A Panoramic View of Hebrew Astronomical Tables in the Middle Ages

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Abstract: This survey of the corpus of sets of astronomical tables in Hebrew from the twelfth to the early sixteenth century is intended to complement surveys of astronomical tables in Arabic and Latin. In addition to translations of sets of astronomical tables into Hebrew from Arabic, Latin, and Greek, there were also translations of Hebrew astronomical tables into these languages. But the focus here is on adaptations and original works rather than on translations. Some sets of tables in Hebrew only address the motions of the Sun and the Moon as well as eclipses, and many do not include any astrological material. The impact of the zijes by al-Battānī and Ibn al-Kammād is examined, as well as the impact of the Toledan Tables and the Parisian Alfonsine Tables. A topic in many sets of tables in Hebrew is the determination of the time from mean to true syzygy, for which various solutions were offered. Of particular interest are the tables by Levi ben Gerson of Orange, France (d. 1344), which include a table for the lunar equation based on his innovative lunar model. Another major figure is Abraham Zacut of Salamanca, Spain (d. 1514), whose tables were composed in Hebrew, but were printed in 1496 in Latin without a translation of his Hebrew canons. Rather, some copies have new canons in Latin and others have them in Castilian. Zacut was familiar with astronomical traditions in both Hebrew and Latin, and his tables were translated into Arabic twice.

Keywords: Immanuel ben Jacob Bonfils, Jacob ben David Bonjorn, Isaac Ibn al-Ḥadib, Joseph Ibn Waqār, Judah ben Verga, Ibn al-Raqqām, lunar cycle of 11,325 days

Una visió panorámica de les taules astronòmiques hebrees a l'Edat Mitjana.

Resum: Aquest examen del corpus de conjunts de taules astronòmiques en hebreu dels segles xi al xvi vol complementar els treballs d'aquesta naturalesa sobre taules astronòmiques en àrab i llatí. A més de les traduccions dels conjunt de taules astronòmiques de

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l'hebreu a l'àrab, llatí i grec, es feren traduccions de taules astronòmiques hebrees a aquestes llengües. El centre d'atenció, no obstant, radica a les adaptacions i obres originals més que a les traduccions. Alguns conjunts de taules en hebreu només tracta els moviments del sol i de la lluna, així com dels eclipsis, i molts d'ells no contenen cap mena de materials astrològics. S'examina l'impacte de les taules de les d'al-Battānī i Ibn al-Kammād, juntament amb la influència de les Taules Toledanes i les Taules Alfonsines parisenques. Un dels temes en molts conjunts de taules en hebreu es la determinació del temps des de la sizígia mitjana a la vertadera, per a la qual s'oferien diverses solucions. Són especialment interessants les taules de Levi ben Gerson d'Orange (m. 1344), que inclouen una taula per a l'equació lunar basada en un model lunar innovador. Una altra figura destacada es Abraham Zacut de Salamanca (m. 1514), les taules del qual van ser escrites en hebreu però impreses en llatí al 1496 sense una traducció dels canons en hebreu. Algunes còpies presenten nous cànons en llatí i altres en castellà. Zacut estava familiaritzat amb les tradicions astronòmiques tant en hebreu com en llatí i les seves obres van ser traduïdes a l'àrab dues vegades.

Paraules clau: Immanuel ben Jacob Bonfils, Jacob ben David Bonjorn, Isaac Ibn al-Ḥadib, Joseph Ibn Waqār, Judah ben Verga, Ibn al-Raqqām, cicle llunar de 11,325 díes.

1. Introduction

The aim of this paper is to highlight the corpus of Hebrew astronomical tables, including references to detailed studies of individual topics. It is not intended to summarize the relevant literature. This survey is mostly based on my own examination of the manuscripts that contain these tables.[1]

Astronomy in the Middle Ages included the study of a variety of topics, such as astronomical models, astronomical instruments, observations, trigonometry, introductions to astronomical theories, chronology, astrological matters, cosmological issues such as the sizes and distances of the planets, etc. But here we focus on sets of astronomical tables produced in Hebrew that are analogous to sets of tables in Arabic, Persian, Latin, and Greek, as well as a few in vernacular languages. These sets of tables (Heb. *luḥot*) may be called zijes,

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[1] Since the 1990s I have worked closely with José Chabás on matters concerning medieval astronomical tables, and most of what is presented here is based on our joint efforts. In particular, he wrote first drafts of the sections on the Tables of Barcelona, Jacob ben David Bonjorn, Isaac Ibn al-Ḥadib, and Jacob ben Makhir, and provided useful comments on the entire text.

from the Arabic term *zīj* for an astronomical handbook, that is, a set of tables together with their instructions, or canons.[2] E. S. Kennedy (1956) called attention to this genre in the Islamic world by listing over 100 zijes from the eighth to the fifteenth century with detailed descriptions of a few of them. Pioneering work had already been done, notably by H. Suter (1914) who published the Latin version of the zij of al-Khwārizmī (Baghdad, fl. 840) and C. A. Nallino (1899–1907) who edited and translated into Latin the zij of al-Battānī (Raqqa, d. 929). The structure of zijes is modeled on Ptolemy's *Handy Tables* (Stahlman 1960) which, in turn, is based on Ptolemy's *Almagest* (Toomer 1984). In general, Ptolemy's parameters were not challenged in medieval zijes, but there are exceptions, notably, the parameters for mean motions and the solar model, the motion of the fixed stars, and the obliquity of the ecliptic. Some other parameters were also modified, such as the eccentricity and apogee of Venus to conform with the parameters for the Sun. In the ninth century changes in the parameters were based on specific dated observations, but subsequently (with a few notable exceptions) changes mainly affected presentation with the goal of making the tables more user-friendly (on user-friendliness see, e.g., Chabás and Goldstein 2012, pp. xix, 1, 5, 80, 147, 210).

Compiling zijes in Hebrew began with Abraham Bar Ḥiyya in the twelfth century and, for purposes of this paper, the end-point is the early sixteenth century. Most of this activity took place in the Iberian peninsula and southern France (Samsó 2002, and Freudenthal 1995). Some of the Hebrew zijes were translated or adapted from Arabic, Latin, Greek, or Persian, but most are original works, that is, they are not translations. The Hebrew zijes, excluding translations, depend on (or take as their point of departure) a variety of sources: Ptolemy's *Almagest*, the zij of al-Battānī, the zij of Ibn al-Kammād (fl.

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[2] The term *zīj* occurs in at least one medieval Hebrew astronomical text. In the introduction to his *Ḥuqqot šamayim* (*Ordinances of the heavens*), Judah ben Asher II (d. 1391) refers to the scholar Isaac b. Sid who composed «the *z[ī]j* of Alfons[o] (זג' אלפנש)» (Vatican, Biblioteca Apostolica Vaticana [BAV], MS Heb. 384, fol. 285b:2–3; Langermann 1983, p. 623 [in Hebrew]; see also sections 11 and 12). The zij of Alfonso refers to the Alfonsine Tables composed in Castilian in Toledo ca. 1272 by Isaac ben Sid and Judah ben Moses ha-Cohen under the patronage of King Alfonso X (d. 1284: see Chabás and Goldstein 2003). Despite the importance of the zij compiled by these two Jewish scholars, it is not addressed here, for it was not in Hebrew. The same applies to the tables compiled by Ibn Ezra (Tudela, d. 1167), which are not extant in Hebrew. For zijes in the Islamic world, see King et al. 2001.

