral.[8] It is either «basic», that is, beginning with 1 (and continuing) to the completion of the square, or it is «not basic». The *basic magic sum* is either for one row or for all the rows.[9]

To obtain it (one proceeds in the following way).[10] One multiplies the (number of) cells on the side of the figure by itself; the result will be the quantity of all cells of the figure. Adding 1 to it gives the *equating number* [which is the number occurring in two opposite cells of a bordered (square), as seen in what follows].[11] Consider now the equating number. If it is even, one multiplies its half by the total number of cells on one side, and if it is odd, one multiplies the whole of it by half the number of cells on the side; the result will be the basic magic sum for one

6. In the translation, the figures are inverted, thus left-to-right; some are supplemented. Our additions in the text, to ease comprehension, are in parentheses; in the subsequent Arabic text, presumed interpolations are in square brackets, lacunas in angular brackets (with only the significant ones indicated in the translation). We have disregarded insignificant copying mistakes or repetitions. Minor lacunas recurring at the end of the description of the square of order ten have been indicated in the Arabic text only because they bear witness to the weariness of a copyist faced with a tediously repetitive text. In the first part, where the author describes the basic squares he will use, he sometimes refers to the horizontal row (here: «line»), sometimes (the majority of cases) to the column, and sometimes to the «diagonal», which means its extremity (first, second, third, fourth, the first and the fourth being the ends of the first diagonal). No «line» or «column» is specified for an inner diagonal cell («the fourth of the fourth» is indeed sufficient). Finally, in a horizontal move, where right and left refer to the (Arabic) direction, there is an asterisk in the translation since the opposite direction is to be considered in the transliterated figures.

7. This may have originally been a marginal gloss. It seems to be out of place here.

8. Whether its cells are occupied by figures or alphabetical numerals, the magic square is always connected with a numerical quantity, namely its magic sum.

9. The term *wafq* is used for both «magic square» and «magic sum».

10. The subsequent calculations for the equating number (n^2+1) and the magic sum $(\frac{1}{2}\,n(n^2+1))$ apply to a magic square filled with the first natural numbers.

11. Clearly an interpolation; it is irrelevant here, since bordered squares have not been defined yet.

row.[12] Multiplying that by the number of cells on one side, the result will be the basic magic sum for all the rows. [This does not hold unless a name is placed in the square, as seen in what follows.][13]

Example. Considering the cells on one side of the square of (order) three, we find 3. Multiplying 3 by itself gives 9, which is the quantity of all the cells of the three by three (square). Adding then 1 to it gives 10, which is the equating number, (here) even. Its half is 5. Multiplying 5 by 3 gives 15, which is the basic magic sum for one row. Multiplying 15 by 3 gives 45, which is the basic magic sum for all the rows.

Likewise for the square of 4. Here the equating number is odd, namely 17; multiplying it by half the number of cells in one row, thus 2, gives 34, which is the basic magic sum for one row. Multiplying it by the quantity of cells in one row gives 136, which is the basic magic sum for all the rows. [Then subtracting from this product, namely 136, and sharing the remainder among four cells of one side, if it is equal to an odd, among the cells (of) 9.][14] Likewise for the other figures.

Furthermore, the magic square may be *ordinary*, namely when it displays the magic sum for (each row of) the whole square, be it(s order) odd or even, but does not display a magic sum for the square inside it. [In this case, the equating number is not considered.][15] [But in it, the knight's move is considered] [and the series of the magic square are increased];[16] [and it is called the corner(s) of the circle.][17] Or it may be *bordered*, which is different, and better, than the ordinary one: if its border is removed, the remainder is a complete magic square.[18]

Let us describe, for each of these two (kinds), a method (of construction) for the figure of (each of the) eight basic ones.[19] But (only one) for the three by three square since there is only one method.[20]

12. The purpose of this distinction is merely to avoid the occurrence of a fraction.

13. This is badly expressed: it must mean that this definition of the equating number is applicable only to the basic magic squares, in particular not to ones involving names (Section II).

14. Lacunary. Presumably an attempt to return, in both cases, to the magic sum for one row and the equating number.

15. Meaning: unlike in bordered squares, opposite border cells do not display this quantity. But the equating number remains the same anyway since it depends only on the (whole) square's order.

16. Allusion to the construction of ordinary squares found later on.

17. Expression and meaning nonsensical.

18. Our author uses for «border» the unusual term *milban*, meaning a doorframe (Dozy's dictionary).

19. Thus for the orders 3 (smallest possible magic square) to 10. Hardly a method, more an enumeration of the individual places.

20. No border here; furthermore, there is just one possible placing with the first nine consecutive numbers.

(1. Square three by three)

(The method) is the following. 1 is put in the middle cell of the first column, 2 in the corresponding (lower) knight's cell, 3 **(44^{r})** in the knight's cell of the latter, 4 next to it above [which is the knight's (cell) of 1][21], 5 in its queen's cell, 6 likewise, 7 next to it, 8 in its knight's cell, 9 likewise. [It is possible to have the beginning in any middle (side cell), the remainder (of the placing) being as you know.][22]

8	3	4
1	5	9
6	7	2

(2. Square five by five)

1	1	14	22	10	18
2	25	8	16	4	12
3	19	2	15	23	6
4	13	21	9	17	5
5	7	20	3	11	24
	1	2	3	4	5

0,1	2,4	4,2	1,5	3,3
4,5	1,3	3,1	0,4	2,2
3,4	0,2	2,5	4,3	1,1
2,3	4,1	1,4	3,2	0,5
1,2	3,5	0,3	2,1	4,4

23	8	5	22	7
20	16	11	12	6
1	9	13	17	25
2	14	15	10	24
19	18	21	4	3

For the ordinary magic square five by five, 1 is put in the first cell of the first row; then one moves to the rows below with the knight's move twice (2, 3), then, successively, to the second cell of the fourth column (4), to the fourth cell of the fifth column (5); then next to it above (6), to the fifth of the first column (7), to the second of the second (8), to the third of the fourth line (9), to the fourth of the first line (10); then to the fifth of the fourth column (11), to the second of the fifth column (12), to the fourth of the first column (13), to the second of the first line (14), to the third of the third (15); then next to it above (16), to the fourth of the fourth (17), to the fifth of the first line (18), to the third of the first column (19), to

21. This removes the ambiguity. Still, it must be an addition.
22. The square obtained is simply rotated.

the second of the fifth line (20); then next to it above (21), to the third of the first line (22), to the third of the fourth column (23), to the fifth of the fifth (24), to the second of the first column (25). It is then complete.[23]

You may, in that placing, put the beginning in the first cell of each series of five, indicated by (the word) «then» after entering the tour for the series of five: an ordered succession for the series of five is not necessary, except for the tour within each series.[24] [The same holds for any of the ordinary magic squares where the knight's move increases [parts of four, six, seven] to other (numbers), maintaining (both) the (existence of) series and knight's move.][25] (Such it is,) according to this figure.[26]

As to the bordered magic square for the five by five, 1 is put in the middle cell of the first column, the second (2) below it, and, successively, in the fourth diagonal (3), next to it on the right* (4), in the central cell at the top (5), in the cell above the middle one on the left* (6), next to it above thus in the second diagonal (7), in the cell to the right* of the (upper) central cell (8). Half of the (cells of the) border are thus filled. [This is the guiding principle for all bordered magic squares.][27] You complete the three by three inside the border as we have taught you. Then for filling the remainder of the border cells there are two ways: one is to begin with the equating number and move from it with decreasing (numbers and) in the opposite

23. For each series of five numbers (clearly separated by «then» in the text), the descent is from one column to the next by the knight's move; since after placing five numbers the next cell is occupied, we change the course, here upwards one cell for the next sequence. The square obtained is even pandiagonal, that is, with pairs of broken diagonals making the magic sum. Consequently the initial point, our 1, could be set in any cell. Usually, for larger (odd-order) squares, it is placed in the middle cell of the top row; indeed, the squares thus constructed will also remain magic (but not pandiagonal) when the odd order is divisible by 3. On that, see *Les carrés magiques*, pp. 35–37 (Russian edition, pp. 44–46) or *Magic squares*, pp. 36–39.

24. This means that the starting points of the series of five consecutive numbers may be exchanged, with the series thus following one another in a different order. As seen from the median figure (our addition), this will not affect the repartition of the units but merely operate a displacement of the multiples of 5; consequently, each horizontal, vertical and (main) diagonal row will still contain each unit and each multiple of 5, that is, the required magic sum.

25. The sequences of numbers linked by the knight's move (and thus the number of series) may increase together with the order (not divisible by 3, see note 23).

26. Reference to the figure of the 5 by 5 square (above, left); this also suggests that what precedes is an addition.

27. Meaning: to fill half the border (including two consecutive corners). The opposite cells are left empty for the complements to the equating number.

increasing, the other with increasing (numbers and) in the opposite decreasing, till the empty cells of the border are filled.[28]

For instance, the number of cells of the five by five border is 16, half of which is 8. So (half) the border has been filled (with the numbers) from 1 to 8. Then the three by three inside it has been filled (beginning with 9); this gives the cells filled to 17. The remaining numbers **(44^v)** are then placed in the remaining (border) cells, thus from 18 opposite 8 by addition up to 25 opposite 1, or from 25 opposite 1 by subtraction up to 18 opposite 8. [But the first (way) is the main one, for it follows the natural order.][29] [These two ways apply to all magic squares with a border.] [Likewise, this placing is possible for all figures of odd order above the five by five.][30] [One may begin this placing from any middle (cell) and put the remainder as you know].[31] Such (is the figure).

(3. Square seven by seven)

For the ordinary seven by seven, one puts 1 in the first diagonal, then moves to the left* repeatedly by the knight's move to the edge (2, 3, 4), and, successively, to the fifth cell of the second column (5), to the left* twice by the knight's move (6, 7); then to the third cell above it (8),[32] to the sixth of the first column (9), below it by the knight's move (10), to the fifth of the first line (11), to the left* by the knight's move (12), to the second of the third line (13), to the fourth of the fourth (14,

28. In a bordered square the sum of two opposite cells (horizontally, vertically, for the corner cells diagonally) must equal the equating number, here 26. We may take the largest number in the border, thus 25, and place opposite, by following the descending sequence, the ascending sequence of the complements, thus here from 1 on; but we may as well proceed inversely, beginning with the complement of 18. The author seems to find this rather banal distinction important, for it is encountered later on.

29. The sequence of numbers is placed continuously (18 following 17). This and the subsequent gloss refer to the earlier explanation of the 'two ways'.

30. This refers to the placing in the outer border, which is indeed applicable to all borders of odd orders. But asserting this here is useless since the general placing could not be inferred from this example; by the way, and oddly enough, the next examples, of orders 7 and 9, are filled differently. On the generality of the above arrangement, see *Les carrés magiques*, pp. 120 (Fig. 185), 124–125 and (justification) 128–129 (Russian edition pp. 131, 134–135, 139–141), or *Magic squares*, pp. 144, 148–149, 152–154.

31. We have seen a similar remark (probably by the same interpolator) for the square of order 3.

32. Counting the initial cell.

knight's move); then to the third above it (15), to the left* by the knight's move (16), to the fourth of the first column (17), to the left* by repeated knight's moves until reaching the fourth diagonal (18, 19, 20), on the side of the first diagonal (21); then to the sixth [opposite to it] ⟨of the second column⟩ (22), to its knight's (23), to the sixth of the first line (24), to the second of the first column (25), by knight's moves repeatedly to the fifth of the seventh column (26, 27, 28); then to the third above it (29), to the fourth of the second column (30), to the left* twice by the knight's move (31, 32), to the third diagonal (33), to the third of the first line (34), to the left* by the knight's (35); then to the seventh of the fifth column (36), to the second diagonal (37), to the second of the second (38), to the left* by the knight's twice (39, 40), to the fifth of the first column (41), to the left* by the knight's (42); then to the third above it (43), to the left* by the knight's twice (44, 45), to the seventh of the second column (46), to the fourth of the first line (47), to the left* by the knight's (48), to the third of the first column (49). Such is (the figure).[33]

For the bordered seven by seven, one is to put 1 below the first diagonal, (then successively) in the third of the seventh column (2), in the fourth of the first column (3), in the fifth of the seventh column (4), in the sixth of the first column (5), in the second diagonal (6), in the seventh of the sixth column (7), in the fifth of the first line (8), in the fourth of the seventh line (9), in the third of the first line (10), in the second of the seventh line (11). Half of the surrounding border is thus filled.[34] Next we operate **(45^{r})** likewise for the border inside, namely of the five by five, and that inside it, of the three by three. [One places the remaining (next) number in the cell of 1 of this three by three (19), (then) in the corresponding knight's cells up (20) and down (21), in the fourth diagonal (22), in the queen's ⟨repeatedly⟩ up to the first diagonal (23 to 28).][35] Then one completes the empty cells with the remaining numbers as done (before) for the five by five. [You may also begin

33. For each series of seven numbers, the descent is from one line to the next by the knight's move; since after placing seven numbers the next cell is occupied, go upwards two cells to start the next sequence. The square obtained is even pandiagonal, that is, with pairs of broken diagonals making the magic sum. As mentioned in a previous note, such a method could be used for arranging other squares of odd orders not divisible by 3.

34. Half of the border cells *less one*. This is precisely the particular feature of this method (see note below).

35. A reader thus completed the inner square of order three and the cells in the whole descending diagonal. He left out the border of order 5, filled here just like that of order 7, thus with the alternate placing between opposite rows.

from any side.][36] [This is likewise possible for all magic squares of odd orders.][37] Here it is, according to this figure.[38]

	1	2	3	4	5	6	7
1	1	21	34	47	11	24	37
2	25	38	2	15	35	48	12
3	49	13	26	39	3	16	29
4	17	30	43	14	27	40	4
5	41	5	18	31	44	8	28
6	9	22	42	6	19	32	45
7	33	46	10	23	36	7	20

28	39	10	41	8	43	6
1	27	32	17	34	15	49
48	12	26	29	20	38	2
3	37	19	25	31	13	47
46	14	30	21	24	36	4
5	35	18	33	16	23	45
44	11	40	9	42	7	22

(4) Square nine by nine

We have put for each of the ordinary and bordered squares one example. We did not confine ourselves to the previous odd-order placings, for the objective is to explain different methods for a general rule, neither have we gone into detail since the example is sufficient after paying careful attention to the previous examples.[39] Thus here are the two illustrations. Understand (that) and the methods we have explained, and treat in the same way all other odd-order magic squares larger than 9.[40]

36. Same early reader as before (note 31).

37. Same (competent) reader as earlier (note 30) and below (notes 69, 72).

38. This is a further, different general method for filling borders of odd orders, with an alternate placing and (starting with the lower left-hand corner cell) a continuous sequence in a diagonal. See *Les carrés magiques*, pp. 126 and (justification) 132 (Russian edition pp. 137, 142–143), or *Magic squares*, pp. 150–151, 156–157.

39. Strange justification! First, the previous methods for ordinary magic squares are not always general since they will fail with all odd orders divisible by 3. In this particular construction for order 9 we may observe — disregarding the rearrangement of the columns in order to obtain the magic sum — that the placing has retained the vertical knight's move within each series.