1116), the Toledan Tables (twelfth century), the Parisian Alfonsine Tables, and the tables of Nicholaus de Heybech (Erfurt, fl. 1392). The *Almagest* was translated from Arabic into Hebrew ca. 1235 by Jacob Anatoli (Steinschneider 1893, p. 523), but it has not been included in this survey of Hebrew zijes and, in general, little attention is given to translations. In several cases, later Hebrew zijes explicitly depended on earlier Hebrew zijes: see, e.g., Abraham Zacut (secton 15).

It is also important to distinguish tables based on an algorithm from lists where the entries are not based on such a procedure. The focus here is on tables based on algorithms that are usually represented by mathematical functions in the secondary literature. In an astronomical context the most common lists that are not based on an algorithm are those for fixed stars with their celestial coordinates and those for cities with their geographical coordinates. In the corpus of Hebrew zijes there are relatively few lists not based on an algorithm: one such list is for the colors of eclipses that derives from an Indian tradition: see, e.g., Ibn al-Kammād, the Tables of Barcelona, Juan Gil, Ibn al-Ḥadib, and the Parisian Alfonsine Tables (Goldstein 2005; Chabás and Goldstein 2015b, pp. 629–630). For star lists see, e.g., Abraham Bar Ḥiyya, Levi ben Gerson, the Tables of Barcelona, and Abraham Zacut. For geographical lists see, e.g., Isaac Israeli, the Tables of Barcelona, Judah ben Verga, and Abraham Zacut.[3] Both the star catalogue and the geographical list in the *editio princeps* of the Parisian Alfonsine Tables (Ratdolt 1483) are found in two manuscripts of the Hebrew translation. There is also a Hebrew manuscript that includes a list of cities in Italy with their coordinates (Goldstein 2018, pp. 269–270).

Some Hebrew zijes include astrological lists as well as astrological tables, notably Abraham Zacut's *Ḥibbur*, but most do not include any astrological material. On the role of astrology in medieval Jewish thought as well as the controversies concerning its practice, see Leicht 2011 and Sela 2011. Also, trigonometric and planetary tables are absent in many Hebrew zijes.

In addition to translations into Hebrew, several Hebrew zijes were translated into other languages, notably, Latin, Greek, and Arabic. Translations and adaptations in Latin are the most common, e.g., Abraham Bar Ḥiyya, Jacob ben Makhir, Immanuel ben Jacob Bonfils, Levi ben Gerson, and Abraham Zacut. Levi's tables

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[3] A geographical list has been incorrectly ascribed to Bar Ḥiyya: it properly belongs to the *Tables of Barcelona* (Goldstein 2019c).

were also translated into Provençal (Mancha 1998). Tables translated into Byzantine Greek include those by Bonfils, Bonjorn, and Ibn al-Ḥadib (Tihon 2017). And the tables of Abraham Zacut were translated into Arabic twice (Samsó 2002–2003, 2004). In one case, a set of tables in Latin, the Tables of Novara (a variant of the Toledan Tables), was translated into Arabic written in Hebrew characters (Goldstein 1979, pp. 34–35). A manuscript worthy of special attention, Munich, Bayerische Staatsbibliothek, MS Heb. 343, contains a small «library» of Hebrew astronomical tables (Steinschneider 1895, pp. 188–195).

Arabic zijes available in al-Andalus and the Maghrib were often the basis for Hebrew zijes compiled in western Europe, but the situation in the eastern Mediterranean was different. The *Īlkhānī* zij of Naṣīr al-Dīn al-Ṭūsī (Iran, d. 1274) was translated into Hebrew by Solomon ben Elijah (Thessalonika, late fourteenth century) as the Persian Tables via a Byzantine Greek intermediary (Goldstein 2020–2021); the *Šamil* zij, probably by Athīr al-Dīn al-Abharī (Iran, fl. 1240), was translated anonymously from Arabic into Hebrew (Goldstein 2020–2021); and the zij of Ulugh Beg (Samarkand, ca. 1440) was translated anonymously into Hebrew ca. 1500 from either Persian or Arabic (Goldstein 1979, pp. 38–39; Goldstein 2020). In the canons to his Hebrew zij, Abraham ben Yom Ṭov Yerushalmi (Istanbul, fl. 1510) remarked that his tables (not extant) were based on Ulugh Beg's zij (Goldstein 2020). These zijes in Arabic and Persian compiled in the eastern Islamic world were not available in al-Andalus or the Maghrib.[4]

A topic that still needs attention is the variety and evolution of technical vocabulary in Hebrew zijes as well as in Hebrew astronomy generally. Sarfatti (1968) is the best account of Hebrew mathematical terminology and it could serve as a model for usages in astronomy. For examples of the diversity in Hebrew terminology in trigonometric and planetary tables, see Goldstein 2019a and 2019b.

Although chronological order is generally followed for the sequence of sections, we begin with Levi ben Gerson for two reasons. First, he was one of the most innovative astronomers in the Middle Ages and, second, his *Astronomy* contains more than just tables and canons (see, e.g., Goldstein 1997). An extract of many of the tables in his *Astronomy* with canons is extant in both He-

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[4] A copy of Kūshyār ibn Labbān's *Jāmiʿ zīj* is extant in Hebrew characters: see Dalen 2021, p. 31.

brew and Provençal versions (Goldstein 1974, p. 77; Mancha 1998), and this extract meets our criteria for a zij. Here we address his tables without restriction to those in the extract (for a list of Levi's tables in Hebrew manuscripts, see Goldstein 1974, pp. 78–79). In 1974 I analyzed the tables of Levi ben Gerson that seemed to be most unusual by medieval standards, for they include many innovations. I can now say that, after examining the whole corpus of medieval Hebrew zijes, they are indeed as unusual as I had considered them over 45 years ago.

2. LEVI BEN GERSON: D. 1344

Levi ben Gerson (Gersonides; Leo de Balneolis) lived in Orange, southern France, and was among the most original Jewish thinkers of the Middle Ages. He contributed to philosophy, mathematics, and astronomy, as well as to Biblical exegesis: see, e.g., Freudenthal 1992. Our focus here is on his astronomical tables, one aspect of his contribution to astronomy: for other aspects of his work in astronomy, see, e.g., Goldstein 1985a, Goldstein and Hon 2020, and Glasner 2015. These tables are mostly embedded in his *Astronomy* which forms Book 5, Part 1, of his *Wars of the Lord*. There was an incomplete Latin version, probably made with Levi's assistance, that sometimes includes information missing in the Hebrew original: Mancha 1992. The edition of Levi's tables contains 45 tables (some with variants), all of which seem to have been recomputed by Levi even in cases where the underlying parameters were used by previous astronomers: Goldstein 1974. The tables are restricted to trigonometry and the daily rotation, equation of time, solar motion, lunar motions and syzygies, parallax, and eclipses. There is no star list in the Hebrew version of Levi's *Astronomy*, but there is such a list for 1336 in paragraph form in the Latin version (Mancha 2002), and a list for 1325 in Levi's *Ḥug shamayim* (Goldstein 1985b, pp. 200–202). The text of Levi's *Astronomy* also describes a table for converting dates in the Julian calendar to dates in the Hebrew calendar, but the corresponding table is not extant. Rather, a different table for this conversion appears in one manuscript (Paris, Bibliothèque nationale de France, MS Heb. 724), and it is for the cycle that began in 5036 AM (= 1275/1276): Goldstein 1974, pp 104–105, 168–169. The source for this table has not been determined, although a marginal note in the manuscript claims that it is due to Jacob ben Makhir (see section 4).

2.1. Trigonometry and the daily rotation

The sine function is tabulated at intervals of 0;15° and, when compared with modern recomputation, Levi's values are generally correct to within a second. Levi's table for declination is based on an obliquity of 23;33,8°. This value for the obliquity is already ascribed to Abraham Ibn Ezra in the tables of Abraham Bar Ḥiyya, but the entries in their respective tables are not the same: in fact, Levi's entries are more accurate, indicating that he recomputed them. There are also tables for right and oblique ascension for various geographical latitudes, and a table for the half-increment in longest daylight for these geographical latitudes.

2.2. Equation of time

Levi tabulated the equation of time, defined by the formula:

$$\text{apparent time} = \text{mean time} + E,$$

where E is the equation of time, and $E \geq 0$. The entries are in time degrees, where 1h = 15°. The underlying parameters are: solar apogee Cnc 3°; solar eccentricity 2;14; obliquity 23;33°; and $E_0 = 4;2°$ at Aqr 20°.

2.3. Solar motion

The mean solar motion is 0;59,8,20,8,44,6,3,14°/d, and the radix is given for epoch noon, last day of February 1301 at Orange. The table is arranged for the Julian calendar: single years, steps of 60 years, and steps of 1800 years; months, days, hours, and minutes of an hour. There are two tables for the solar equation, one with a maximum of 2;8,0° (corresponding to an eccentricity of 2;14), and a revised version with a maximum of 2;17,0° (corresponding to an eccentricity of about 2;23), based on eclipse observations reported in chap. 100 of his *Astronomy*: see Goldstein 1979b. There is also a table for the daily solar positions in 1321 (old style, where the year began on March 1, 1321 and ended on February 28, 1322), with rules for adjusting this table for other years. The recomputed underlying parameters are: solar apogee Cnc 3°, and maximum equation 2;17°. One Hebrew manuscript and the Provençal version include a variant table for the daily solar positions, where the recomputed underlying parameters are: solar apogee Cnc 2;37°, and maximum equation 2;8° (Goldstein 1974, pp. 146–148, 244–245; Mancha 1998, p. 322). Since Levi explicitly updated the maximum solar equation from 2;8° to 2;17°, it follows that the variant table was compiled by Levi at an earlier date.

2.4. Lunar motion and syzygies

The lunar mean motions are arranged as the solar mean motion, but there are four columns of entries: motion in longitude, anomaly, double elongation from the Sun, and mean lunar node: for the daily motions, see Goldstein 1974, pp. 106–108.

Levi's greatest achievement in this domain was his construction of an original «complete» lunar theory based on his own observations, and then compiling a table for ease of computing positions based on it. A «complete» lunar theory refers to a theory that applies to all lunar phases, whereas a «simple» lunar theory only applies to syzygies (conjunctions and oppositions of the Sun and the Moon). Levi argued that a lunar epicycle entails seeing both sides of the Moon, contrary to experience; hence, his theory only used eccentrics. Moreover, he argued that Ptolemy's lunar theory produces a variation in the distance and the apparent size of the Moon in excess of what is observed, and that there is a systematic discrepancy at the octants between observations and Ptolemaic theory. Levi's lunar theory met all these requirements and was at least as successful as Ptolemy's theory; surprisingly, Levi's theory was not discussed by subsequent astronomers, although there was nothing comparable to it in the Middle Ages in Hebrew, Latin, or Arabic: Goldstein 1974, pp. 53–74, 212–217.

Particularly noteworthy are the two variants of tables for lunar velocity, one of which is based on Levi's simple lunar theory, and the other seems to be based on applying what is now called Regiomontanus's rule to Levi's complete lunar theory. It was most unusual in the Middle Ages to apply Ptolemy's complete lunar theory to computing velocities: see Goldstein 1974, pp. 108–116, 182; Goldstein 1992; Goldstein 1996. Following the general practice in the Middle Ages, the basis for these tables (that is, the underlying parameters and the rationale for the algorithm) is not discussed and has to be reconstructed.

Tables of syzygies display all mean syzygies beginning March 14, 1321, with a column indicating whether the syzygy is a conjunction or an opposition; and columns for the date, weekday number, hour, and motion in anomaly. There are four tables: the first table is for syzygies in the 8 years (corresponding to 99 mean synomic months) beginning in 1321, the second table extends the first table for the accumulated motion after each of 69 periods of 99 mean synodic months, the third and fourth tables extend the first two tables to a total of 273,240 mean synodic months or more than 600,000 years! Levi's value for the mean synodic month is based on comparing a lunar eclipse that he observed in 1335 with an eclipse reported in the *Almagest* that took place in 134: his result is 29;31,50,7,54,25,3,32d (Goldstein 2003a, pp. 70–71).

For the time from mean to true syzygy, Levi has four tables: the argument in the first two tables is the day of the year (corresponding to the solar longitude), the argument of the third table is the lunar anomaly, and the two arguments of the fourth table are the day of the year and the lunar anomaly. The first two tables just depend on the solar longitude and include the equation of time, the third table depends on the lunar anomaly, and the fourth table displays the remainder that results from using only one of the variables in each of the previous tables. This set of four tables is an unusual solution to a problem involving two variables. In one Hebrew manuscript and the Provençal version each of the four tables is represented by a variant, but the algorithm underlying these variant tables has not been discovered: Mancha 1998, pp. 320–322. For a survey of medieval procedures for finding the time from mean to true syzygy, see Chabás and Goldstein 2015a, pp. 40–56; see also the section 10.

Levi included a double argument table for lunar positions, requiring him to carry out a great many computations, based on the table for his complete lunar model: one argument is the number of days since the most recent syzygy from 1d to 14d, and the other is the lunar position in anomaly at intervals of 10°. The number of days since since syzygy corresponds to the double elongation of the Moon from the Sun, which is one of the usual arguments for computing the correction to a lunar position. The total number of entries, given to seconds, is 504 (= 36 × 14). To find the true lunar longitude one then needs to add the entry found in this table (with interpolation) to the mean lunar longitude at the preceding syzygy.

2.5. *Parallax*

Levi has a preliminary table for the total parallax as a function of the Moon's zenith distance, from which the components in longitude and latitude are computed. The underlying formula is:

$$\pi \approx 0;53,20 \sin z$$

where π is the total parallax, 0;53,20° is the total horizontal parallax, and z is the Moon's zenith distance. For Levi the total adjusted horizontal parallax is 0;53,20°, whereas for Ptolemy and al-Battānī it is 0;50,43°. Levi's parallax tables are arranged in the same form as in al-Battānī's zij (as well as in Ptolemy's *Handy Tables* which al-Battānī simply copied) for a distinct set of geographical latitudes, but Levi displayed the parallax components in longitude and latitude in minutes and seconds rather than just minutes (Nallino 1899–1907, 2:95–101; Goldstein 1974, pp. 117–122, 185–207). Levi also modified the underlying parameters which meant that all the entries had to be recomputed. Although Levi does not say so explicitly, the entries are for the adjusted parallax, that is, the difference between lunar and solar parallax, which is only valid at the time of a solar eclipse.

2.6. Eclipses

There are two tables for both solar and lunar eclipses that display the digits of eclipse and the arcs corresponding to the duration of phases. In the case of lunar eclipses there is also a column for the time between true opposition and eclipse-middle. For both kinds of eclipses there is one table for the Sun at apogee and one for the Sun at perigee. An additional table is provided for interpolating between solar apogee and perigee, where the maximim solar equation is 2;8°. In contrast to Ptolemy's lunar theory, in Levi's lunar theory there is hardly any variation in lunar distance; hence, the lunar distance and the size the lunar disk are taken to be constant in all the eclipse tables: Goldstein 1974, p. 131. Levi compared the circumstances of eclipses he observed with computations based on his tables, and adjusted his parameters accordingly: Goldstein 1979b.