40. On this construction of *bordered* squares, see *Les carrés magiques*, pp. 122–123 and (justification) 133–134 (Russian edition pp. 132–133, 144–145), or *Magic squares*, pp. 146–147, 157–158. The above construction for the *ordinary* square of order 9 is not general.

9	15	21	31	37	52	59	65	80
55	70	76	5	11	26	36	42	48
32	38	53	60	66	81	1	16	22
12	27	6	43	49	28	71	77	56
67	73	61	17	23	2	39	54	33
44	50	29	72	78	57	13	19	7
24	3	18	46	34	40	74	62	68
79	58	64	20	8	14	51	30	45
47	35	41	75	63	69	25	4	10

8	80	78	76	75	16	14	12	10
71	22	64	62	61	28	26	24	11
69	57	32	52	51	36	34	25	13
67	55	47	38	45	40	35	27	15
73	56	49	43	41	39	33	26	9
5	19	29	42	37	44	53	63	77
3	17	48	30	31	46	50	65	79
1	58	18	20	21	54	56	60	81
72	2	4	6	7	66	68	70	74

(5. Square four by four)

The (magic squares of) even (orders) are of two kinds [as mentioned before],[41] namely evenly-even and evenly-odd. The first evenly-even is the square of four by four. We have for it four fundamental (placings).[42] To these are related many (other) placings, as we shall make known soon.[43]

1	14	11	8
12	7	2	13
6	9	16	3
15	4	5	10

[Spring]

12	1	13	8
14	7	11	2
3	10	6	15
5	16	4	9

[Summer]

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

[Autumn]

1	15	14	4
10	8	5	11
7	9	12	6
16	2	3	13

[Winter]

For the first, 1 is put in the first diagonal, then one moves (successively) by the knight's and the queen's (2, 3), by the knight's to the right* (4). [Four cells are thus filled, one in each row and diagonal.][44] [In this placing four series have

41. This early reader had in mind the distinction ordinary/bordered.
42. Their (interpolated) association with the four seasons has no justification, neither here nor in Section III. The second and third figures (together with their attribution) are exchanged in the Arabic text. Note also that only the first construction is relevant here (placing by series).
43. Allusion to the enumeration at the end of this section.
44. True, but one would expect, for «diagonal», the dual instead of the plural (*aqṭār*). Only this arrangement displays this feature, thus only this placing will be generally applicable in the case of names.

been considered.][45] Then next to it (5), towards the right* ascending, as one sees, until one reaches the second diagonal (6, 7, 8). At this point, half of the cells are completed, and the sum of each pair of cells horizontally is 9, separated in the first and third rows and consecutive in the second and fourth. Such is the fundament of this placing.[46] One then puts in the remaining cells the remaining numbers: 9 in the bishop's of 8, (and so on) increasing, in the opposite decreasing, up to the bishop's of 1; or (otherwise) with 16 in the bishop's of 1 (and so on) decreasing, in the opposite increasing, up to the bishop's of 8.[47] [This is general for all the figures with that.][48] But it is possible here to exchange the series of four, as we have pointed out previously in the five by five.[49] [Then one places what is to fall in the bishop's cell, or on the side, or opposite].[50] As in this example.

1	6	15	12
16	11	2	5
10	13	8	3
7	4	9	14

For the second,[51] 1 is written in the second cell of the first line, then, successively, to the left* by the knight's (2), to the third of the first column (3), to the left* by the knight's (4), to the ⟨fourth of the first column (5), to the third of the third (6), to the⟩ second of the second (7), to the second diagonal (8). Half of the cells are filled. Such is the fundament of this placing. The remaining numbers are written

45. True, but this observation should have come later.

46. The essential point is indeed that the bishop's cells are empty, for they are to receive the complements.

47. The author seems once again (note 28) to be making much of the minor distinction of filling the remaining bishop's cells by taking the numbers already placed in descending or ascending sequence.

48. That is, placing initially the first eight numbers and then filling the remainder by associating pairs of conjugate cells (here distant by a bishop's move, opposite vertically in the next and the last arrangements, diagonally in the third one).

49. See above, note 24. The text illustrates that with the example below: the series I, II, III, IV of the first example are replaced here by I, III, IV, II, respectively. This leaves, as before, one term of each series in each row, but the complements are no longer in bishop's cells.

50. This refers to the placing of the complements (see above, note 48).

51. Remember (note 42) that the figure of this and the next are exchanged in the Arabic.

with the equating number, (namely) increasing (and) decreasing in the (vertically opposite) counterpart, (respectively) decreasing (and) increasing in the counterpart.

For the third, 1 is put in the first diagonal, then one (successively) moves to the fourth of the third column (2), to its side on the right* (3), to the second diagonal (4), to above the fourth diagonal **(45^v^)** [which is opposite][52] (5), to the knight's upwards (6), to its left* side (7), to the knight's on the right* downwards (8), to below the second diagonal (9), to the knight's downwards (10), to (its) left* side (11), to the knight's upwards (12), to the third diagonal (13), to the right* side of the second diagonal (14), to its right* side (15), to the fourth diagonal (16). It is complete [the rule is that you count, then place to the end of the numbers from the two (opposite) sides, as you will know by observing the example].[53]

The fourth is a mixture of the second and the third.[54] [The second way is in keeping with the first[55] for the opposite, as the second is in keeping for the opposite likewise; (the third) participates with the second for the comparison.][56]

So it is in their figures.

With these four figures and with these four fundaments many figures are inferred, the number of which rises to 2688: 1920 are (from) the first fundament, 768 (from) the three other fundaments.[57] Indeed, to the first fundament belong six figures, with regard to the occurrence of 1 in the first (cell) and the moving forward of some of the lines without modifying the columns.[58] Likewise for each of the sixteen numbers; so multiplying 6 by 16 gives 96.[59] Likewise considering

52. Superfluous.

53. The third and fourth squares are filled directly, by pairs of opposite rows. These are well-known squares. This third square, in particular, was one of the squares transmitted to Europe by 14th-century Latin translations. See *Les carrés magiques*, p. 43 (Russian edition p. 52), or *Magic squares*, p. 45.

54. Same elements in the columns and lines, respectively.

55. Rather: the fourth.

56. Probably another way to express my note 54.

57. As said in our introduction, only few of these configurations will be magic squares. The purpose of this computation is thus unclear; but its author seems to be proud of it, see his concluding words.

58. That is, keeping the first line as it is and switching the other three. This indeed gives six possibilities, but only one of these new squares will meet the magic condition for the two diagonals. Indeed, a pandiagonal square, as the first is, remains magic (and pandiagonal) only with a cyclical permutation of its rows. On this, see *Les carrés magiques*, pp. 2–3 (Russian edition pp. 10–11), or *Magic squares*, pp. 3–4.

59. If each of the sixteen numbers occupies the place of 1, with lines and columns maintaining their elements.

the lines, so on the whole 192 figures.[60] The permutation of the series of four alluded to above gives 960 figures.[61] Thus the total from the first principle results in, considering the two, 1920 figures since to each of these four figures there is another figure, resulting from the border of each being exchanged relative to this fundament.[62] To each of the remaining fundaments there are eight figures [they were the six for the first fundament].[63] So multiplying 8 by 16 gives 128 figures relative to the columns; and relative to the lines as well. So there will be for each of the three (fundaments) 256 figures, thus altogether 768.[64] The addition with the result from the first fundament makes 2688 figures. This is what we have sought and discovered, and there is nothing to add to what we have said. Understand it and bear in mind the truth of this observation, and the number of all possible arrangements [and for intricate names their placing in common figures, such as *qābiḍ*, *bāsiṭ*, *jāmi'* and the like.][65]

(6. Square eight by eight)

For the ordinary magic square eight by eight, one begins at the first diagonal (1), moves (successively) to the second cell of the seventh column (2), to the right* twice with the knight's (3, 4) and (once) with the queen's (5), to the eighth of the sixth line (6), to the right* twice with the knight's (7, 8); then to its right* side (9), to the left* upwards with two knight's (10, 11), to the fifth of the first column (12), to the fourth of the fourth (13), to the left* (upwards) with two knight's (14, 15), to the second of the first line (16); then to the first of the seventh column (17), to the right* with three times a knight's (18, 19, 20), to the fifth of the fourth column (21), to the right* downwards by a knight's (22), **(46^r)** to the seventh of the eighth column (23), to the sixth of the eighth line (24); then to the fourth of the eighth column (25), to the third of the second column (26), to the left* upwards twice

60. Exchanging this time the columns without changing the content of the lines.

61. See notes 24, 49. The four series 1, 2, 3, 4; 5, 6, 7, 8; 9, 10, 11, 12; 13, 14, 15, 16 then changing their places. But the result is inappropriate anyway.

62. Presumably a rotation of the border by 90 degrees. In any event that does not make sense.

63. Attentive reader. The eight figures mentioned here are presumably the original and its inverse relative to (say) the horizontal axis, and then those resulting from the three rotations by 90 degrees.

64. Appropriately, no exchanges of series here. See note 42.

65. As to the placing of these three names (anyway here irrelevant), that is indeed problematic because they contain two of the first three letters of the alphabet. See notes 85, 108.

with the knight's (27, 28), to the seventh of the eighth line (29), to the seventh of the first column (30), to the left* upwards twice with the knight's (31, 32); then to the fourth (from it) in this same line (33), then to the right* downwards by the knight's twice (34, 35), followed by one more time (36), to the third of the first line (37), to the right* by a knight's (38), to the third of the seventh column (39), to the right* by a knight's (40); then to the third diagonal (41), to the left by one knight's followed by two (42, 43, 44), to the right* by a queen's (45), to the left* upwards again by a knight's (46), to the second of the second (47), to the left* upwards by a knight's (48); then to the fifth of the sixth column (49), to the right* downwards twice by the knight's (50, 51), to the fourth diagonal (52), to the fifth of the first line (53), to the right* twice by the knight's (54, 55), to the fourth of the seventh column (56); then to the sixth (from it) in this same line (57), to the left* upwards by a knight's thrice (58, 59, 60), to the eighth of the fifth column (61), to the seventh of the seventh (62),[66] to the sixth of the first column (63), to the left* by a knight's (64). With 'then' we have pointed out the series.[67] [For any name is interrupted in the magic square (where) its condition is found.][68] Such it is, according to that figure.

1	1	16	37	48	53	28	17	60
2	38	47	54	27	18	59	2	15
3	55	26	19	58	3	14	39	46
4	20	57	4	13	40	45	56	25
5	12	5	64	21	32	49	44	33
6	63	22	31	50	43	34	11	6
7	30	51	42	35	10	7	62	23
8	41	36	9	8	61	24	29	52
	1	2	3	4	5	6	7	8

9	54	53	1	63	60	14	6
3	15	48	46	44	24	18	62
61	49	25	39	38	28	16	4
58	20	34	32	29	35	45	7
8	22	31	33	36	30	43	57
10	42	40	26	27	37	23	55
52	47	17	19	21	41	50	13
59	11	12	64	2	5	51	56

For the bordered eight by eight, 1 is put in the first of the fourth column, (then successively) in the eighth of the fifth column (2), below the first diagonal (3),

66. Again a knight's move.
67. This was already the case in the previous squares of odd orders 5 and 7.
68. Presumably an allusion to Section II and the placing of names by series. Whatever the case, it is obvious that the readers were mainly interested in the applications described in the next section.

in the third of the eighth column (4), in the eighth of the sixth column (5), in the second diagonal (6), in the fourth of the eighth column (7), in the fifth of the first column (8), in the first diagonal (9), in the sixth of the first column (10), in the eighth of the second column (11), on its left* side (12), in the seventh of the eighth column (13), in the seventh of the first line (14). Half of the cells of the surrounding border are filled and the remaining half is empty. [This is the general fundament for all magic squares with an evenly-even border.][69] Next one fills the remaining half using the equating (number), before filling the inside six by six and the four by four [or after it].[70]

[But the second (way) is the main one, for it follows the natural placing [namely relative to six].[71] The six by six inside is filled by placing 15, which is left from the surrounding border as (new) beginning in the first diagonal of this six by six;[72] then (successively) below the second diagonal (16/2), in the sixth of the second column

69. With such a placing of the first fourteen numbers, the sum in each border row is the same, namely 30; with the complements in place opposite, the required magic sum will be found. Now this placing is indeed applicable to any larger border of evenly-even order. For then there will remain a quantity of empty cells divisible by 4, which is easy to complete by «neutral placings»: take each tetrad of subsequent consecutive smaller numbers and put the extremes on one side and the median ones on the other side, then place their complements in the cells opposite. See *Les carrés magiques*, pp. 139–141 (Russian edition pp. 150–152), or *Magic squares*, pp. 163–166.

70. Interpolation, meaning: the inner six by six and the innermost square (to prevent the confusion due to the double meaning of *murabb'a*, «square» and «four by four»). Concerning this «first» way (see what follows), we fill successively, and completely, each border starting from the outer one. Another possibility would be to fill successively half the cells of each border, and that is what an interpolator will now explain. We have seen a remark of the same kind — surely by the same interpolator — after considering the bordered square of order 5.

71. This second interpolator pointed out that the next filling is for the six by six border.

72. Place of 1 in the figure we have supplied. As said further on, this placing for the six by six is generally applicable to evenly-odd orders, for it leaves tetrads of empty cells to be filled as before. See *Les carrés magiques*, pp. 142–143, 147 (Russian edition pp. 154–155, 158–159), or *Magic squares*, pp. 167–168, 171–172.

1				10	4
					2
6					
8					
					9
	3	5	7		

(17/3), in the second diagonal (18/4), in the sixth of the third column (19/5), **(46v)** in the third of the first column (20/6), in the fourth of the sixth line (21/7), in the fourth of the first column (22/8), in the fifth of the sixth column (23/9), in the fifth of the first line (24/10). Thus half of the border is filled. This method is general for all evenly-odd (orders). The remainder of the surrounding border is completed, and the inner square of 4 by 4 is filled by any of the foregoing placings you wish [the first of them, the second and the third, however].[73]] Such it is, according to that figure.

(7. Square six by six)

For the ordinary six by six, which is the first evenly-odd, one begins in the first diagonal (1) and moves (successively) to the second of the fourth column (2), to the third of the fourth line (3), to the fifth of the sixth column (4), downwards to the queen's (5), to the third of the second column (6); then to the left* side of the third diagonal (7), to the third of the fourth column (8), to the fourth of the sixth column (9), to the third of the first line (10), to the fifth of the fifth (11), to below the first diagonal (12); then to the fourth of the first column (13), to its queen's (14), to the third of the fifth column (15), to the knight's of 1 (16), to the fourth diagonal (17), to the fourth of the first line (18); then to its left* side (19), to the bishop's cell (20), to below the second diagonal (21), to the fifth of the fourth column (22), to the second of the fourth line (23), to the third diagonal (24); (then) to the sixth of the fourth column (25), to the second of the second (26), to the fourth of the fifth column (27), to the second diagonal (28), to the fifth of the third column (29), to the third of the first column (30); then to the fifth in this same row (31), to the third of the sixth column (32), to the third of the sixth line (33), to the second of the fifth column (34), to the first of the second column (35), to the fourth of the fourth (36). It is finished.