3. ABRAHAM BAR ḤIYYA: D. CA. 1140

Abraham Bar Ḥiyya of Barcelona also known as *ha-nasi'* (the prince) was responsible for the first works in Hebrew on mathematics and astronomy based on Arabic sources, creating a new technical vocabulary in Hebrew for this purpose (Millás 1959, pp. 19–20; Sela 2003, pp. 96–104). The focus here is restricted to his zij, called *Luḥot ha-nasi'*: the canons were edited and translated into Castilian by Millás (1959) together with a list of the tables and transcriptions of a few of them. As Langermann (1999, essay 1, p. 15) noted, the canons and the tables do not appear in the same manuscripts. Millás (1959, p. 110) suggested that Bar Ḥiyya's tables were simply an adaptation of the corresponding tables in the zij of al-Battānī. However, as we shall see, a close inspection of a few of these tables reveals a more complex relationship with the previous literature.

The tables themselves have not been edited, but many of them have been discussed by Mercier (2014, pp. 164–181) who compared them with a Latin translation. The zij begins with a set of tables for the Hebrew calendar and conversion between this calendar and others, followed by tables for sines, declinations, shadows (= cotangents), and oblique ascensions. The table for the equation of time is unusual and differs from the corresponding table in the zij of al-Battānī; in fact, the only other known occurrence of this table is in Bonfils's *Tables for 1340* (see section 6; Chabás and Goldstein 2012, pp. 39–41; and Goldstein and Chabás 2017a, pp. 76–78). Oddly, there are two tables for declination: Abraham Bar Ḥiyya is named in the heading of the first table with a maximum 23;51,20°, and Abraham Ibn Ezra is named in the heading of the second table with a maximum of 23;33,8° (Goldstein 1974, p. 96).

The tables of mean motion use Egyptian years of 365 days (12 months of 30 days plus 5 extra days) and months of 30 days, but the era used is that of Creation (as in the Hebrew calendar), and the epoch of these tables is the beginning of 19-year cycle 257 corresponding to Sept. 21, 1104 (= Elul 29, 4865 AM; Mercier 2014, 165–168). This differs from al-Battānī's presentation, which uses the Hijra calendar (Nallino 1899–1907, 2:19–28). The mean solar motions were rounded from the entries in the corresponding tables in the *Almagest*, which indicates that Bar Ḥiyya accepted Ptolemy's tropical year length of 365;14,48d (= 365¼ – 1/300). The mean motions of the Moon in longitude, anomaly, and elongation, as well as the mean motions of the planets, were also derived from the corresponding tables in the *Almagest*. For a comparison of the mean motions in the *Almagest* with those in the zij of al-Battānī, see Chabás and Goldstein 2012, pp. 57–59. Mercier added that «there are serious errors» in the Hebrew version of mean motion for the lunar node (Mercier 2014, pp. 170–172). However, the radices for the mean motions were derived from the zij of al-Battānī for noon on Sept. 21, 1104 CE. According to Bar Ḥiyya, these radices are for Jerusalem (Millás 1959, p. 16, Heb. text p. 50), but Mercier demonstrated that they were computed from the zij of al-Battānī without a correction for the difference in time between Raqqa (used by al-Battānī) and Jerusalem (Mercier 2014, p. 174, 176). It seems that Bar Ḥiyya may have assumed (incorrectly) that al-Battānī's tables were computed for the meridian of Jerusalem or, more likely, he assumed that Raqqa and Jerusalem lie on the same meridian. Bar Ḥiyya's values for the planetary apogees are close to those derived from the zij of al-Battānī for 1104, but the value for the solar apogee is 10° less than al-Battānī's value (Mercier 2014, p. 180). The tables for the equations of the Moon and planets are all taken from al-Battānī, but the solar equation has 2;23° as its maximum, as in the *Almagest* (Mercier 2014, p. 181). There follow tables for lunar and solar eclipses and parallax as well as for planetary latitudes and planetary visibility that correspond to tables in the zij of al-Battānī (Millás 1959, pp. 113–114). Bar Ḥiyya's list of positions for 1104 for stars of the first and second magnitude is given in Millás 1959, pp. 124–125, and in Goldstein 1985b, pp. 186–191; see also Sela 2016–2017. The entries were taken from the zij of al-Battānī, with an increment for precession of 3;20° in their longitudes.

A careful analysis of all of Bar Ḥiyya's tables, based on all the manuscripts, remains to be undertaken.

4. Jacob ben Makhir: d. ca. 1305

Jacob ben Makhir Ibn Tibbon belonged to the prominent Ibn Tibbon family who moved from al-Andalus to Languedoc, southern France, in the twelfth century. Members of this family over several generations are well known for their many contributions to the transmission of Arabic culture in the Iberian Peninsula into Hebrew. Jacob was also known by his vernacular name, Profeit Tibbon, later rendered in Latin as Profatius. Although probably born in Marseille, he was active in Montpellier both as an author and translator.

Jacob compiled a set of tables entitled *Almanac*, consisting of a series of tables to compute the true positions of the planets and the two luminaries that are accompanied by a text including a prologue and canons (Boffito and Melzi d'Eril 1908; Chabás and Goldstein 2021). It was originally written in Hebrew and diffused widely in Latin. The Hebrew prologue (Steinschneider 1876, pp. 607–614) differs significantly from the prologue in Latin, of which there are two versions. It has been suggested that the translator was Armengaud Blaise (Montpellier, ca. 1264–1312), with the help of Jacob himself. These two scholars also collaborated in the translation from Hebrew into Latin of a text on an astronomical instrument called the quadrant of Israel, composed by Jacob.

The tables of the *Almanac* use signs of 30° and were computed for the coordinates of Montpellier, beginning in 1300 or 1301. Tropical coordinates are also used. Toomer (1973) claimed that the planetary longitudes in Jacob's tables depend on the Toledan Tables, despite the fact that those tables use sidereal coordinates. This claim was recently confirmed by computing the appropriate values for precession, that is, the difference between a sidereal longitude and a tropical longitude at the given time, by means of tables ascribed to Thābit Ibn Qurra (d. 901: Goldstein 1994; Chabás and Goldstein 2022).

The entries in the tables for the true positions of the celestial bodies are given at intervals of a few days, in cycles of 59 years (Saturn), 83 years (Jupiter), 79 years (Mars), 8 years (Venus), and 46 years (Mercury), as well as the corrections to be added or subtracted to the initial positions after completion of these cycles, following the pattern set in other almanacs, such as that ascribed to Azarquiel (al-Andalus, eleventh century: Millás 1943–1950; Boutelle 1967). Jacob's tables include more than 12,000 entries, given to minutes. There are four tables for the Sun displaying daily positions for the period 1301–1304, given to seconds. Inspection of successive entries in this table shows that Jacob computed only one out of five entries, and found the intermediate values by interpolation.

The tables for the Moon are far more interesting (Chabás and Goldstein 2022). The table for the daily true lunar anomaly is the largest in this set and contains more than 8,500 entries, from March 1300 to December 1323. It consists of 294 columns, one for each lunar month of 30 or 29 entries. The entries in this table serve as argument in a double argument table, also unprecedented, displaying the complete lunar equation. The entries in this table have a vertical shift of 7;40°, resulting from the addition of the maximum values of the equation of anomaly (5;1°) and the lunar increment (2;39°), with the goal of avoiding subtractions. This table is the first known example of a double argument table in the West with a vertical shift, and was later used by Joseph Ibn Waqār (see section 8) and by Moses Farissol Botarel (see section 14). These two tables for the Moon are clear examples of a common approach to table making in the Middle Ages: neither the model nor the underlying parameters are changed; rather, user-friendliness is enhanced by means of innovation in the presentation and the inclusion of more entries.

There are also tables for the motion of the lunar node and lunar latitude, and for computing the circumstances of solar and lunar eclipses. It is worth noting that the lunar eclipses are addressed in a single table, in contrast to most zijes dealing with eclipses which present two separate tables. The table for the hourly velocities of the Moon and the Sun also displays entries for the length of half-daylight. The table for the hourly velocity of the Moon relative to the Sun at any time consists of 13, 14, or 15 columns, depending on the manuscript, where the first column displays the difference between the hourly velocities of the luminaries, from 0;27,50°/h to 0;33,20°/h (or 0;33,30°/h) at intervals of 0;0,10°/h, and the rest of the columns display multiples of them. This kind of table is already found in the zij of Ibn al-Kammād (see section 8).

Parallax is addressed in a single table for geographical latitude 43°, corresponding to Montpellier, and it is intended to be used in computing the circumstances of a solar eclipse; it corrects the true position of the Moon to its apparent position for an observer at a given geographical latitude. It consists of 12 subtables, one for each zodiacal sign, displaying the components of parallax in longitude and latitude as a function of time before or after noon. But, unlike the tables which give the components of parallax to minutes, Jacob gives them to minutes and seconds. This unusual precision also occurs in the tables of Levi ben Gerson (see section 2). In contrast to the table for lunar eclipses, there are two subtables for solar eclipses, one for greatest distance of the Moon and another for least distance; the argument is the lunar latitude, following the pattern of Ptolemy's *Handy Tables*. The same table (with variants) appears in Bar Ḥiyya's tables (see section 3).

In short, Jacob ben Makhir computed his *Almanac* with the Toledan Tables rather than with any other zij available at the time, such as the *Almanac* of Azarchiel or the zij of al-Battānī. In his *Almanac*, Jacob used tropical coordinates, and thus he had to compute precession to adjust the sidereal coordinates found in the Toledan Tables or a version of them. Jacob computed the positions of the planets to minutes, thus increasing the precision found in previous almanacs and enhancing the user-friendliness of the table. Moreover, Jacob adjusted the true positions of the celestial objects to Montpellier, by applying a distance from that city to Toledo that has not been found in the previous literature.

5. ISAAC ISRAELI OF TOLEDO: EARLY FOURTEENTH CENTURY

Isaac ben Joseph Israeli of Toledo (also known as Isaac Israeli the Younger) is best known for his *Yesod ʿolam* (*Foundation of the world*; ed. Goldberg and Rosenkranz 1846–1848), a treatise on astronomy and chronology, in which he mentions four eclipses observed by Isaac ben Sid, one of the authors of the Castilian Alfonsine Tables working under the patronage of King Alfonso X (reigned 1252–1284): Chabás and Goldstein 2003, pp. 141–143. Here the focus is on his astronomical tables of which there are three versions: *Yesod ʿolam* dated 1310, *Šaʿar ha-šamayim* (*The gate of heaven*) dated 1320, and *Šaʿar ha-milluʿim* (*The supplementary gate*) dated 1330: see Goldstein and Chabás 2017b. Isaac includes tables for oblique ascensions and parallax for the latitude of Toledo, agreeing with the corresponding tables in the Toledan Tables. There is also a list of planetary apogees «in our time» that seems to depend on Ibn al-Kammād (see section 8). Isaac's tables for the planetary mean motions are given in the Julian calendar for intervals of 28 years beginning in 1301, and then for single years from 1 to 28, months, days, and hours. In general, the mean motions are close to those in the *Almagest*, and the solar lunar, and planetary equations agree with those in al-Battānī's zij. In *Yesod ʿolam* there is a list of 75 cities with their coordinates, where longitude is measured from the «meridian of water», 17;30° west of the Canary Islands (Ptolemy's prime meridian): see Goldstein 2018. The tables for solar and lunar eclipses agree with those in the *Almagest*, but the argument of lunar latitude is measured from the lunar node whereas in the *Almagest* it is measured from the northern limit. Of special interest are tables for conjunctions and oppositions of the Sun and the Moon at intervals of 30 years in the Hijra calendar, beginning in 601 AH, that is, August 29, 1204, followed by tables for years and months in the Hijra calendar. These tables were taken from the Toledan Tables: this borrowing is surprising since the Toledan Tables are sidereal whereas elsewhere Isaac used tropical coordinates. In conclusion, there are no innovations in Isaac's tables. He depended on a variety of sources which are not specified in any of his texts, notably Ptolemy, al-Battānī, Ibn al-Kammād, and the Toledan Tables. It is noteworthy that none of Isaac's tables is closely related to those in the Alfonsine Tables, despite his familiarity with Isaac ben Sid's observations.

6. Immanuel ben Jacob Bonfils: fl. 1340–1365

Immanuel ben Jacob Bonfils lived in Tarascon-sur-Rhône, southern France, and is best known for his zij called *Six wings* (*Šeš kenafayim*), which was among the most popular zijes in Hebrew, judging from the large number of surviving copies. This zij was published in Hebrew in Zhitomir in 1872. It was also translated into Latin in 1406, and into Byzantine Greek in 1435 (Solon 1970, p. 1), as well as into Russian in the late fifteenth century (Taube 1995, pp. 174, 177): the Greek version was edited by Peter Solon in his unpublished thesis of 1968 and a summary of it appeared in Solon 1970. There is little information about Bonfils's biography: in one manuscript he is said to have taught astronomy in the school (*beyt ha-midraš*) in Orange, where Levi ben Gerson had lived a generation earlier (Goldstein 1974, p. 28). For an extensive discussion of Bonfils's works, see Renan and Neubauer 1893, pp. 692–697; for his contributions to mathematics see, e.g., Lévy 2003. In addition to his *Six wings*, Bonfils compiled a zij for epoch 1340 that will be discussed below.

The *Six wings* consist of 6 tables, each called a wing, and they are restricted to the motions of the Sun and the Moon and eclipses.[5] Wing 1 displays mean times and positions at mean syzygy, arranged for the Hebrew calendar. Wing 2 is a double argument table that displays the correction in time and longitude from mean to true syzygy. Wing 3 displays a table for the length of daylight. Wing 4 is a table for lunar eclipses. Wing 5 displays a table for parallax. And Wing 6 displays a table for solar eclipses. Specifically, in Wing 1, there are separate tables for conjunction and opposition for 19-year cycles, single years from 1 to 19, and months in the Hebrew calendar (distinguishing ordinary years from embolismic years). The entries are the time of mean conjunction/opposition, mean position of the luminaries, solar anomaly, lunar anomaly, and the argument of lunar latitude. Solon showed that the underlying length of the mean synodic month is 29;31,50,8,20 days, a Babylonian value that was also used by Ptolemy (see Goldstein 2003a, p. 71). Wing 2 has two arguments: (1) the argument of the Sun, and (2) the mean lunar anomaly; the entries are the times between mean and true syzygy plus 24;16h, and the differences in longitude between mean and true syzygy plus 3°. The canons tell the user that 24h and 3° are to be subtracted from the tabular entries, a procedure for avoiding negative quantities. The value 0;16h indicates that the equation of time is included in the entries, as stated in the canons. As we have seen, the problem of the differences in the time and longitude between mean and true syzygy was of great interest to medieval astronomers, and various solutions were offered (Chabás and Goldstein 2015a, pp. 2–56; on Bonfils, see pp. 51–52). Although Bonfils's table is structured differently from the analogous tables by Levi ben Gerson, the initial value of 24;16h (Levi: 24;17h) for solar and lunar anomalies equal

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[5] In a private communication, Richard L. Kremer indicated that he is working together with Josefina Rodríguez-Arribas on an edition, with translation and commentary, of the Hebrew text of Bonfils's *Six wings*.

to 0° and the subtraction of 24h at the end of the computation are common to both of them (see Goldstein 1974, pp. 141–142).