The bordered six by six has preceded, inside the eight by eight.[74] Though it is sufficient, we have given a further example of it in order that one does not think of (the existence of) some restriction.[75]

73. Indeed, the figure presented uses the fourth one. Remember that series are irrelevant for bordered squares.

74. See above figure and, for the border, our note 72.

75. For larger evenly-odd borders, this initial placing of the first ten numbers and their complements will also leave sets of four empty cells, to be filled with consecutive numbers using «neutral

So it is, according to these two figures.

1	35	10	18	19	28
12	26	16	2	34	21
30	6	20	8	15	32
13	23	3	36	27	9
31	14	29	22	11	4
24	7	33	25	5	17

5	33	31	30	10	2
1	23	20	17	14	36
34	18	13	24	19	3
8	12	15	22	25	29
28	21	26	11	16	9
35	4	6	7	27	32

(8. Square ten by ten)

For the ordinary ten by ten, one begins in the first diagonal (1), then moves to the ninth of the sixth column (2), to the third of the fifth column (3), to the left* by a knight's (4), to the second of the seventh line (5), to the fifth of the ninth column (6), to its queen's (7), to the eighth of the fourth column (8), to the third of the second line (9), to the tenth of the eighth column (10); then to its queen's (11), to below the second diagonal (12), to the seventh of the eighth column (13), to the sixth **(47^r^)** of the fourth column (14), to the seventh of the fifth line (15), to the third of the first column (16), to the sixth of the fourth line (17), to the eighth of the third column (18), to the fifth of the tenth line (19), to the first of the second column (20); then to the second of the fifth column (21), to the third of the first line (22), to the sixth of the eighth line (23), to the third of the seventh column (24), to the fifth of the first column (25), (downwards) to its queen's (26), to the fourth diagonal (27), to the seventh of the fourth column (28), to the ninth of the eighth column (29), to the ninth of the fourth line (30); then to its left* side (31), to the sixth of the fifth line (32), to the ninth of the first column (33), to the sixth of the eighth column (34), to the second of the eighth line (35), to the fourth of the first line (36), to the seventh of the fifth column (37), to the third of the third (38), to the seventh of the second line (39), to the

placings». The above arrangement is just one of 140 possibilities of placing the first ten numbers; a complete list is given in our *Magic squares in the tenth century*, pp. 63–64 (the above one is the 35th, that of note 72, the 6th).

ninth of the tenth line (40); then to the fifth of the first line (41), to the second of the ninth column (42), to the second of the third line (43), to the fourth of the ninth line (44), to the fourth of the third column (45), to the fifth of the eighth column (46), to the knight's on the right* downwards (47), to the tenth of the seventh column (48), to the eighth of the tenth column (49), to above it (50); then (in the same line) to the tenth from it (51), below it (52), to the fourth of the tenth line (53), to the sixth of the fifth column (54), to the third of the fifth line (55), to the fourth of the eighth column (56), to the seventh of the ninth line (57), to the ninth of the third line (58), to the second of the second (59), to the sixth of the first line (60); then to the tenth of the second column (61), to the second of the fourth column (62), to the third of the eighth column (63), to the sixth of the seventh line (64), to the seventh of the first line (65), to the eighth of the ninth column (66), to the sixth of the third column (67), above the fourth diagonal (68), to the fifth of the fifth (69), to the fourth of the first column (70); then (in the same line) next to it (71), to the ninth of the third column (72), to the seventh of the seventh (73), to the third diagonal (74), to the ninth of the sixth line (75), to its queen's (76), to the third of the fourth column (77), to the fifth of the eighth line (78), to the eighth of the first line (79), to the knight's on the right* (80); then beside the second diagonal (81), to the sixth of the tenth line (82), to the eighth of the eighth (83), to the fourth of the fifth column (84), to the third of the tenth column (85), to the fifth of the fourth column (86), to the sixth of the seventh **(47$^{\mathbf{v}}$)** column (87), to the third of the seventh line (88), to the second of the first column (89), to the ninth of the second column (90); then to its queen's (91), to the eighth of the second line (92), to the eighth of the seventh column (93), to the sixth of the first column (94), to its queen's (95), to the ninth of the seventh line (96), to the fourth of the fourth (97), to the third of the sixth column (98), to the ninth of the fifth column (99), to the second diagonal (100). So it is, according to this figure.[76]

76. The square of order 10 thus filled is very particular: all pairs of horizontally opposite numbers add up to 101, namely the equating number for this order. It is surprising that no interpolator, some of whom seem to be acquainted with magic squares, noticed that. There is, however, a problem with this construction: the two middle series of ten numbers have two of their terms in the same column (51, 52 and 50, 49); this square will therefore be of limited use for the inscription of words. See note 98.

1	1	20	22	36	41	60	65	79	81	100
2	89	59	9	62	21	80	39	92	42	12
3	16	43	38	77	3	98	24	63	58	85
4	70	71	45	97	84	17	4	56	30	31
5	25	95	55	86	69	32	15	46	6	76
6	94	26	67	14	54	47	87	34	75	7
7	51	5	88	28	37	64	73	13	96	50
8	52	35	18	8	78	23	93	83	66	49
9	33	90	72	44	99	2	57	29	11	68
10	74	61	91	53	19	82	48	10	40	27
	1	2	3	4	5	6	7	8	9	10

The bordered ten by ten is determined from the explanation given for the example of the six by six.[77] But we have put the following further example of it, whereby it will appear clearly (that there are) arrangements not shown before.[78] Consider, according to what we have explained, the other even (orders) above ten.

33	66	64	62	42	49	48	54	51	36
67	19	79	78	76	29	71	32	20	34
38	80	9	90	88	86	18	12	21	63
40	24	91	1	99	98	4	10	77	61
60	75	14	94	8	5	95	87	26	41
43	27	16	7	93	96	6	85	74	58
57	28	84	100	2	3	97	17	73	44
56	70	89	11	13	15	83	92	31	45
46	81	22	23	25	72	30	69	82	55
65	35	37	39	59	52	53	47	50	68

You are to know that what we have explained until now are basic placings, which are also valid for the composite (magic squares); indeed, the parts in the

77. See above, note 72.
78. The placing in the six by six is the same as within the eight by eight. That of the eight by eight arranges the first fourteen numbers in a different way (same sums, before and after writing the complements, in opposite sides); see note 69. But the main difference is that here the filling begins with the innermost square. The concluding remark serves to point out that the arrangements presented here are not unique. See a similar remark above, note 75.

composite have the same status as the (terms in the corresponding) basic magic square. So one fills the squares in the composite as one fills the basic ones.[79] For instance, the nine by nine is compound of three by three square(s). So one arranges first the three by three (subsquare) corresponding to the cell set as 1 in the basic three by three square, then the three by three corresponding to the cell set as 2, and so on to the end of the three by three (squares). Consider likewise the composites from the square of 4 by 4, from the square of 5 by 5, from the square of 6 by 6, and so on indefinitely.

[There may well arise various compositions, so this placing is possible with whatever you need; but you will choose the adequate figure and the square fitting the need (and) ascribed to the planet belonging to that figure.][80]

There is for the evenly-even (order) another kind of composite, namely dividing the square and placing the numbers in each part by means of the equating number [as in the bordered ones].[81]

Selected aspects have been unveiled for who (is willing to) consider them, and the veil opened for who (is willing to) further consider them. Allah tells the truth and shows the way.

Section (11): How to place the names in the squares

There are here two kinds. The first is placing the numbers in such a manner that the constant in each row vertically, horizontally and diagonally is the quantity of the name (i.e. the sum of the numerical values of the name) of which the magic square is sought. The second is that the individual letters of the name be written in

79. Composite squares are magic squares comprising subsquares, each of which is itself magic, with its own magic sum (thus the arrangement of these subsquares is not arbitrary). Composite squares must be an early discovery: they made it possible, with the arrangement for a few squares of smaller orders known, to construct arbitrarily large squares. See *Les carrés magiques*, pp. 105–110 (Russian edition pp. 115–120), or *Magic squares*, pp. 107–115.

80. Rather confusing; refers to Sections II-III.

81. Each subsquare of the same size displays then the same sum, so their arrangement is arbitrary. See *Les carrés magiques*, pp. 78–84 (Russian edition pp. 89–95), or *Magic squares*, pp. 79–86; the treatise translated from the Greek makes extensive use of this kind of construction. The reader's analogy here is superficial: in bordered squares only cells facing each other add up to the equating number, whereas in the subsquares in question it is each pair of conjugate numbers.

a row and that the quantity of this name be placed in the other cells in accordance with the series of the (basic) magic square; thereby the quantity of this name will appear in each row vertically, horizontally and diagonally, as we shall make known in our account, God Most High willing.[82]

First kind

The condition is that the quantity of the name be larger than the basic magic sum for the figure in which one wants to place the name,[83] and that this (quantity of the) name have an integral third if it is to be placed in the three by three [but not so for the larger ones generally] [because the placing of the fragmented name, by considering it, is not correct, unlike with the other figures].[84] [Because for some name(s) like, for instance, *qābiḍ*, there is an integral third although that is not appropriate in this respect].[85]

The method of placing consists in (first) subtracting from the name, or the phrase,[86] the basic magic **(48^{r})** sum for the figure (considered) and distribute the remainder among the cells of one row of the figure.[87] Then the shares for the cells are either equal or not. In the first case, one takes what resulted for one cell, adds

82. This has been discussed at length in our introduction.

83. The smallest possible magic sum is that of a magic square filled with the consecutive numbers from 1 on.

84. Two remarks, by the same or different readers: first, the condition of the integral third does not apply in the case of squares of orders larger than 3; second, placing by series is not relevant to the square of order 3 — anyway a «fragmented name», thus considering its individual letters, is not the subject here.

85. This is quite confusing. Considering first the sum of the name *qābiḍ* (100, 1, 2, 800 = 903, see note 65), it could appear according to the first kind in a three by three square: we shall put, say, 100, 601, 202 in the upper row, then 301, a third of 903, in the central cell, whereby the content of the remaining cells is determined. Next, if the name is to be written as in the second kind (which is not the subject here), thus in a four by four square with, say, each letter in the upper row, the condition of the integral third is irrelevant; still, it is true that this will not be possible, and the interpolator must have seen this mentioned elsewhere. The impossibility is clear from the data, even without having recourse to our general figure: we should have here two pairs of different positive integers making the sum 3; for in any four by four magic square the sum in the upper middle cells *must* be equal to the sum in the extreme cells below. See indeed *Les carrés magiques*, pp. 192–193 (Russian edition p. 204), or *Magic squares*, p. 225.

86. Usually from the Koran.

87. The numerical value of the name (not that of its letters) will then appear in each row, column and diagonal.

1 to it, places (the result) in the cell of 1 and moves as for the natural placing, according to the method (either) for bordered or ordinary.[88] In the second case one is to consider the quantity arising from the subtraction of the (number of) lesser shares from the (number of) cells of the figure's side [the number of series of the figure].[89] One then begins with the lesser portion, by adding 1 to it and moving to the end of the lesser parts; then another 1 is added to make up for the deficit, and one moves (further) to the end of the figure.[90]

Example.[91] We wish to place the name «king» (*mlk*) in the three by three according to the first kind. Its sum is $(40+30+20=)$ 90, which has an integral third. We subtract from it the basic magic sum, which is 15. We divide the remainder (namely 75) among the cells of one row, which are three; the share of each cell is 25. Let us take it and add 1 to it.[92] So we place in this way according to that figure.

33	28	29
26	30	34
31	32	27

8	3	4
1	5	9
6	7	2

Likewise if we wish to place this name in the five by five. We subtract 65 from it and divide the remainder (namely 25) into five parts, (thus obtaining) five for each cell. We add 1 to it, and fill then like that.

6	19	27	15	23
30	13	21	9	17
24	7	20	28	11
18	26	14	22	10
12	25	8	16	29

1	14	22	10	18
25	8	16	4	12
19	2	15	23	6
13	21	9	17	5
7	20	3	11	24

88. Only in the case of equal parts may a bordered square be used.

89. There are as many series in the square as cells in one row. The «lesser shares» are those which will not receive a supplementary increment.

90. This is rather succinct, but is clear from our introduction. The lesser shares are treated as in the previous case, whereas the others have a supplement of 1.

91. We find here three examples of the first case (parts equal), for the orders 3, 5, 4. We have provided, next to the figures of the examples, the arrangement corresponding to the the basic square.

92. This will be the number replacing 1 in the basic square, all being thus simply increased by 25.

Likewise if we wish to place this name in the square of 4. We subtract from it 34, which is the basic magic sum, and divide the remainder (namely 56) by four; the due of each cell is 14. We add 1 to it and fill. Such is the figure.

15	28	25	22
26	21	16	27
20	23	30	17
29	18	19	24

1	14	11	8
12	7	2	13
6	9	16	3
15	4	5	10

We wish to place the name *jalīl* (3, 30, 10, 30) in the square.[93] We find its parts: after dividing by 4, we have for three of them 10 each, and for one, 9. We add then 1 to 9, and place (the numbers) in each row until the first series of four is completed. Then, we add to it 1 another time, so as to make up for the deficit, then fill until completing the figure, as here.

10	24	21	18
22	17	11	23
16	19	26	12
25	13	15	20

1	14	11	8
12	7	2	13
6	9	16	3
15	4	5	10

We wish to place this name in the five by five. After subtraction of the basic magic sum, there remains $(73 - 65 =)$ 8. We divide it into 5, which gives for each of three 2 and for two, 1. We take 1, which is one of the two lesser parts, add 1 to it[94] and place (this) in the cell of 1, and (then) move to complete the (first) two series of five. Then we add to it another 1, in order to make up for the deficit, and move from it to complete the remaining series of five, as here.

93. This is an example of the «other» case since there is a remainder after division. We are to obtain the magic sum 73, thus 39 more than the basic one. Divided by 4, it gives the quotient 9 with the remainder 3. So 9 will be added to the numbers of the first series in the ordinary 4 by 4 square, while 10 will be added to the cells of the three other series.

94. 1 of the basic square and 1 of the part.

2	16	24	11	20
27	9	18	5	14
21	3	17	25	7
15	23	10	19	6
8	22	4	13	26

1	14	22	10	18
25	8	16	4	12
19	2	15	23	6
13	21	9	17	5
7	20	3	11	24

Proceed in this way for the other figures: (indeed) the placing of names in the other figures proceeds in the same way.

[You may fill the square in another way, namely by taking the basic magic sum and subtract from it the quantity of cells in a row; you fill, after the division, without addition, and add the larger fractional part after placing.][95]

You are to know that if the parts following the subtraction (of the basic magic sum) are equal and equally shared, the placing of the fragmented name will be correct both in the ordinary and the bordered; otherwise it will be correct only for the ordinary in which are contained the series[96] [pointed out (by) «then» in the explanation],[97] as I have explained to you above. **(48$^{\mathbf{v}}$)** [Now the example of the ordinary for the ten by ten does not allow the placing of the fragmented name with deficient shares after the division.][98]

For placing the name in the ordinary (square) there is an easier method. It consists in placing the numbers until there remains a single series of the magic square. Then one calculates the constant quantity in a row of this (incomplete) figure, and considers with which amount the quantity of the name in question will

95. To mean: In the previous four by four square, e.g., take 30 instead of the basic magic sum 34, and subtract it from 73; the division by four gives 10 and the remainder 3. So fill the square in the usual way beginning with 10 and add a unit to the three later series. In the next example the initial term would be $73 - 60 = 13$; the division by 5 giving 2, this will be the first term of the first series, the second beginning with 7 and the third with 13. All this is of little interest, except for the reader who added it.