Wing 3 has as its argument the solar longitude, where the entry is the length of half-daylight with maximum 7;40h at Gem 30° (= Cnc 0°). The underlying parameters, according to Solon, are 23;51°, Ptolemy's value for the obliquity of the ecliptic, and geographical latitude 44°, Bonfils's value for the latitude of Tarascon. Wing 4 for lunar eclipses has an unusual presentation, but its entries can be obtained directly from the *Almagest* (VI.8–9), using the solar and lunar velocities in al-Battānī's tables (Nallino 1899–1907, 2:88). The vertical argument is the argument of lunar latitude in steps of 0;30° from 12° to 0°; the columns are headed 0s, 1s/11s, 2s/10s, ..., 6s. The entries are the digits of eclipse, the time from outer contact to inner contact or eclipse-middle, and the time from inner contact to eclipse-middle. Wing 5 for the components of parallax in longitude and latitude has 12 subtables, one for each zodiacal sign, and follows the pattern in Ptolemy's *Handy Tables* and in al-Battānī's zij. The entries are all in hours and minutes. Wing 6 is for solar eclipses and displays the digits of eclipse and the time from outer contact to eclipse-middle, where the argument is the argument of lunar latitude from 7° to 0° in steps of 0;30°.

A second zij by Bonfils for epoch 1340 survives in only four copies, and the canons are related to another work by Bonfils that has often called in modern times (without support in the text) *ʿErekh ha-ḥilluf* (the equation of time); in one manuscript of this text it is called «My Treatise on the Reasons for the Tables». Here we will refer to it simply as Bonfils's *Treatise* (Goldstein and Chabás 2017a, p. 72). In the *Treatise* Bonfils mentioned the following astronomers: Ptolemy, al-Battānī, Abraham Bar Ḥiyya, Abraham Ibn Ezra, and Levi ben Gerson. In the canons to the *Six wings* only Ptolemy, al-Battānī, and Ibn Ezra were mentioned. In the canons to the tables for epoch 1340, Bonfils mentions Ptolemy, al-Battānī, Ibn Ezra, and Levi ben Gerson. The three astronomical works, the *Six wings*, the *Treatise*, and the *Tables for 1340* are closely related (for textual issues, see Goldstein 2017). Some of the columns in Wing 1 are the same as those in one set of mean motions in the *Tables for 1340*, except that the entries in the former are to minutes (of arc and time) whereas in the latter to seconds (of arc and time). In the *Treatise*, Bonfils refers to the *Six wings*, and passages in the *Treatise* are also found in the canons to the *Tables for 1340*. Bonfils's name appears several times in one manuscript of the *Tables for 1340*, but not at all in another manuscript of these tables that is missing much of the text of the canons (Goldstein and Chabás 2017a, p. 74). In the

Treatise, the coordinates of Tarascon are given as longitude 146;30° (from the eastern limit) and latitude 44° less 14 seconds, and there are several tables for geographical latitude 44° in the *Tables for 1340*. In short, there can be no doubt that the *Tables for 1340* are due to Bonfils, despite claims to the contrary: see Mercier 2014, but previously Mercier (1998, pp. 125 and 127 n. 5) acknowledged Bonfils's authorship.

The list of tables in the *Tables for 1340* is quite extensive, and includes tables for the planets as well as tables for the equation of time, the Sun, the Moon, eclipses and, in one of the four manuscripts, a few tables of astrological significance (Goldstein and Chabás 2017a, pp. 75–76). In three of the manuscripts the equation of time is given in minutes and seconds of an hour, whereas in the fourth manuscript the equation of time is given equivalently in degrees and minutes. The source is the corresponding table in Bar Ḥiyya's tables (Goldstein and Chabás 2017a, pp. 76–78; for a list of maxima and minima in tables for the equation of time, see Chabás and Goldstein 2012, p. 40).

Bonfils included two sets of mean motion tables. The first is a «standard» set where mean motions are displayed for Julian years for 1340 to 1380, cycles of 40 years from 1 to at least 8 (the number of cycles varies in different manuscripts), Julian months (beginning in March), days from 1 to 30, hours from 1 to 24, and minutes of an hour from 2 to 60 at intervals of 2 minutes. In the canons the epoch is given as noon, the last day of 1340. Since the year begins on March 1, the last day of 1340 corresponds to Feb. 28, 1341. The heading in one manuscript for the mean motions of Saturn is: «Saturn: Table of the mean motion of Saturn according to the reckoning of al-Battānī by Immanuel ben Jacob [Bonfils]», with variants in the other manuscripts (Goldstein and Chabás 2017a, p. 78). The mean motion tables for all five planets were indeed computed from al-Battānī's parameters (ibid., p. 79). Of special interest is a table listing the distances of the apogees of the five planets from the solar apogee (ibid. pp. 81–82), for which there is no counterpart in al-Battānī's zij. Precession is applied to the solar apogee and, hence, these values for the planetary apogees can be used for any date simply by adding to them the position of the solar apogee on that date. This treatment of the planetary apogees reflects an Andalusian tradition.

The second set of mean motion tables is preserved in two manuscripts with the title: «Table of the remainders of the cycles, years, and months, for conjunction and opposition in days, hours, and minutes, and mean motions for the seven planets, the ascending node, and the apogee, according to the reckoning of al-Battānī by Immanuel ben Jacob [Bonfils]». This mean motion table for all the planets is arranged for the Hebrew calendar and consists of three parts: a subtable for 19-year cycles from cycle 269 to cycle 276, a subtable for years from 1 to 19, and a subtable for the months in the Hebrew calendar, beginning with Tishri. The entries in the columns for the Sun and the Moon agree with those in Bonfils's *Six wings* (but for the higher precision in this table). However, the columns for the planets in this table have no counterpart in the *Six wings*. The column for the «remainder» displays the weekday (from (1) to (7), where (1) = Sunday, etc.) and the time of conjunction in hours and minutes. It is the remainder in the sense that multiples of 7 days are cast out, and only the remainder is displayed. The entries for time in the column of remainders are very close to those in the corresponding column in Wing 1, but for the greater precision in the *Tables for 1340*. The column labeled «remainder» in the *Tables for 1340* was adapted from a table in Bar Ḥiyya's *Sefer ha-ʿibbur* (*On the Hebrew calendar;* lit. *Book of intercalation*), which includes a table for mean conjunctions (*luaḥ ha-moladot*), where the same information is given but, instead of minutes of an hour, it lists parts of an hour such that 1080 parts is equal to 1h (ed. Filipowski 1851, p. 51). For example, in Bar Ḥiyya, the entry for year 1 of the 19-year cycle is (4) 8h 876p, and in Bonfils it is given equivalently as (4) 8;48,40h (*Tables for 1340*), or (4) 8;49h (*Six wings*). And for year 19, Bar Ḥiyya has (2) 16h 595p, while Bonfils equivalently has (2) 16;33,3h, rounded to (2) 16;33h in *Six wings*. Bonfils's table of remainders is found in his two zijes but there is nothing like it in any other zij.