96. The presence of the series is indeed essential.

97. See note 23.

98. This is an acute observation since it asserts the impossibility of placing a fragmented name with unequal shares in the given ten by ten square (see above, note 76). It is unlikely that this remark originated with the author himself since it removes any justification for the presence of the ten by ten, the description of which was already questionable since it is not associated with any of the planets.

be completed. One places the complement to the quantity of the name in the row containing the first cell of this series and proceeds with the addition to the end of the cells. Here is its figure.[99]

1		11	8
12	7	2	
6	9		3
	4	5	10

1	*53*	11	8
12	7	2	*52*
6	9	*55*	3
54	4	5	10

1	14	11	8
12	7	2	13
6	9	16	3
15	4	5	10

Second kind

It consists in writing the letters of the name or phrase in a row, then considering the link of each of the cells filled with a series of the magic square. Then one fills the remaining cells, (proceeding) by increments and decrements in accordance with the known way for the natural placing.[100]

Example. We want the magic square for the name *jamīl* by this method in the four by four.[101] We write (the) different letters in four of its consecutive cells. The situation is such that each letter falls in one of four series of the four by four. There remains for each (series) three empty cells. Each of these filled cells must be, considering the series of the magic square, beginning or end or intermediate. If it is the beginning of the series, then (one will proceed) by addition; if it is the end, by subtraction; if it is intermediate, by addition and subtraction, in accordance

99. Name *jalīl*, thus 73, with the first cell of the fourth series being that of 13. The sum in this row being 21, thus in deficit of 52, that will be the first term of the fourth series. We have completed the figure of the manuscript (on the left) and added the basic square. For this case, see *Les carrés magiques*, p. 206 (Russian edition p. 218), or *Magic squares*, pp. 236–237. Here again it becomes evident that each series must have one cell in each row, horizontal, vertical or diagonal.

100. The «natural» placing is that of the basic square. There are as many series as is the order, with points of departure in various rows; within their sequences, the numbers will therefore increase and decrease relative to their term in (say) the first horizontal row.

101. The values in question, 3, 40, 10, 30, with the sum 83, are «different» as specified just after. The previous example *jalīl* could not be considered since two of the given values are identical. Although this is clear from the outset, the author should have pointed out at the beginning of Section I that the numbers filling a magic square must all be different.

with the natural placing.[102] Now the j (= 3) is, in the example, at the beginning of the first series of the four by four; so we proceed by addition as far as the cell of 4.[103] The l (= 30) is at the end of the second series; so (one proceeds from it) by subtraction, to the cell of 5. The y (= 10) is in the third cell of the third series; so, (first), by addition, to the cell of 12 and (then), by subtraction, to the cell of 9. The m (= 40) is in the second cell of the fourth series; so, by subtraction, to the cell of 13 and, by addition, to that of 16. Here is the figure, with 83 in each row.

3	40	10	30
11	29	4	39
28	8	42	5
41	6	27	9

1	14	11	8
12	7	2	13
6	9	16	3
15	4	5	10

Proceed likewise, by this method, for the other figures, except for the three by three: indeed, for the placing in the three by three the name or the phrase is (first) written in the first row; then a third of the sum of the(se) quantities is put in the centre; next one places in each of the two diagonals what completes the quantity of the name; then one places what completes the centre.[104]

66	129	84
111	93	75
102	57	120

s_1	s_2	s_3
$r+s_3-s_1$	r	$r+s_1-s_3$
$2r-s_3$	$2r-s_2$	$2r-s_1$

Example. We have placed the phrase *Allah is kind to his servants*[105] in the first row, as here. A third of it is 93, (which we place) in the centre of the square. Then we consider the diagonal (descending to the) right*, and find that what completes

102. We have added the basic four by four square (first «fundament», as need be: see note 42). See also Fig. 7 in our introduction.
103. This refers to the basic square. Same in what follows.
104. That is, in the three middle cells in the border. As mentioned in our introduction, this will not always work since negative quantities may occur (see the appended figure, same as in our introduction). Further conditions are thus required, some of which are pointed out by Arabic authors; see *Les carrés magiques*, pp. 183–184 (Russian edition pp. 196–197), or *Magic squares*, pp. 216–217 (with the same example as here, namely 66, 129, 84, sum 279).
105. Koran 42.19.

it is 102, (which) we then place in it. Then we consider the left*-hand diagonal, and find it completed with 120, (which) we then place in it. Next we complete the columns one after the other. Such is its figure.

This explanation ought to be sufficient for whoever is of sound mind or has paid attention. [And He is a witness].[106] Whoever knows well what we have reported and has paid attention to what we have precisely presented, and carefully follows the threads we have traced, will attain what we have left out and achieve by means of the placing we had in mind each figure where **(49^r^)** it does not go beyond the limit of possibility.[107]

[Like *bāsiṭ*, *Aḥmad*, and the like, of course without repetition.][108] [Completing *ṭālib*, *maṭlūb*, *maḥab(b)ah* without (including) increment and decrement, as in this figure in the four by four.][109] [And *muqtadir*, *muḥammad*, *kāmilan*, without repetition, in the five by five and the six by six and the like.][110] The quick-witted does not need an explanation on them and the fool will not go around it. And Allah is the guide to the path of right belief.

1	56	**87**	45
88	44	**2**	**55**
43	85	58	**3**
57	**4**	**42**	86

1	s_2+1	$\boldsymbol{s_1}$	s_3+3
s_1+1	s_3+2	2	$\boldsymbol{s_2}$
s_3+1	s_1-2	s_2+3	3
s_2+2	4	$\boldsymbol{s_3}$	s_1-1

106. This ought to refer to Allah rather than to the mortal reader of this treatise.
107. A real opportunity for interpolators to intervene. We shall merely point out the reason why the placing of some of the words mentioned is «beyond the limit of possibility».
108. Classical examples where the usual methods of placing each letter in a cell of the top row of the 4 by 4 square and filling as in our general figure (see introduction) will not work: in *bāsiṭ* (2, 1, 60, 9, sum 72), a zero and two pairs of identical numbers occur; in *Aḥmad* (1, 8, 40, 4, sum 53), the two extreme series produce identical terms. On the various impossible cases, see our *Magic squares*, pp. 234–238. In the 13th century, Zanjānī explains how to circumvent the difficulty for *Aḥmad*; see *Herstellungsverfahren II, II′*.
109. Thus we are given one series (1 to 4) and three terms each in one place of the three other series (87, 55, 42; bold in our figure). We have completed this figure and added the figure of the general arrangement.
110. With *muqtadir* the *d* (=4) is a problem in the five by five; *Muḥammad* cannot be placed according to the second kind (identical letters); with *kāmilan*, *a* (=1) is a problem.

Section 111: How to prepare the squares

This preparation necessarily depends on certain circumstances to be elucidated. The maker must be pious, reserved, godfearing and abstaining from what is prohibited; nor must his (work) be directed against any person or any desire, but towards the pursuer of truth and legitimate end, feeling in his heart joy and benevolence.[111] The time of his request must be in accordance with (propitious or) unpropitious events and (the data in) the lines of the (astronomical) table, the division of the (astrological) houses must be equal, the numerals and the letters written in must be complete, in particular each of the figures (of squares) described previously, each of the days and each of the nights, each of their hours, each of the categories of writings pertaining to one of the practicable things.[112]

Figure of the three by three. Saturday, night of Wednesday, first of their hours. Information, planting, extraction of water, increase of income, granting of request, fame among rulers, recovery from chronic diseases. All of this is connected with Saturn.

Figure of the four by four. Thursday, night of Monday, first of their hours. Carrying on difficult affairs, attaining a goal, facilitating the search for wealth and riches, means for acquiring intelligence and happiness. All of this is connected with Jupiter.

Figure of the five by five. Tuesday, night of Saturday, first of their hours. All kinds of hostility, search for conquest and victory, plundering, bloodshed. All of this is connected with Mars.[113]

111. Here at least our author is being naïve or idealistic. Magic squares as amulets were mostly used for the benefit of the holder and misfortune of others. See our *Magic squares for daily life*.

112. To each of the seven planets is attributed one magic square (with an order from three to nine), a particular hour of a given day (day-light) and night in the week, and specific qualities. This will be indicated now.

113. This list seems to contain contradictions. In fact, the effect depends on the course of the planet and the material. Thus we are told in Cornelius Agrippa's *De occulta philosophia*, II.xxii, that if the square is engraved, *fortunato Marte, in lamina ferrea* (...)*, potentem facit in bello & iudiciis & petitionibus, & terribilem adversariis suis, & victoriam pręstat adversus hostes;* (...) *si vero infortunato Marte sculpatur in lamina æris rubri, impedit ædificia, deiicit potentes a dignitatibus & honoribus & divitiis, generat discordiam & lites & odia hominum*. See also *Magic squares in daily life*, pp. 717, 721–722.

Figure of the six by six. Sunday, night of Thursday, first of their hours. Search for honour, high rank, prestige, increasing authority and dignity. All of this is connected with the Sun.

Figure of the seven by seven. Friday, night of Tuesday, first of their hours. All kinds of love, search for happiness, joy, sensual pleasures. All of this is connected with Venus.[114]

Figure of the eight by eight. Wednesday, night of Sunday, first of their hours. Understanding, memorization, eloquence, elimination of idle fantasies. All of this is connected with Mercury.

Figure of the nine by nine. Monday, night of Friday, first of their hours. Being healthy, even-tempered, recovery from an illness, easing of pain, protection from the evil eye and malignant glance, warding off fear. All of this is connected with the Moon.

[As for the other hours of the days and nights, they are for the other celestial bodies, in accordance with the succession of their orbits.][115] [The figure of the ten by ten is for the constellations of the celestial sphere.] [There are here two further figures (**49$^{\mathbf{v}}$**) to be explained. One is the square of 11, for the head (of the Dragon), and the other is the square of 12, for its tail.] [The figures which are beyond are considered according to that order to infinity.]

For each of these moving bodies there is an appropriate fumigation and an adequate metal on which is put the figure ascribed to that body for the matter connected with that body, and you will fumigate it during the operations at its specific time.[116]

114. This seems idyllic. But, as Agrippa warns, *si infortunata Venere in œre formetur, omnibus prœdictis contraria facit* (the same for the other planets).

115. Set of four presumed interpolations, which may not all originate with different readers.

116. All the elements necessary for making the amulets will now be specified: woods, seeds, powder, liquids or resins for the fumigation, metal on which to engrave the square, most appropriate time to do it (the «exaltation» of the planet is its place in the zodiac when its influence becomes optimal). Since the vocabulary of some of the substances for fumigation is unusual, we have referred in these cases to lexical sources, namely the Latin mediaeval *Picatrix* edited by Pingree (P) and the

Saturn. Fumigation: aloe,[117] ladanum,[118] saffron,[119] costus,[120] grains of olibanum.[121] Metal: lead. Special time (that is) exaltation: 21° of Libra.

Jupiter. Fumigation: aloe, sandal,[122] saffron, storax,[123] sandarac,[124] pine-resin,[125] berries of the (female) laurel-tree.[126] Metal: tin.[127] Exaltation: 15° of Cancer.

Mars. Fumigation: aloe, red sandal,[128] olibanum,[129] aloe juice,[130] long pepper.[131] Metal: iron.[132] Exaltation: 28° of Capricorn.

Sun. Fumigation: aloe, musk,[133] saffron, storax, male olibanum,[134] pomegranate flower,[135] (root of the) yellow turpeth.[136] Metal: gold. Exaltation: 19° of Aries.

Vocabulista edited by Schiaparelli (V), then the modern Arabic dictionaries by Freytag (F), Lane (L), Dozy (D), Kazimirski (K), Wehr (W).

117. *ʿūd*, ἀλόη. P: *lignum aloe*; F: 435*b*; L: 2190*c*; D: II,186*b*; K: II,401*a*; W: 586*b*.

118. *lādān* (normally: *lādan*), λάδανον, λῆδον. P: *laudanum*; V: 170*a*, 253 (*lā<u>dh</u>an*, «*aroma*»); D: II,524*a*; K: II,985*b*; W: 768*b*.

119. *zaʿfarān*, κρόκος. P: *crocus*; V: 112*a*, 325 («*croceus*»); (*et al.*).

120. *qusṭ*, κόστος. P: *costus*; V: 161*a*, 323 (*costum*); L: 2523*a*; K: II,736*b*.

121. *qu<u>sh</u>ūr al-kundur*. P: *grana alben*, *grana thuris*. See note 129.

122. *ṣandal*. P: *sandalus*; F: 352*a*; L: 1732*b*; D: I,846*a*; K: I,1375*b*.

123. *maiʿa*, στύραξ. P: *storax*; F: 595*a*; D: II,629*b*–630*a*; K: II,1174*a*; W: 832*b*.

124. *sandarūs*, σανδαράκη. P: *classa* (*cassa*, *casia* [κασία]); D: I,693*a*; K: I,1151*b*; W: 397*a*.

125. *sam<u>gh</u> ṣanaubar*. P: *resina pini*; (*ṣanaubar* only:) F: 351*b*; L: 1731*b*; K: I,1374*a*; W: 478*b*.

126. *ḥabb al-<u>gh</u>ār*. P: *bacce lauri*; V: 265 (*bacca*); L: 2308*a* («berries (...) female laurel-tree»); (*<u>gh</u>ār* only:) W: 615*a*.

127. Text: lead and copper.

128. P: *sandalus rubeus*.

129. *kundur* (see note 121). P: *thus* (θύος), *incensum*; F: 546*b*; L: 2633*a*; D: II,492*b*; K: II,934*a*; W: 750*a*.

130. *ṣabir*. P: *aloe*; V: 128*b*, 238 (*aloes*); F: 335*b*; L: 1645*a*; D: I,815*a* («suc d'aloès», «aloès»); K: I,1306*a*; W: 455*b*.

131. *dārufilfil*. P: *piper*, *macropiper*; V: 156*b* (*fulfala*, *piper*), 523 (*piper*, *dār fulfal* = *piperis locus* [*sic*, instead of *piper longum*]); L: 2435*a*; D: II,279*b*; K: II,631*b*; W: 649*a*.

132. Text: lead and copper.

133. *misk*, μόσχος (from Persian). P: *muscus*; V: 186*a* (*miska*, «*muscum*»), 486 (*misk*, *miska*, «*musqatum*»); F: 581*b*; L: 3020*a*; D: II,592*b*; K: II,1106*a*; W: 810*a*.

134. *lubān <u>dh</u>akar*. P: *incensum*, *thus* (note 129); D: II,515*a* («encens mâle»); (*lubān* only:) F: 553*a*; L: 3007*b*; D: II,492*b*; K: II,962*b*; W: 763*a*.

135. *jullanār*, βαλάυστιον (lat. *balaustium*). P: *balaustia*; F: 86*b*; L: 446*a*; D: I,209*b* («fleur double de balaustier ou grenadier sauvage»); W: 118*b*.

136. *turbid aṣfar*. P: *turbith*; D: I,143*b* («convolvulus turpethum»).

Venus. Fumigation: aloe, camphor,[137] musk, mastic,[138] costus, saffron, ladanum. Metal: copper.[139] Exaltation: 27° of Pisces.

Mercury. Fumigation: olibanum, ilex,[140] Carmanian cumin,[141] almond-shells.[142] Metal: quicksilver.[143] Exaltation: 15° of Virgo.

Moon. Fumigation: aloe, ladanum, grains of olibanum,[144] lemongrass,[145] carob beans,[146] talc.[147] Metal: silver. Exaltation: 3° of Taurus.

[(Dragon's) head. Fumigation as for Jupiter, so its metal. Exaltation: 3° of Gemini.]

[(Dragon's) tail. Fumigation as for Saturn, so its metal. Exaltation: 3° of Sagittarius.]

If you envisage some operation connected with one of the planets, take a tablet in the metal appropriate to that body, or a blank parchment [or a piece of silk, or a suitable coloured cloth],[148] and write on it the names or the phrase [or the letters corresponding to these words][149] during the exaltation of that planet, or on its day, or its night, or its hour, or during its time, and see that the writing be done when the moon is in application with Venus, be this application trine, sextile or conjunction,[150] and write around the square the relevant phrase, and the name of the request or the required thing on the back of the tablet. When you have finished this operation, if the ascendent which you have selected is a fiery sign,[151] bury

137. *kāfūr*. P: *camphora*; (*et al*).
138. *maṣṭakā*, μαστίχη. P: *mastix*; V: 188*b*, 471; D: II, 597*b*; K: II, 1116*b*; W: 812*b*.
139. Text: silver and copper.
140. *ushna*. P: *ilex*, *ilex muscata*; L: 62*c*; K: I, 35*b* («mousse d'arbre»); W: 16*a*.
141. *kammūn*, κύμινον (here: *cuminum Carmaniae*, «*kirmānī*» *sc*. from Kerman); (*kammūn* only): V: 166*a* (*kamūn*, «*ciminum*»); F: 545*b*; W: 749*b*.
142. *qishr al-lauz*. P: *cortices amigdalarum*, ἀμύγδαλον, ἀμυγδάλη; (*lauz* only:) V: 174*b*, 241 (*lauza*, «*amigdalum*»); L: 2681*a*; W: 787*a*.
143. *al-zaibaq al-maqtūl*, thus «calomel» or mercurous chloride. See D: I, 576*b*–577*a*.
144. *ḥabb al-lubān*. P: *grana thuris*, *grana alben*; K: I, 363*b* («noix muscade»); see note 134.
145. *idhākhir*, pl. (camel grass, Cymbopogon schœnanthus). P: *squinantus*, *iusquiamus*; F: 199*b*; L: 956*c* («*juncus odoratus; or schœnanthum*»); K: I, 19*a* («jonc»; writes: *idkhir*).
146. *ḥabb khurnūb*. (*khurnūb* only:) V: 93*b*, 405 (*kharnūb(a)*, «*garofa*»); L: 716*c*–717*a*; D: I, 357*a*; K: I, 553*a*; W: 213*a*.
147. *ṭalq*.
148. Our author did not mention which colours are attributed to the planets.
149. Thus isolated letters. This looks like an addition.
150. Thus when the two bodies are, respectively, 120, 60, 0 degrees apart.
151. Aries, Leo, Sagittarius.

this tablet in a fire-place. If the sign is airy,[152] hang it in a windy place, or on the seeker himself. If the sign is watery,[153] bury it under a place in which water flows, or rinse it in water and give it to the owner of the thing to drink.[154] If the sign is earthy,[155] bury it in a place to which the person [the seeker] approaches. [The best, in our opinion, is to deal with the name according to its numerical value, then make the intercessory prayer with due benevolence and spiritual **(50^r)** attention, (and) the desire will be realized without alteration.][156]

[If you wish, mix the name of the asker, the name of the (thing) asked, the letters of the planet, and add to that the numerical value of one of the attributes of God in relation with that treatment; write it on the tablet, or whatever one has managed to find instead, and take an invocation with a numerical value equivalent to the numerical value of the magic square which after fumigation the asker carries. You will make this invocation after the five times (of prayer), until the required occurs, and it will occur faster.][157]

Such is the general principle for working with the squares, be it in numbers or in letters. Praise be to God.

152. Gemini, Libra, Aquarius.
153. Cancer, Scorpio, Pisces.
154. Rather than «the owner», «the seeker»; see also subsequent correction.
155. Taurus, Virgo, Capricorn.
156. Indeed less demanding. So also the next.
157. This also seems to be a later addition.

Arabic text

(43^{v}) بسم الله الرحمن الرحيم

امّا الاوفاق فثلثة اقسام امّا مجرّد ومطوّق وحرفى

امّا القسم الاوّل ففى الاوفاق العددية

والمراد فى الوفق العددى ان يكون المدلول عددًا سواء كان الدالّ حرفًا او رقمًا وهو امّا مطلق وهو الذى يكون مبدأه فى الواحد الى تمام المربّع او غير مطلق وهو بخلافه و⟨الوفق⟩ المطلق امّا ان يكون بسطر واحد او بجميع السطور.

ومعرفة ذلك ان تضرب بيوت ضلع الشكل فى نفسها فما حصل فهو عدد بجميع البيوت فى الشكل فاذا زيد عليه واحد يكون عددًا عدلًا [وهو العدد الواقع فى البيتين المتقابلين من المطوّق على ما سيجىء]. ثمّ انظر الى العدد العدل ان كان زوجًا تضرب نصفه فى جميع بيوت ضلع واحد وان كان فردًا تضرب كلّه فى نصف البيوت ⟨من الضلع⟩ فما خرج منه فهو الوفق المطلق بسطر واحد فاذا ضُرب هذا الحاصل فى جميع بيوت ضلع واحد من الاضلاع فما حصل فهو الوفق المطلق بجميع السطور [لا يكون الّا بوضع الاسم فى المربّع كما سيجىء] .

مثلًا اذا نظرنا الى بيوت ضلع من مربّع ثلثة وجدنا ثلثة فاذا ضربنا الثلثة فى نفسها يصير تسعة وهو مقدار جميع البيوت للمثلّث فزدنا عليه واحدًا فيكون عشرة وهو العدد العدل زوج ونصفه خمسة فضربنا الخمسة فى الثلثه فيكون خمسة عشر وهو الوفق المطلق بسطر ⟨واحد⟩ فاذا ضربنا الخمسة عشر فى الثلثة تكون خمسة واربعين وهو الوفق المطلق بجميع السطور.

وكذا الحال فى مربّع اربعة لانّ العدد العدل فيه فرد وهو سبعة عشر فيُضرب فى نصف البيوت ⟨من الضلع⟩ وهو اثنان فيحصل اربعة وثلثون وهو الوفق ⟨المطلق⟩ بسطر واحد فاذا ضُربت فى جميع بيوت ضلع واحد من الاضلاع يحصل مائة وستّة وثلثون وهو الوفق المطلق بجميع السطور. [ثمّ اخرج من هذا المضروب وهو مائة

وستّة وثلثون ويُقسم الباقى على اربعة بيوت ضلع واحد فان كان مساويًا فرد فى بيوت تسعة.] وقسّ عليها باقى الاشكال.

والوفق ايضًا امّا مجرّد وهو الذى يعطى الوفق للمربّع الكبير فردًا كان او زوجًا ولا يعطى الوفق للمربّع الداخل فى ذلك [ولا يعتبر فيه عدد العدل] [بل يعتبر فيه سير الفرس] [ويراعى اجزاء الوفق] [ويسمّى زاواية (sic) الدائرة] وامّا مطوّق وهو بخلاف المجرّد وافضل منه لانّه اذا ازيل منه الملبن بقى الباقى وفقًا تامًّا.

فلنبيّن لكلّ منهما طريقًا فى شكل من الاصول الثمانية غير المثلّث فانّ له طريق واحد فقط.

وهو ان يوضع الواحد فى البيت الاوسط من السطر الاوّل الطولى والاثنان فى بيت فرسه والثلثة (44^{r}) فى بيت فرس ذلك البيت والاربعة فى جنبه الاعلى [وهو فرس الواحد] والخمسة فى فرزانه والستّة كذلك والسبعة فى جنبه والثمانية فى فرسه والتسعة كذلك. [ويجوز الابتداء فى اىّ وسط كان ويوضع الباقى على ما عرفتَ] .

٤	٣	٨
٩	٥	١
٢	٧	٦

وامّا الوفق المجرّد للمخمّس فهو ان يوضع الواحد فى البيت الاوّل من السطر الاوّل وينتقل منه الى اسفل السطور فرسًا فرسًا ومنه الى البيت الثانى من السطر الرابع الطولى ومنه الى البيت الرابع من السطر الخامس الطولى ثمّ منه الى جنبه الاعلى ومنه الى الخامس من الاوّل الطولى ومنه الى الثانى من الثانى ومنه الى الثالث من الرابع العرضى ومنه الى الرابع من الاوّل العرضى ثمّ منه الى الخامس من الرابع الطولى ومنه الى الثانى من الخامس الطولى ومنه الى الرابع من الاوّل الطولى ومنه الى الثانى من الاوّل العرضى ومنه الى الثالث من الثالث ثمّ منه الى جنبه الاعلى ومنه الى الرابع ⟨من الرابع⟩ ومنه الى الخامس من الاوّل العرضى ومنه الى الثالث من الاوّل الطولى ومنه الى الثانى من الخامس العرضى ثمّ منه الى جنبه الاعلى ومنه الى الثالث من الاوّل العرضى ومنه الى الثالث من الرابع الطولى ومنه الى الخامس من

الخامس ومنه الى الثانى من الاوّل الطولى. فقد تمّ.

١٨	١٠	٢٢	١٤	١
١٢	٤	١٦	٨	٢٥
٦	٢٣	١٥	٢	١٩
٥	١٧	٩	٢١	١٣
٢٤	١١	٣	٢٠	٧

ويجوز لك فى هذا ⟨الوضع الابتداء⟩ فى اوّل كلّ خمس اشير اليه بثمّ بعد الضبط للسير فى الاخماس ولا يلزم الترتيب فى الاخماس بل فى سير كلّ خمس [وكذا الحال فى كلّ من الاوفاق المجرّدة التى يراعى فيها سير الفرس [اجزاء ربعا وسدسا وسبعا] الى غير ذلك ممّا يحفظ فيه الاجزاء وسير الفرس.] على هذه الصورة.

وامّا الوفق المطوّق للمخمّس فهو ان يوضع الواحد فى البيت الاوسط من السطر الاوّل الطولى والثانى الى تحته ومنه الى القطر الرابع ومنه الى جنبه الايمن ومنه الى البيت الاوسط الاعلى ومنه الى البيت فوق الوسط الايسر ومنه الى جنبه الاعلى وهو ⟨القطر⟩ الثانى ومنه الى الجنب الايمن من البيت الاوسط الاعلى. فقد امتلأ نصف الملبن المحيط [وهو اصل فى جميع الاوفاق المطوّقة] وفى داخل الملبن ⟨يبقى مثلّث⟩ تتمّه كما عرّفناك. ثمّ فى تكميل باقى البيوت فى الملبن وجهان احدهما ان يبتدأ بالعدد العدل وينتقل منه بالنقصان فى مقابلة الزيادة وثانيهما بالزيادة فى مقابلة النقصان حتّى يمتلأ البيوت الخالية من الملبن.

٧	٢٢	٥	٨	٢٣
٦	١٢	١١	١٦	٢٠
٢٥	١٧	١٣	٩	١
٢٤	١٠	١٥	١٤	٢
٣	٤	٢١	١٨	١٩

مثلًا عدد ⟨بيوت⟩ الملبن المحيط للمخمّس ستّة عشر ونصفه ثمانية فملئ الملبن من واحد الى ثمانية ثمّ ملئ المثلّث فى داخله فبلغ البيوت المملوءة الى سبعة عشر فيوضع

فی (44^v) باقی البیوت باقی العدد وهو من ثمانیة عشر فی مقابلة الثمانیة بالزیادة الی خمسة وعشرین فی مقابلة الواحد او من الخمسة والعشرین فی مقابلة الواحد بالنقصان الی ثمانیة عشر فی مقابلة الثمانیة. [لکن الاوّل اوّلی لانّه علی الوضع الطبیعی] [وهذان الوجهان یجریان فی جمیع اوفاق المطوّق] [کما ان هذا الوضع جاز فی جمیع الاشکال الافراد فوق المخمّس] [ویجوز لك الابتداء فی هذا الوضع من کلّ وسط ویوضع الباقی علی ما عرفتَ] هکذا.

٣٧	٢٤	١١	٤٧	٣٤	٢١	١
١٢	٤٨	٣٥	١٥	٢	٣٨	٢٥
٢٩	١٦	٣	٣٩	٢٦	١٣	٤٩
٤	٤٠	٢٧	١٤	٤٣	٣٠	١٧
٢٨	٨	٤٤	٣١	١٨	٥	٤١
٤٥	٣٢	١٩	٦	٤٢	٢٢	٩
٢٠	٧	٣٦	٢٣	١٠	٤٦	٣٣

٦	٤٣	٨	٤١	١٠	٣٩	٢٨
٤٩	١٥	٣٤	١٧	٣٢	٢٧	١
٢	٣٨	٢٠	٢٩	٢٦	١٢	٤٨
٤٧	١٣	٣١	٢٥	١٩	٣٧	٣
٤	٣٦	٢٤	٢١	٣٠	١٤	٤٦
٤٥	٢٣	١٦	٣٣	١٨	٣٥	٥
٢٢	٧	٤٢	٩	٤٠	١١	٤٤

وامّا المجرّد للمسبّع فهو ان یوضع الواحد فی القطر الاوّل وینتقل الی الیسار فرسًا فرسًا الی نهایته ومنه الی البیت الخامس من السطر الثانی الطولی ومنه الی الیسار فرسًا مرّتین ثمّ منه الی البیت الثالث فوقه ومنه الی السادس من السطر الاوّل الطولی ومنه الی اسفله فرسًا ومنه الی الخامس من الاوّل العرضی ومنه الی الیسار فرسًا ومنه الی الثانی من الثالث العرضی ومنه الی الرابع من الرابع ثمّ منه الی الثالث فوقه ومنه الی الیسار فرسًا ومنه الی الرابع من الاوّل الطولی ومنه الی الیسار فرسًا فرسًا حتّی ینتهی الی القطر الرابع ومنه الی جنب القطر الاوّل ثمّ منه الی السادس [الذی یقابله] ⟨من الثانی الطولی⟩ ومنه الی فرسه ومنه الی السادس من الاوّل العرضی ومنه الی الثانی من الاوّل الطولی ومنه فرسًا فرسًا الی الخامس من السابع الطولی ثمّ منه الی الثالث فوقه ومنه الی الرابع من الثانی الطولی ومنه الی الیسار فرسًا مرّتین ومنه الی القطر الثالث ومنه الی الثالث من الاوّل العرضی ومنه الی الیسار فرسًا ثمّ منه الی السابع من الخامس الطولی ومنه الی القطر الثانی و منه الی الثانی من الثانی ومنه الی الیسار

فرسًا مرّتين ومنه الى الخامس من الاوّل الطولى ومنه الى اليسار فرسًا ثمّ منه الى الثالث فوقه ومنه الى اليسار فرسًا مرّتين ومنه الى السابع من الثانى الطولى ومنه الى الرابع من الاوّل العرضى ومنه الى اليسار فرسًا ومنه الى الثالث ⟨من الاوّل الطولى⟩ هكذا.