The tables for solar and lunar velocity as well as the equations for the Sun, the Moon, and the five planets all derive from al-Battānī's zij. The exception is col. 3 for Venus (equation of center), where Bonfils has a maximum of 2;23° as in *Almagest* XI.11, but in al-Battānī's zij the maximum is 1;59° (Nallino 1899–1907, 2:126–131). Al-Battānī replaced the equation of center for Venus in order for it to conform with his revised solar equation (see Goldstein and Sawyer 1977, pp. 167–168). The tables for planetary latitude are also the same as those in al-Battānī's zij (Goldstein and Chabás 2017a, pp. 88–90).

The table for parallax is for geographical latitude 44°, and the entries are the same as in Levi ben Gerson's table (Goldstein 1974, pp. 196–199). There is no parallax table for this latitude in al-Battānī's zij. The solar and lunar eclipse tables are derived from the *Almagest* and they also also appear among Bar Ḥiyya's tables; they differ from those in al-Battānī's zij (Goldstein and Chabás 2017a, pp. 93–95).

Two mansucripts have a table for normed right ascension (beginning with Capricorn), but the entries differ: in one the entries agree with those in al-Battānī's zij (based on an obliquity of 23;35°), and in the other the entries agree with those in

Ptolemy's *Handy Tables* (based on an obliquity of 23;51°). One manuscript also has a table for oblique ascension for latitude 44° whose entries are identical with those in Levi ben Gerson's table (Goldstein and Chabás 2017a, p. 96).

7. Jacob ben David Bonjorn: late fourteenth century

Jacob ben David Bonjorn (also referred to as *ha-poʻel* [the table maker], Yomtob, Bona diei, or Bonet Bonjorn) was probably born in 1333 in Perpignan, but the date of his death is unknown. In 1356 he entered the service of Pere el Cerimoniós (1319–1387), King of Aragon and Catalonia, whose court was then based in Perpignan, southern France (Chabás 1991 and 1992). He is the author of a set of tables on the solar and lunar motions and the computation of eclipses, accompanied by canons explaining their use. This is the only known work produced by him, and it was most probably written in Hebrew. Bonjorn's tables and canons had a great success and were widely diffused, given the great number of extant copies in Hebrew, Latin, Catalan, and Greek.

In the canons a few scholars are mentioned: Ptolemy, Abraham bar Ḥiyya, Jābir ibn Aflaḥ, Maimonides, Levi ben Gerson, and King Alfonso. Several astronomers used Bonjorn's tables or wrote commentaries on them, notably Samuel d'Escola (Provence, fourteenth century), Isaac Ibn al-Ḥadib (see section 10), and Samuel Foto (Constantinople, second half of the fifteenth century), a disciple of Mordecai Comtino (Steinschneider 1964, pp. 197–198). Among the users of Bonjorn's tables, perhaps the most outstanding is the astronomer from Salamanca, Abraham Zacut (see section 15).

Bonjorn's work depends on a list of 767 consecutive true syzygies, that is, true conjunctions and oppositions of the Sun and the Moon, for a period of about 31 years, with year 1361 as epoch, and computed for the latitude of Perpignan (see Millás 1943 and 1949; Chabás 1991). In his tables, Bonjorn combined an innovative and accurate lunar cycle with an ingenious chain of tables that can be grouped into different categories.

There are no tables for the planets in Bonjorn's work, and nothing concerning astrology is addressed in them. Noteworthy is also the fact that there is no trace of Alfonsine astronomy in his zij, and no direct dependence on Arabic sources, despite his awareness of some of them.

7.1. Syzygies

Bonjorn's list of syzygies cover a period of 383.5 synodic months lasting approximately 31 Julian years and 2 days, that is, 11,325 days (Chabás and Goldstein 2020; Samsó 1997). The list is presented as 31 yearly subtables, for the time interval from 1361 to 1391, following the Julian calendar and starting on March 1. For each syzygy we are given its date and time, the true solar position, and the argument of lunar latitude, as well as a time correction to be subtracted (Heb. *ha-tiqqun li-groʿa*; Lat. *equatio substraenda*). This correction, for which there is no known precedent, is used to compute the time of true syzygies in cycles before or after the basic one, that is, before 1361 or after 1391. It is given in minutes of time and parts of it, where 17 parts are equal to a minute (Heb. *šeniyim mi-YZ*: seconds of 17). This original unit is reminiscent of the *ḥeleq* that is used in computations of the Hebrew calendar, corresponding to 1/1080h = 1/18 min.

The length of the mean synodic month underlying Bonjorn's tables is 29;31,50,7,54 days, a value consistent with that used by Levi ben Gerson (see section 2), a scholar highly praised by Bonjorn in his canons.

To determine the day of other syzygies before (after) the basic cycle, Bonjorn adds a specific 3 × 3 matrix to each of the 31 subtables. The central row is always 1, 2, 3, and the entries in the upper (lower) rows are the number of days to be subtracted (added) to a date of a syzygy in the basic cycle to obtain that of the corresponding syzygy 1, 2, or 3 cycles of 31 years before (after). Such a compact and clever scheme for this purpose is not found in any previous zij.

To further extend the usefulness of his list of true syzygies to dates before and after those in the basic cycle 1361–1391, Bonjorn designed a procedure to correct the positions of both luminaries. To that aim, he generated a double argument table where the argument at the head of the columns is the time correction to be subtracted and the argument at the beginning of each row displays 24 dates evenly spaced throughout a year. For each value of the time correction and each date, we are given two entries, one for the correction to the solar position and another for the correction to the argument of lunar latitude. The parameters for the mean motions of the Moon and the Sun agree with those determined by Levi ben Gerson.

7.2. Parallax

Bonjorn's 12 monthly subtables for parallax were computed for the latitude of Perpignan, 42;30°. They are arranged according to the format used by Ptolemy in the *Handy Tables*, and accepted by al-Battānī, Bar Ḥiyya, Levi, etc., and display the components of adjusted parallax in longitude and latitude as a function of local time. The parameter for the obliquity of the ecliptic, 23;51,20°, was borrowed from Ptolemy. For parallax in longitude, Bonjorn took the basic parameters from Levi and for parallax in latitude those from Ptolemy. In contrast to most of the previous tables of this kind, the component in longitude is displayed in units of time (hours and minutes) rather than in minutes of arc.

7.3. Eclipses

Both solar and lunar eclipses are computed for mean distance of the Sun between apogee and perigee. For solar eclipses, Bonjorn provides a double argument table with parallax in latitude as the argument at the head of the columns and the argument of lunar latitude as the argument at the beginning of each row. Two entries are given for each pair of values (parallax in latitude and argument of lunar latitude): the eclipsed portion of the solar disk and the half-duration of the eclipse.

For lunar eclipses a single table is provided. The entries are the eclipsed portion of the lunar disk, the half-duration of the eclipse, and the half-duration of totality, as a function of the argument of lunar latitude.

8. The impact of Ibn al-Kammād (fl. 1116) on Hebrew zijes

Abū Jaʿfar Aḥmad b. Yūsuf Ibn al-Kammād was probably born in Seville and worked in Córdoba (Chabás and Goldstein 1994). He composed three zijes that are only partially preserved in Arabic, but one of them, al-*Zīj al-muqtabis*, is extant in a Latin version by John of Dumpno (dated 1260) and a Hebrew version by Solomon Franco (fl. 1375: Goldstein 2013b), each preserved in a single manuscript. This zij was quite influential with a considerable impact on Hebrew table-makers, including two astronomers most of whose tables have not been addressed in the secondary literature, namely, Juan Gil of Burgos (fl. 1350) about whom little is known, and Joseph Ibn Waqār of Toledo who was a member of a prominent Jewish family. Ibn Waqār's zij was composed in Arabic written in Hebrew characters in 761 AH (= 1359/60), and the canons were subsequently translated by the author himself into Hebrew 37 years later (both versions are preserved in the same manuscript now in Munich). The epoch for Ibn Waqār's zij is 720 AH (= 1320/21), whereas the epoch for Juan Gil's zij is 1310 of the Spanish era (= 1273: Chabás and Goldstein 2015b, pp. 582–583). Each of these two zijes is preserved in a single manuscript.