وامّا المطوّق للمسبّع فهو ان يوضع الواحد تحت القطر الاوّل ومنه الى الثالث من السابع الطولى ومنه الى الرابع من الاوّل الطولى ومنه الى الخامس من السابع الطولى ومنه الى السادس من الاوّل الطولى ومنه الى القطر الثانى ومنه الى السابع من السادس الطولى ومنه الى الخامس من الاوّل العرضى ومنه الى الرابع من السابع العرضى ومنه الى الثالث من الاوّل العرضى ⟨ومنه⟩ الى الثانى من السابع العرضى. فقد امتلأ نصف الملبن المحيط. ثمّ نعمل (45^r) فى الملبن الداخل كذلك وهو للمخمّس وفى داخله مثلّث. [فيوضع ⟨فى⟩ بيت الواحد من ذلك المثلّث العدد الباقى ومنه الى بيت فرسه صاعدًا وهابطًا ومنه الى القطر الرابع ومنه الى فرزانه ⟨وكذلك⟩ الى القطر الاوّل] ثمّ يكمل البيوت الخالية بالعدد الباقى كما فى المخمّس. [ويجوز لك الابتداء فى جميع الجوانب] [وهو جاز فى جميع الاوفاق الفردية كذلك] هكذا على هذه الصورة.

٨٠	٦٥	٥٩	٥٢	٣٧	٣١	٢١	١٥	٩
٤٨	٤٢	٣٦	٢٦	١١	٥	٧٦	٧٠	٥٥
٢٢	١٦	١	٨١	٦٦	٦٠	٥٣	٣٨	٣٢
٥٦	٧٧	٧١	٢٨	٤٩	٤٣	٦	٢٧	١٢
٣٣	٥٤	٣٩	٢	٢٣	١٧	٦١	٧٣	٦٧
٧	١٩	١٣	٥٧	٧٨	٧٢	٢٩	٥٠	٤٤
٦٨	٦٢	٧٤	٤٠	٣٤	٤٦	١٨	٣	٢٤
٤٥	٣٠	٥١	١٤	٨	٢٠	٦٤	٥٨	٧٩
١٠	٤	٢٥	٦٩	٦٣	٧٥	٤١	٣٥	٤٧

١٠	١٢	١٤	١٦	٧٥	٧٦	٧٨	٨٠	٨
١١	٢٤	٢٦	٢٨	٦١	٦٢	٦٤	٢٢	٧١
١٣	٢٥	٣٤	٣٦	٥١	٥٢	٣٢	٥٧	٦٩
١٥	٢٧	٣٥	٤٠	٤٥	٣٨	٤٧	٥٥	٦٧
٩	٢٦	٣٣	٣٩	٤١	٤٣	٤٩	٥٦	٧٣
٧٧	٦٣	٥٣	٤٤	٣٧	٤٢	٢٩	١٩	٥
٧٩	٦٥	٥٠	٤٦	٣١	٣٠	٤٨	١٧	٣
٨١	٦٠	٥٦	٥٤	٢١	٢٠	١٨	٥٨	١
٧٤	٧٠	٦٨	٦٦	٧	٦	٤	٢	٧٢

وامّا الوفق المجرّد والمطوّق للمتسّع فوضعنا لهما مثالين ولم نكتف بما سبق من الاوضاع الفردية لانّ الغرض بيان الطرق المختلفة للضابط ولم نفصّل لانّ المثال كافٍ بعد التيقّظ فيما قبله من الامثلة. هكذا على هذين الصورتين فافهم وما ذكرنا من الطرق وقسّ عليها باقى الافراد التى فوق التسعة الى غير النهاية.

وامّا الازواج فهى على قسمين [كما سبق اليه الاشارة] احدهما زوج الزوج والآخر زوج الفرد. فاوّل زوج الزوج المربّع اربعة فى اربعة وله عندنا اصول اربعة ينسب منها اوضاع كثيرة سنشير اليها عن قريب.

اوّلها ان يوضع الواحد فى القطر الاوّل وينتقل منه فرسًا وفرزانه ومنه فرسًا الى اليمين. [فقد امتلأ بيوت اربعة من كلّ ضلع من الاضلاع والاقطار ⟨بيت واحد⟩] [وقد روعى فى هذا الوضع ارباع] ثمّ الى جنبه ومنه الى اليمين صعودًا كما ترى حتّى تنتهى الى القطر الثانى فعند ذلك كمل نصف البيوت وكان عدد كلّ بيتين عرضًا تسعة متباعدين فى السطرين الاوّل والثالث ومتقاربين فى الثانى والرابع هذا هو الاصل فى ذلك الوضع ثمّ يوضع فى البيوت الباقية باقى العدد امّا ان يوضع التسعة فى فيل الثمانية بالزيادة فى مقابلة النقصان الى فيل الواحد او بستّة عشر فى فيل الواحد بالنقصان فى مقابلة الزيادة الى فيل الثمانية [وهو مطّرد فى جميع الصور بهذا] ولكن يجوز فيه ان تبدّل الارباع كما اشرنا اليه سابقًا فى المخمّس [ثمّ يوضع ما يقع فى بيت الفيل ⟨او⟩ فى الجنب او فى المقابلة] على هذا المثال.

٨	١١	١٤	١
١٣	٢	٧	١٢
٣	١٦	٩	٦
١٠	٥	٤	١٥

[ربيع ١]

١٢	١٥	٦	١
٩	٢	١١	١٦
٣	٨	١٣	١٠
١٤	٥	٤	٧

وثانيها ان يكتب الواحد فى البيت الثانى من السطر الاوّل العرضى ومنه الى اليسار فرسًا ومنه الى الثالث من الاوّل الطولى ومنه الى اليسار فرسًا ومنه الى ⟨الرابع من الاوّل الطولى ومنه الى الثالث من الثالث ومنه الى⟩ الثانى من الثانى ومنه الى

القطر الثانى وقد كمل نصف البيوت. وهو اصل فى هذا الوضع. ويكتب باقى الاعداد بالمعادلة بالزيادة فى مقابلة النقصان او بالنقصان ⟨فى مقابلة⟩ الزيادة.

٨	١٣	١	١٢
٢	١١	٧	١٤
١٥	٦	١٠	٣
٩	٤	١٦	٥

[خريف ٣]

٤	١٤	١٥	١
٩	٧	٦	١٢
٥	١١	١٠	٨
١٦	٢	٣	١٣

[صيف ٢]

٤	١٤	١٥	١
١١	٥	٨	١٠
٦	١٢	٩	٧
١٣	٣	٢	١٦

[شتاء ٤]

ثالثها ان يوضع الواحد فى القطر الاوّل وينتقل منه الى الرابع من الثالث الطولى ومنه الى جنبه الايمن ومنه الى القطر الثانى ومنه الى فوق القطر الرابع (45ᵛ) [الذى يقابله] ومنه الى الفرس فوق ومنه الى جنبه الايسر ومنه الى الفرس يمينًا هابطًا ومنه الى تحت القطر الثانى ⟨ومنه الى الفرس هابطًا ومنه⟩ الى الجنب الايسر ومنه الى الفرس صاعدًا ومنه الى القطر الثالث ومنه الى الجنب الايمن للقطر الثانى ومنه الى جنبه الايمن ومنه الى القطر الرابع. وقد كمل. [والضابط ان تعدّ فتضع الى منتهى العدد من الجانبين ⟨كما⟩ تعرف بالنظر فى المثال.]

ورابعها ممتزج من الثانى والثالث. [ويوافق الوجه الثانى فى الاوّل فى مقابلة كما يوافق الثانى فى المقابلة كذلك ويشترك فى الثانى فى المقارنة.] هكذا فى صورها.

وبهذه الصور الاربع وبهذه الاصول الاربعة ⟨يحصل كثرة صور عددها⟩ يرتقى الى الفين وستّ مائة وثمان وثمانين صورةً. الف وتسعمائة وعشرون منها الاصل الاوّل وسبعمائة وثمان وستّون منها الاصول الثلثة الباقية لانّ الاصل الاوّل يحصل له ستّ صور باعتبار وقوع الواحد فى الاوّل مع تقدّم بعض السطور والتأخّر طولًا وكذلك بكلّ من الاعداد الستّة عشر فاذا ضربت الستّة فى ستّة ⟨عشر⟩ صار ستّة وتسعون وباعتبار العرض كذلك فيكون المجموع مائة واثنتين وتسعين صورةً ويكون تبديل الارباع كما سبق اليه الاشارة تسعمائة وستّين صورةً فمجموع الحاصل من الاصل الاوّل باعتبارين الف وتسعمائة وعشرون صورةً لانّ لكلّ من هذه الصور الاربع صورة اخرى تحصل بتبديل ملبن كلّ واحد منها باعتبار هذا الاصل. ويحصل لكلّ

واحد من الاصول الباقية ثمانى صور [فكان الستّ فى الاصل الاوّل] فاذا ضربت الثمانية فى ستّة عشر خرج مائة وثمان وعشرون صورةً باعتبار الطول وباعتبار العرض كذلك فيكون لكلّ من الثلثة مائتان وستّ وخمسون صورةً فالمجموع الحاصل منها سبع مائة وثمانية وستّون فاذا جُمع مع ما حصل من الاصل الاوّل بلغ ⟨المجموع⟩ الفين وستّمائة وثمانيًا وثمانين صورةً. هذا ما تبعنا واستخرجنا ولا يحصل بعد فيما ذكرنا. فافهم ولاحظ حقّ الملاحظة وقدر وضع جميع ما يمكن من الاوضاع [ومن الاسماء العسيرة وضعها فى ⟨الاشكال⟩ المشتركة كالقابض والباسط والجامع وامثالها.]

٦٠	١٧	٢٨	٥٣	٤٨	٣٧	١٦	١
١٥	٢	٥٩	١٨	٢٧	٥٤	٤٧	٣٨
٤٦	٣٩	١٤	٣	٥٨	١٩	٢٦	٥٥
٢٥	٥٦	٤٥	٤٠	١٣	٤	٥٧	٢٠
٣٣	٤٤	٤٩	٣٢	٢١	٦٤	٥	١٢
٦	١١	٣٤	٤٣	٥٠	٣١	٢٢	٦٣
٢٣	٦٢	٧	١٠	٣٥	٤٢	٥١	٣٠
٥٢	٢٩	٢٤	٦١	٨	٩	٣٦	٤١

وامّا الوفق المجرّد للمثمّن فهو ان يبتدأ فى القطر ⟨الاوّل⟩ وينتقل الى البيت الثانى من السابع الطولى ومنه الى اليمين فرسًا مرّتين وفرزانه ومنه الى الثامن من السادس العرضى ومنه الى اليمين فرسًا ⟨مرّتين⟩ ثمّ منه الى جنبه الايمن ومنه الى اليسار فرسين متصاعدًا ومنه الى الخامس من الاوّل الطولى ومنه الى الرابع من الرابع ومنه الى اليسار فرسين ومنه الى الثانى من الاوّل العرضى ثمّ منه الى الاوّل من السابع الطولى ومنه الى اليمين فرسًا ثلث مرّات ومنه الى الخامس من الرابع الطولى ومنه الى اليمين فرسًا ⟨هابطًا⟩ (**46r**) ومنه الى السابع من الثامن الطولى ومنه الى السادس من الثامن العرضى ثمّ منه الى الرابع من الثامن الطولى ومنه الى الثالث من الثانى الطولى ومنه الى اليسار فرسًا مرّتين متصاعدًا ومنه الى السابع من الثامن العرضى

ومنه الى السابع من الاوّل الطولى ومنه الى اليسار ⟨فرسًا⟩ مرّتين متصاعدًا ثمّ منه الى الرابع من هذا السطر العرضى ومنه الى اليمين فرسًا مرّتين هابطًا بعد ⟨مرّة⟩ اخرى ومنه الى الثالث من الاوّل العرضى ومنه الى اليمين فرسًا ومنه الى الثالث من السابع الطولى ومنه الى اليمين فرسًا ثمّ منه الى القطر الثالث ومنه الى الشمال فرسًا مرّة بعد مرّتين ومنه الى اليمين فرزانه ومنه الى اليسار فرسًا متصاعدًا ايضًا ومنه الى الثانى من الثانى ومنه الى الشامل فرسًا متصاعدًا ثمّ منه الى الخامس من السادس الطولى ومنه الى اليمين فرسًا مرّتين هابطًا ومنه الى القطر الرابع ومنه الى الخامس من الاوّل العرضى ومنه الى اليمين فرسًا مرّتين ومنه الى الرابع من السابع الطولى ثمّ منه الى السادس من هذا السطر العرضى ومنه الى اليسار فرسًا متصاعدًا ثلث مرّات ومنه الى الثامن من الخامس الطولى ومنه الى السابع من السابع ومنه الى السادس من الاوّل الطولى ومنه الى اليسار فرسًا. بثمّ اشرنا الى اجزاء [فانّه ينفصل فى وفق كلّ اسم وجد شرطه] هكذا على هذه الصورة.

وامّا المطوّق للمثمّن فانّ يوضع الواحد فى الاوّل ⟨من⟩ الرابع الطولى ومنه الى الثامن من الخامس الطولى ومنه الى تحت القطر الاوّل ومنه الى الثالث من الثامن الطولى ومنه الى الثامن من السادس الطولى ومنه الى القطر الثانى ومنه الى الرابع من الثامن الطولى ومنه الى الخامس من الاوّل الطولى ومنه الى القطر الاوّل ومنه الى السادس من الاوّل ⟨الطولى ومنه⟩ الى الثامن من الثانى الطولى ومنه الى جنبه الايسر ومنه الى السابع من الثامن الطولى ومنه الى السابع من الاوّل العرضى. فقد امتلأ نصف البيوت من الملبن المحيط وبقى نصفها خاليًا. [وهو الاصل المطّرد فى جميع الاوفاق للمطوّق من زوج الزوج] ثمّ تكمل النصف الباقى بالمعادلة قبل امتلأ ما فى جوفه من المسدّس والمربّع [او بعده].

[لكن الثانى اوّلى لانّه على الوضع الطبيعى [الذى هو انّ سدسًا باتّبر] ويُملأ المسدّس الداخل بان يوضع الخمسة عشر التى بقى من الملبن المحيط ابتداءً فى القطر الاوّل من هذا المسدّس ومنه الى تحت القطر الثانى ومنه الى السادس ⟨من⟩ الثانى الطولى ومنه الى القطر الثانى ومنه الى السادس من الثالث الطولى (46ᵛ) ومنه الى

الثالث من الاوّل ⟨الطولى⟩ ومنه الى الرابع من السادس العرضى ومنه الى الرابع من الاوّل الطولى ومنه الى الخامس من السادس الطولى ومنه الى الخامس من الاوّل العرضى. فقد امتلأ نصف الملبن. هذا الطريق مطرّد فى كلّ زوج فرد. فيكمل الباقى من الملبن المحيط وفى جوفه مربّع اربعة فى اربعة فيُملأ باىّ وضع تريد من الاوضاع المذكورة له [لكن الاوّل منها الثانى والثالث] .] هكذا على هذه الصورة.

٦	١٤	٦٠	٦٣	١	٥٣	٥٤	٩
٦٢	١٨	٢٤	٤٤	٤٦	٤٨	١٥	٣
٤	١٦	٢٨	٣٨	٣٩	٢٥	٤٩	٦١
٧	٤٥	٣٥	٢٩	٣٢	٣٤	٢٠	٥٨
٥٧	٤٣	٣٠	٣٦	٣٣	٣١	٢٢	٨
٥٥	٢٣	٣٧	٢٧	٢٦	٤٠	٤٢	١٠
١٣	٥٠	٤١	٢١	١٩	١٧	٤٧	٥٢
٥٦	٥١	٥	٢	٦٤	١٢	١١	٥٩

وامّا المجرّد للمسدّس وهو اوّل زوج الفرد فيبدأ فى القطر الاوّل وينتقل منه الى الثانى من الرابع الطولى ومنه الى الثالث من الرابع العرضى ومنه الى الخامس من السادس الطولى ومنه الى فرزانه هابطًا ومنه الى الثالث من الثانى الطولى ثمّ منه الى الجنب الايسر للقطر الثالث ومنه الى الثالث من الرابع الطولى ومنه الى الرابع من السادس الطولى ومنه الى الثالث من الاوّل العرضى ومنه الى الخامس من الخامس ومنه الى تحت القطر الاوّل ثمّ منه الى الرابع من الاوّل الطولى ومنه الى فرزانه ومنه الى الثالث من الخامس الطولى ومنه الى فرس الواحد ومنه الى القطر الرابع ومنه الى الرابع من الاوّل العرضى ثمّ منه الى جنبه الايسر ومنه الى بيت الفيل ومنه الى تحت القطر الثانى ومنه الى الخامس من الرابع الطولى ومنه الى الثانى من الرابع العرضى ومنه الى القطر الثالث ثمّ منه الى السادس من الرابع الطولى ومنه الى الثانى من الثانى ومنه الى الرابع من الخامس الطولى ومنه الى القطر الثانى ومنه الى الخامس من الثالث الطولى ومنه الى الثالث من الاوّل الطولى ثمّ منه الى الخامس من هذا السطر ومنه الى الثالث من السادس الطولى ومنه الى الثالث من السادس العرضى ومنه الى

الثانى من الخامس الطولى ومنه الى الاوّل من الثانى الطولى ومنه الى الرابع من الرابع. فقد تمّ.

وامّا المطوّق للمسدّس فقُدّم فى داخل المثّمن وهو ان كان كافيًا فلكن فوضعنا له مثالًا آخر لئلا يتهّم الانحصار. هكذا على هاتين الصورتين.

٢٨	١٩	١٨	١٠	٣٥	١
٢١	٣٤	٢	١٦	٢٦	١٢
٣٢	١٥	٨	٢٠	٦	٣٠
٩	٢٧	٣٦	٣	٢٣	١٣
٤	١١	٢٢	٢٩	١٤	٣١
١٧	٥	٢٥	٣٣	٧	٢٤

٢	١٠	٣٠	٣١	٣٣	٥
٣٦	١٤	١٧	٢٠	٢٣	١
٣	١٩	٢٤	١٣	١٨	٣٤
٢٩	٢٥	٢٢	١٥	١٢	٨
٩	١٦	١١	٢٦	٢١	٢٨
٣٢	٢٧	٧	٦	٤	٣٥

وامّا المجرّد للمعشّر فهو ان يبتدأ فى القطر الاوّل وينتقل الى التاسع من السادس الطولى والى الثالث من الخامس الطولى والى اليسار فرسًا والى الثانى من السابع العرضى والى الخامس من التاسع الطولى والى فرزانه والى الثامن من الرابع الطولى والى الثالث من الثانى العرضى والى العاشر من الثامن الطولى ثمّ الى فرزانه والى تحت القطر الثانى والى السابع من الثامن الطولى والى السادس (47r) من الرابع الطولى والى السابع من الخامس العرضى والى الثالث من الاوّل الطولى والى السادس من الرابع العرضى والى الثامن من الثالث الطولى والى الخامس من العاشر العرضى والى الاوّل من الثانى الطولى ثمّ الى الثانى من الخامس الطولى والى الثالث من الاوّل العرضى والى السادس من الثامن العرضى والى الثالث من السابع الطولى والى الخامس من الاوّل الطولى والى فرزانه والى القطر الرابع والى السابع من الرابع الطولى والى التاسع من الثامن الطولى والى التاسع من الرابع العرضى ثمّ الى جنبه الايسر والى السادس من الخامس العرضى والى التاسع من الاوّل الطولى والى السادس من الثامن الطولى والى الثانى من الثامن العرضى والى الرابع من الاوّل العرضى والى السابع من الخامس الطولى والى الثالث من الثالث والى السابع من

الثانى العرضى والى التاسع من العاشر العرضى ثمّ الى الخامس من الاوّل العرضى والى الثانى من التاسع الطولى والى الثانى من الثالث العرضى والى الرابع من التاسع العرضى والى الرابع من الثالث الطولى والى الخامس من الثامن الطولى والى الفرس الايمن هابطًا والى العاشر من السابع الطولى والى الثامن من العاشر الطولى والى فوقه ثمّ ⟨الى⟩ العاشر منه والى تحته والى الرابع من العاشر العرضى والى السادس من الخامس الطولى والى الثالث من الخامس العرضى والى الرابع من الثامن الطولى والى السابع من التاسع ⟨العرضى⟩ والى التاسع من الثالث العرضى والى الثانى من الثانى والى السادس من الاوّل العرضى ثمّ الى العاشر من الثانى الطولى والى الثانى من الرابع الطولى والى الثالث من الثامن الطولى والى السادس من السابع العرضى والى السابع من الاوّل العرضى والى الثامن من التاسع الطولى والى السادس من الثالث الطولى والى فوق القطر الرابع والى الخامس من الخامس والى الرابع من الاوّل

١٠٠	٨١	٧٩	٦٥	٦٠	٤١	٣٦	٢٢	٢٠	١
١٢	٤٢	٩٢	٣٩	٨٠	٢١	٦٢	٩	٥٩	٨٩
٨٥	٥٨	٦٣	٢٤	٩٨	٣	٧٧	٣٨	٤٣	١٦
٣١	٣٠	٥٦	٤	١٧	٨٤	٩٧	٤٥	٧١	٧٠
٧٦	٦	٤٦	١٥	٣٢	٦٩	٨٦	٥٥	٩٥	٢٥
٧	٧٥	٣٤	٨٧	٤٧	٥٤	١٤	٦٧	٢٦	٩٤
٥٠	٩٦	١٣	٧٣	٦٤	٣٧	٢٨	٨٨	٥	٥١
٤٩	٦٦	٨٣	٩٣	٢٣	٧٨	٨	١٨	٣٥	٥٢
٦٨	١١	٢٩	٥٧	٢	٩٩	٤٤	٧٢	٩٠	٣٣
٢٧	٤٠	١٠	٤٨	٨٢	١٩	٥٣	٩١	٦١	٧٤

الطولى ثمّ الى جنبه والى التاسع من الثالث الطولى والى السابع من السابع والى القطر الثالث والى التاسع من السادس العرضى والى فرزانه والى الثالث من الرابع الطولى والى الخامس من الثامن العرضى والى الثامن من الاوّل العرضى والى اليمين فرسًا ثمّ الى جنب القطر الثانى والى السادس ⟨من⟩ العاشر العرضى و⟨الى⟩ الثامن

من الثامن والى الرابع ⟨من⟩ الخامس الطولى والى الثالث ⟨من⟩ العاشر الطولى والى الخامس ⟨من⟩ الرابع الطولى والى السادس من السابع (47^{v}) الطولى والى الثالث ⟨من⟩ السابع العرضى والى الثانى ⟨من⟩ الاوّل الطولى والى التاسع من الثانى الطولى ثمّ الى فرزانه والى الثامن ⟨من⟩ الثانى العرضى والى الثامن ⟨من⟩ السابع الطولى والى السادس ⟨من⟩ الاوّل الطولى والى فرزانه والى التاسع ⟨من⟩ السابع العرضى والى الرابع ⟨من⟩ الرابع والى الثالث ⟨من⟩ السادس الطولى والى التاسع ⟨من⟩ الخامس الطولى والى القطر الثانى هكذا ⟨على هذه الصورة⟩.

والمطوّق للمعشّر فمقنّن عن البيان بالمثال المسدّس لكن وضعنا له مثالًا آخر هكذا ليظهر ما لم يبيّن من الاوضاع الى نيّره. وقسّ على ما ذكرنا باقى الازواج فوق العشرة.

٣٦	٥١	٥٤	٤٨	٤٩	٤٢	٦٢	٦٤	٦٦	٣٣
٣٤	٢٠	٣٢	٧١	٢٩	٧٦	٧٨	٧٩	١٩	٦٧
٦٣	٢١	١٢	١٨	٨٦	٨٨	٩٠	٩	٨٠	٣٨
٦١	٧٧	١٠	٤	٩٨	٩٩	١	٩١	٢٤	٤٠
٤١	٢٦	٨٧	٩٥	٥	٨	٩٤	١٤	٧٥	٦٠
٥٨	٧٤	٨٥	٦	٩٦	٩٣	٧	١٦	٢٧	٤٣
٤٤	٧٣	١٧	٩٧	٣	٢	١٠٠	٨٤	٢٨	٥٧
٤٥	٣١	٩٢	٨٣	١٥	١٣	١١	٨٩	٧٠	٥٦
٥٥	٨٢	٦٩	٣٠	٧٢	٢٥	٢٣	٢٢	٨١	٤٦
٦٨	٥٠	٤٧	٥٣	٥٢	٥٩	٣٩	٣٧	٣٥	٦٥

اعلم ان ما ذكرنا الى آن اوضاع البسائطِ وهى جارية فى المركّبات ايضًا لانّ الاجزاء فى المركّبات بمنزلة واحدة من الوفق البسيط فتملأ مربّعات المركّب كما يملأ البسيط. مثلًا المتسّع مركّب من مربّع ثلثة فى ثلثة فيوضع اوّلًا المثلّث الذى بما ثبت بيت الواحد فى المثلّث البسيط ثمّ المثلّث الذى بما ثبت بيت الاثنين ⟨وهكذا⟩ الى تمام المثلّثات. وقسّ عليه المركّبات من مربّع اربعة فى اربعة ومن مربّع خمسة فى خمسة ومن مربّع ستّة فى ستّة الى ما لا نهاية له.

[وقد يجتمع تراكيب مختلفة فيجوز لك الوضع بالحوائج تريد لكن المختار هو الشكل المناسب والوفق الموافق للحاجة المنسوبة الى كوكب ذلك الشكل.]

وفى ⟨زوج⟩ الزوج نوع آخر من التراكيب وهو ان يقسم المربّع ويوضع العدد فى كلّ قسم بالمعادلة [كما فى المطوّقات] .

وقد كشق القناع عن وجوه المخترات لمَن ينظر اليها وفتح القناع لمَن يدخل فيها. واللّه يقول الحقّ وهو يهدى السبيل.

فصل فى بيان وضع الاسامى فى المربّعات

وهو على نوعين. احدهما ان يوضع العدد بحيث يكون الثابت فى كلّ ضلع طولًا وعرضًا وقطرًا عدد الاسم المقصود وفقه. والآخر ان يُكتب حروف الاسم متفرّقة فى ضلع من الاضلاع ويوضع عدد ذلك الاسم فى باقى البيوت على حسب اجزاء الوفق فيحصل عدد ذلك الاسم فى كلّ سطر طولًا وعرضًا وقطرًا ⟨كما⟩ سنشهر فيما نرسمه ان شاء اللّه تعالى.

وامّا النوع الاوّل فشرطه ان يكون عدد الاسم فيه اكثر من الوفق المطلق لذلك الشكل الذى يراد وضع الاسم فيه وان يكون لذلك الاسم ثُلث صحيح ان اريد وضعه فى المثلّث [لكن فى الاكثر لا كلّيًا] [لانّ الاسم المكسور بالنظر اليه لا يصحّ وضعه فيه بخلاف باقى الاشكال] [ولانّ لبعض الاسم كالقابض مثلًا ثُلث صحيح مع انّه لا يصحّ فيه بهذا الوجه] .

فطريق الوضع ان يطرح من الاسم او الآية عدد الوفق (48r) المطلق للشكل ويقسم الباقى لبيوت ضلع واحد من الشكل فحصص البيوت امّا ان تكون متساوية او لا فان كان الاوّل فيؤخذ ما خرج لبيت ويزاد عليه واحد ويوضع فى بيت الواحد وينتقل كما فى الوضع الطبيعى على طريق المطوّق والمجرّد وان كان الثانى فلينظر الى كمّيّة النقصان من الاجزاء الناقصة من بيوت ضلع من الشكل [عدد اجزاء الشكل] فيبدأ بالحص الناقص بزيادة واحد عليه وينقل الى تمام الاجزاء الناقصة ثمّ يزاد

⟨واحد⟩ آخر جبرًا للنقصان وينقل الى تمام الشكل.

مثلًا اذا اردنا وضع اسم الملك فى المثلّث على النوع الاوّل وعدده ٩٠ وله ثُلث صحيح. طرحنا ⟨منه⟩ وفق المطلق وهو خمسة عشر وقسمنا الباقى فى بيوت الضلع وهى ثلثة وحصّ كلّ بيت خمسة وعشرون فأخذناه وزدنا عليه واحدًا فوضعنا هكذا على هذه الصورة.

وكذا اذا اردنا وضع ذلك الاسم فى المخمّس. طرحنا خمسة وستّين منه فقسمنا الباقى على خمسة حصص لكلّ بيت خمسة وزدنا عليه واحدًا فوضعنا هكذا.

٢٩	٢٨	٣٣
٣٤	٣٠	٢٦
٢٧	٣٢	٣١

٢٣	١٥	٢٧	١٩	٦
١٧	٩	٢١	١٣	٣٠
١١	٢٨	٢٠	٧	٢٤
١٠	٢٢	١٤	٢٦	١٨
٢٩	١٦	٨	٢٥	١٢

٢٢	٢٥	٢٨	١٥
٢٧	١٦	٢١	٢٦
١٧	٣٠	٢٣	٢٠
٢٤	١٩	١٨	٢٩

وكذا اذا اردنا وضع ذلك الاسم فى مربّع اربعة. طرحنا منه اربعة وثلثين وهو الوفق المطلق وقسمنا الباقى على اربعة ويكون حصّة كلّ بيت اربعة عشر فزدنا عليه واحدًا ووضعنا هكذا ⟨وهذه⟩ صورته.

وانّا اذا اردنا وضع اسم الجليل فى المربّع. فوجدنا مكسوره وبعد القسمة على اربعة يكون ثلثة منها عشرة عشرة وواحد تسعة فزدنا على التسعة واحدًا فوضعنا فى كلّ ضلع الى تمام الرُبع ⟨الاوّل⟩ ثمّ زدنا عليه واحدًا آخر جبرًا للنقصان ثمّ وضعنا الى تمام الشكل هكذا.

١٨	٢١	٢٤	١٠
٢٣	١١	١٧	٢٢
١٢	٢٦	١٩	١٦
٢٠	١٥	١٣	٢٥

٢٠	١١	٢٤	١٦	٢
١٤	٥	١٨	٩	٢٧
٧	٢٥	١٧	٣	٢١
٦	١٩	١٠	٢٣	١٥
٢٦	١٣	٤	٢٢	٨

واذا اردنا وضع ذلك الاسم فى المخمّس فبعد طرح الوفق المطلق بقى ثمانية وقسمناه الى خمسة فيكون ثلثة منها اثنين اثنين واثنان واحد واحد وأخذنا الواحد وهو احد الجزئين الناقصين فزدنا عليه واحدًا ووضعنا فى بيت الواحد ونقلنا الى تمام الخُمسين ثمّ زدنا عليه واحدًا آخر جبرًا للنقصان ونقلناه الى تمام الاخماس الباقية هكذا.

وقِسْ عليها باقى الاشكال وعلى هذا الوضع اسامى ⟨فى⟩ سائر الاشكال.

[ويجوز لك الوضع فى المربّع ⟨بطريق آخر وهو ان يؤخذ⟩ الوفق المطلق وينقص منه مقدار بيوت ضلع وتضع بعد القسمة بلا زيادة وتزيد فيه بعد الوضع الكسر الاكثر.]

واعلم ان اذا كان الاقسام بعد الطرح متساوية ومتناصفة كان وضع الاسم المكسور فى المجرّد والمطوّق صحيحًا واذا كان خلاف ذلك لا يصحّ الّا فى المجرّد الذى يُرعى فيه الاجزاء [واشير اليه ثمّ فى البيان] كما بيّنّاك سابقًا. (48ᵛ) [وان مثال المجرّد للمعشّر لا يصحّ فيه وضع الاسم المكسور الذى فيه الناقصة بعد القسمة.]

ولوضع الاسم للمجرّد طريق اسهل وهو ان يوضع العدد الى ان يبقى جزء ⟨واحد⟩ من الوفق ثمّ يحسب العدد الثابت فى ضلع من هذا الشكل وينظر بكم عدد يتكمّل عدد اسم المطلوب ويوضع ما يكمل عدد الاسم فى ذلك الضلع الذى فيه اوّل بيت من هذا الجزء وينتقل بزيادة الى تمام البيوت. هكذا صورته.

٨	١١		١
	٢	٧	١٢
٣		٩	٦
١٠	٥	٤	

وامّا النوع الثانى ان يُكتب حروف الاسم او الآية فى سطر من السطور ثمّ تنظر الى نسبة البيوت المملوءة كلّ واحد منها بجزء من اجزاء الوفق فتملأ البيوت الباقية بالزيادة والنقصان على الوجه المعلوم بالوضع الطبيعى.

مثلًا اذا اردنا وفق اسم الجميل بهذا الوجه فى المربّع كتبنا حروفًا متفرّقة فى اربعة بيوت منه متوالية والحال ان كلّ حرف واقع فى ربع من ارباع المربّع ويبقى لكلّ منها ثلثة بيوت خالية وكلّ واحد من تلك البيوت المملوءة لا يخلو من ان يكون ابتداء اجزاء الوفق او انتهائها او ما بينهما. فان كان ابتداء الاجزاء فبالزيادة وان كان انتهائها فبالنقصان فان كان ما بينهما فبالزيادة والنقصان بحسب الوضع الطبيعى. والجيم فى المثال فى اوّل الجزء الاوّل من المربّع فننقل بالزيادة الى بيت الاربعة. واللام فى آخر الجزء الثانى فبالنقصان الى بيت الخمسة. والياء فى البيت الثالث من الجزء الثالث فبالزيادة الى بيت الاثنى عشر وبالنقصان الى بيت التسعة. والميم فى البيت الثانى من الجزء الرابع فبالنقصان الى بيت ثلثة عشر وبالزيادة الى 〈بيت〉 ستّة عشر. هكذا 〈صورته〉 من كلّ ضلع ٨٣.

٣٠	١٠	٤٠	٣
٣٩	٤	٢٩	١١
٥	٤٢	٨	٢٨
٩	٢٧	٦	٤١

الله لطيف بعباده

٨٤	١٢٩	٦٦
٧٥	٩٣	١١١
١٢٠	٥٧	١٠٢

وقس على هذا باقى الاشكال بهذا الطريق غير المثلّث فانّ الوضع فى المثلّث ان يُكتب الاسم او الآية فى السطر الاوّل ثمّ يوضع ثُلث مجموع الاعداد فى الوسط ثمّ يوضع فى كلّ من القطرين ما يكمل عدد الاسم ثمّ يوضع ما يكمل الاوسط.

مثلًا وضعنا قولة الله لطيف بعباده فى السطر الاوّل على هذا المثال. وثُلثه ثلثة وتسعون فى مركز المربّع ثمّ نظرنا الى القطر الايمن فوجدنا ما يكمله مائة واثنين فوضعنا فيه فنظرنا الى القطر الايسر فوجدنا ما يكمله مائة وعشرين فوضعنا فيه ثمّ كمّلنا طولًا على الترتيب. هكذا صورته.

وهذا القدر من البيان كافٍ لمَن كان له قلب سليم او القى السمع [وهو شهيد] فمَن يتقن بما قرّرناه ويفكر فى ما حرّرناه ويتيقّظ فى السلوك الى ما صوّرناه فقد فاز بما قصّرناه ونال بما اردنا من الوضع فى كلّ شكل حيث (49^r) لا يتجاوز عن حدّ الامكان.

[كالباسط وأحمد ومثالهما فى التقسّر من غير تكرار] [وتتميم الطالب والمطلوب والمحبّة من غير زيادة ولا نقصان هكذا على هذه الصورة فى المربّع] [والمقتدر ومحمّد كاملًا وغير مكرّر فى المخمّس والمسدّس وامثالهما] ولم يصرّح بها الممتحن الزكى ولا يحوّم حوله الغبى. واللّه الهادى الى سبيل الرشاد.

	مطلوب ٨٧		١
محبه	٢		
٣			
	طالب ٤٢	٤	

وامّا القسم الثالث وهو بيان كيفيّة العمل بالمربّعات

فموقوف على بيان الامور لا بدّ منها فى العمل بها امّا ان يكون الواضع متديّنًا ومتورّعًا ومتضرّعًا الى اللّه ومتّقيًا عن الحرام فلا يكتب لاىّ شخص كان ولايّة حاجة كانت بل لطالب الحقّ والامر المشروع ان وجد فى قلبه إنشراحًا وتوجّهًا ووقت مسألته فى المناحس وخطوط الجدول مستقيم وقسمة البيوت متساوية والرقوم والحروف المكتوبة متكاملة ومنها انّ كلّ واحد من الاشكال المذكورة سابقًا وكلّ يوم وليلة من الايّام والليالى وكلّ ساعة منهما وكلّ صنف من اصناف المجلّات منسوب الى الواحد من اليسار.

والشكل المثلّث. يوم السبّت وليلة الاربعاء واوّل ساعتهما. والنباء والزرع واستخراج المياه وزيادة المعاش وحصول الحاجة والجاه عند المشايخ والبرء من الامراض المزمنة. كلّها متعلّقة بزحل.

والشكل المربّع. يوم الخميس وليلة الاثنين واوّل ساعتهما. وتسيير الامور الصعاب وتحصيل المراد وتسهيل المطالب فى التموّل والغناء واسباب الثقافة والسعادة. كلّها متعلّقة بالمشترى.

والشكل المخمّس. يوم الثلاثاء وليلة السبّت واوّل ساعتهما. وانواع العداوة وطلب الفتح والغلبة والنهب وسفك الدماء. كلّها متعلّقة بالمريخ.

والشكل المسدّس. يوم الاحد وليلة الخميس واوّل ساعتهما. وطلب الجاه والرفعة والمهابة وزيادة التمكين والعظمة. كلّها متعلّقة بالشمس.

والشكل المسبّع. يوم الجمعة وليلة الثلاثاء واوّل ساعتهما. وانواع المحبّة وطلب القلب والفرح واللذّات الشهوانية. كلّها متعلّقة بالزهرة.

والشكل المثمّن. يوم الاربعاء وليلة الاحد واوّل ساعتهما. والفهم والحفظ والفصاحة ورفع الخيالات الفاسدة. كلّها متعلّقة بعطارد.

والشكل المتسّع. يوم الاثنين وليلة الجمعة واوّل ساعتهما. وحصول الصحّة واعتدال المزاج والبرء من المرض وتسكين الالم ورفع العين ونظر السوء والامن من الخوف. كلّها متعلّقة بالقمر.

[وامّا سائر الساعات من الايّام والليالى فلكواكب الباقية حسب ما اقتضى ترتيب افلاكها.] [والشكل المعشّر لفلك البروج.] [وههنا شكلان آخران (49^{v}) فى البيان احدهما مربّع احد عشر للرأس وثانيهما مربّع اثنى عشر للمذنّب.] [وما فوق ⟨من⟩ الاشكال يعتبر على ذلك الترتيب الى غير النهاية.]

ولكلّ من الدوّارىّ بخور ملائم ومعدن مناسب يوضع عليه الشكل المنسوب الى ذلك الكوكب للحاجة المتعلّقة بذلك الكوكب وتبخره عند العمل فى وقته المخصوص.

فبخور زحل عود ولادان وزعفران وقسط وقشور الكندر. ومعدنه الرصاص. ووقته المخصوص وشرفه ٢١ من الميزان.

وبخور المشترى عود وصندل وزعفران وميعة وسندروس وصمغ صنوبر وحبّ الغار. ومعدنه الاسرب والنحاس. وشرفه ١٥ من السرطان.

وبخور المريخ عود صندل احمر وكندر وصبر ودارفلفل. ومعدنه الاسرب والنحاس. وشرفه ٢٨ درجة من الجدى.

وبخور الشمس عود ومسك وزعفران وميعة ولبان ذكر وجلّنار وتربد اصفر. ومعدنه الذهب. وشرفه ١٩ درجة من الحمل.

وبخور الزهرة عود وكافور ومسك ومصطكى وقسط وزعفران ولادان. ومعدنها فضّة والنحاس. وشرفها ٢٧ من الحوت.

وبخور عطارد كندر واشنة وكمّون كرمانى وقشر اللوز. ومعدنه الزيبق المقتول. وشرفه درجة ١٥ من السنبلة.

وبخور القمر عود ولادان وحبّ اللبان واذاخر وحبّ خرنوب وطلق. ومعدنه الفضّة. وشرفه ٣ من الثور.

[وبخور الرأس بخور المشترى ومعدنه. وشرفه ٣ من الجوزاء.]

[وبخور المذنّب بخور زحل ومعدنه. وشرفه ٣ من القوس.]

واذا اردت ⟨عمل⟩ من الاعمال المتعلّقة بواحد من السيارات فخذ لوحًا من المعدن الملائم لذلك الكوكب او رقًّا طاهرًا [او ديباج او ثوب ملوّن مناسب] وارسم فيه اسماء او آية [او حروفًا مناسبة لذلك المقول] فى شرف ذلك الكوكب او يومه او ليلته او ساعته او فى وقته واجتهد حتّى تقع الكتابة ⟨فى⟩ وقت كون القمر متّصلًا بالزهرة اتّصالَ التثليث او التسديس او المقارنة واكتب حول المربّع آية مناسبة واسم المطلب او الحاجة المطلوبة على ظهر الجدول. واذا فرغت من العمل فان كان الطالع الذى اخترته برجًا ناريًا ادفن ذلك اللوح فى محلّ وقود النار. وان كان هوائيًا علقه فى مهبّ الرياح او على نفس الطالب. وان كان مائيًا فادفنه تحت مسيل الماء او اغسله فى الماء واسق ذلك الماء صاحب الحاجة. وان كان ترابيًا فادفنه فى موضع يقرب عليه الشخص [الطالب] . [والاحسن عندنا يشغل بالاسم بقدر عدده عقيب الصلوة مع التوجّه وحضور (50^r) القلب يحصل المراد بلا تزوير] .

[ان شئت امزج اسم الطالب واسم المطلوب وحروف الكوكب واضف اليه عدد اسم مناسب ذلك العمل من الاسماء الحسنى وارسمه فى اللوح او ما مَن وجد من غيره على الوسع وخذ دعاء موازيًا عدده عدد ذلك الوفق تحمله الطالب بعد التبخير وتشغل بذلك الدعاء بعد الاوقات الخمس حتّى يحصل المطلوب والموجود هو اسرع.]

فهذا هو الاصل الكلّى بالعمل فى المربّعات عدديًا كان او حرفيًا. والحمد للّه.

Acknowledgements

We wish to express our gratitude to the two editors, Miquel Forcada Nogués and Cristian Tolsa Domènech, as well as to the former editor, Julio Samsó Moya, for their help and advices during the preparation and revision of this article. We are also mostly indebted to the two anonymous referees for reading carefully this study and suggesting numerous corrections and ameliorations. All this has notably improved the final version of this study.

Bibliography

H.C. Agrippa ab Nettesheym: *De occulta philosophia* (2 vols). Antwerp/Cologne 1531-1533.

R. Dozy: *Supplément aux dictionnaires arabes* (2 vols). Leyden 1881.

G. Freytag: *Lexicon arabico-latinum* (2 vols). Halle S. 1837.

A. Kazimirski, de Biberstein: *Dictionnaire arabe-français* (2 vols). Paris 1860.

E. Lane: *An Arabic-English Lexicon* (8 parts). London 1863-1893.

Picatrix. The Latin version of the «Ghāyat al-Ḥakīm», ed. D. Pingree. London 1986.

J. Sesiano : «Herstellungsverfahren magischer Quadrate aus islamischer Zeit (II)», *Sudhoffs Archiv* 65 (1981), pp. 251–265.

— «Herstellungsverfahren magischer Quadrate aus islamischer Zeit (II′)», *Sudhoffs Archiv* 71 (1987), pp. 78–89.

— «Le traité d'Abūl'-Wafā' sur les carrés magiques», *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften* 12 (1998), pp. 121–244.

— *Les carrés magiques dans les pays islamiques*. Lausanne 2004.

— «Magic squares for daily life», in *Studies in the history of the exact sciences in honour of David Pingree* [= *Islamic philosophy, theology and science*, vol. liv], Leyden 2004, pp. 715–734.

— *Магические квадраты на средневековом Востоке*. Saint Petersburg 2014.

— *Magic squares in the tenth century, two Arabic treatises by Anṭākī and Būzjānī*. Cham 2017.

— *Magic squares, their history and construction from ancient times to ad 1600*. Cham 2019.

— *An ancient Greek treatise on magic squares*. Stuttgart 2020.

Vocabulista in Arabico, ed. C. Schiaparelli. Florence 1871.

H. Wehr: *Arabisches Wörterbuch für die Schriftsprache der Gegenwart* (2 vols). Leipzig 1952.