A special feature of Ibn al-Kammād's zij is the preservation of astronomical traditions some of which go back to Greek sources while others build on Indian sources. In Baghdad in the late eighth and early ninth centuries, Indian astronomical traditions prevailed, but they were largely displaced by Greek traditions in the course of the ninth century. However, in al-Andalus, Indian traditions were maintained to a greater extent than in eastern Islam. In al-Andalus the Indian tradition was largely represented by the zij of al-Khwārizmī and the Greek tradition by the zij of al-Battānī. For the impact of Ibn al-Kammād's zij on the zijes of Ibn Waqār and Juan Gil, as well as on the zij of Ben Verga, the anonymous zij of 1400, and the Tables of Barcelona, see Chabás and Goldstein 2015b. For a clear case of Ibn al-Kammād's legacy, consider his maximum solar equation 1;52,44°: in Ibn Waqār and Ben Verga it is 1;53°; in Juan Gil, the Tables of Barcelona, and the anonymous zij of 1400 it is 1;52°. By contrast, in Ptolemy it is 2;23°, in al-Battānī and the Toledan Tables it is 1;59,10°, and in al-Khwārizmī it is 2;14° (Chabás and Goldstein 2015b, pp. 605–606). For a case where Ibn al-Kammād represents both Indian and Greek traditions, consider his two tables for lunar latitude, one with maximum 5;0° (as in the *Almagest* and al-Battānī) and the other with a maximum of 4;29° (a variant of al-Khwārizmī's 4;30°). Tables for both values are also found in the Tables of Barcelona and in Zacut's *Almanach perpetuum* (Chabás and Goldstein 2015b, pp. 614–616; Chabás and Goldstein 2000, pp. 62, 130–131).

Ibn al-Kammād's table for lunar velocity at syzygy is in the Indian tradition and comes from the zij of al-Khwārizmī: the extremal values are 0;30,12°/h (for argument 1°) and 0;35,40°/h (for argument 180°). By contrast, in al-Battānī the extremal values are 0;30,18°/h and 0;36,4°/h. Ibn al-Kammād's table is also found in the zij of Juan Gil and in the Tables of Barcelona (Chabás and Goldstein 2015b, pp. 613–614; Goldstein 1996, p. 190).

The time between mean and true syzygy is another topic where Ibn al-Kammād's zij was influential. He introduced a double argument table to determine this time interval as a function of the elongation between the Sun and the Moon and their relative hourly velocity. The vertical argument, the elongation, is displayed from 0;30° to 12;0° in increments of 0;30°, and the horizontal argument, the velocity in elongation, from 0;27,30°/h to 0;33,30°/h in increments of 0;0,30°/h. The entries, given in hours and minutes, are obtained by dividing the elongation by the relative hourly velocity, but the results are rather crude (Chabás and Goldstein 2015b, pp. 624–625). Similar tables, with slight variations in the arguments, are found in the tables of Juan Gil, in the Tables of Barcelona (Millás 1962, pp. 230–231; Chabás 1996, pp. 507–508), and in the tables of Mordecai Finzi (see section 14). Ibn al-Waqār had a different approach to this problem, using two double argument tables (Chabás and Goldstein 2015a, pp. 42–46).

In his tables for the planetary equations, Ibn Waqār provided separate tables for functions where the argument is the mean longitude and those where the argument is the true anomaly, with two additional tables for interpolation in the second table. An entry in the first table consists of the longitude corrected for the equation of center and minutes of proportion, and an entry in the second table consists of two values for the equation of anomaly for Jupiter and Saturn, one labeled 0 minutes and the other labeled 120 minutes, referring to the values of the equation of anomaly at maximum and minimum distances, with interpolation rules for intermediate values. Similar interpolation rules apply to the other three planets (Chabás and Goldstein 2015b, pp. 611–612). Note that in the corresponding table in the *Almagest* and in al-Battānī, the equation of anomaly is displayed for mean distance together with the increments/decrements at minimum and maximum distances as well as instructions for interpolating for other distances. The planetary apogees derived from Ibn Waqār's tables are truncations of the values for the planetary apogees in the Tables of Barcelona (Chabás 1996, p. 496): Saturn: 8s 1;0° (Barcelona: 8s 1;23°); Jupiter 5s 11;0° (Barcelona: 5s 11; 3°), Mars 4s 2;0° (Barcelona: 4s 2;23°); Venus: 2s 19;0° (Barcelona: 2s 19;25°); and Mercury 6s 21;0° (Barcelona: 6s 21;3°). The entries in Ibn Waqār's first table are the same as those in Ibn al-Kammād with a horizontal displacement of 161°, and agree very closely with those in the Tables of Barcelona. The reason for this displacement is that the argument in Ibn Waqār and the Tables of Barcelona already includes a value for the apogee, whereas Ibn Kammād's argument does not. Moreover, the minutes of interpolation in the first table are generally the same as those in Ibn al-Kammād and the Tables of Barcelona increased by 60 to avoid negative entries (Goldstein 2020–2021, pp. 133–141). In the case of Juan Gil, there are no significant discrepancies between the entries in his tables for the planetary equations and those in the zij of Ibn al-Kammād.

Echoes of Ibn Kammād's astronomy are also found in the Alfonsine Tables (Chabás 2019, pp. 129–130, 262).

9. TABLES OF BARCELONA: LATE FOURTEENTH CENTURY

Jacob Corsuno, a Jew from Seville, was invited by King Pere el Cerimoniós of Aragon and Catalonia (1319–1387), to compile a zij, now called the Tables of Barcelona (Millás 1962; Chabás 1996). Jacob is also the author of a treatise in Arabic on the construction of astrolabes written in Seville in 1376, and translated into Hebrew by the author himself in Barcelona in 1378 (Rodríguez-Arribas 2018). He has also been identified as the astronomer who is mentioned as the teacher of the scribe in the colophon of an Arabic version of the *Almagest*, completed in Zaragoza in September 1381 (Kunitzsch 2001, pp. 32–33).

The Tables of Barcelona were completed around 1381 and they consist of a full set of 53 tables and canons, of which there are three versions, in Catalan, Hebrew, and Latin, all of them reproduced by Millás (1962). It is likely that Jacob Corsuno was responsible for the Hebrew version. The canons consist of 13 chapters and explain the use of the tables. In them the name and coordinates of the city where the zij was compiled are given: Barcelona, latitude 41° and longitude «from the western limit of the inhabited world» 33° (Millás 1962, p. 91: 9–10 Lat.; p. 96: 8–10 Heb.; p. 124: 27–28 Cat.).

The epoch of the Tables of Barcelona, called the era of King Pere, is noon preceding Sunday, March 1, 1321, given as 720 Arabic years 30 days and 11/30 of a day after the Hijra. The tables are arranged for the Julian calendar, use zodiacal signs of 30°, and the positions of the celestial bodies are given in tropical coordinates, whereas the apogees and radices are sidereal. Altogether, this zij builds on the astronomical tradition of Indian origin in the Iberian Peninsula, particularly on the zij *al-Muqtabis* by Ibn al-Kammād (see section 8), also a native of Seville (see Chabás and Goldstein 1994 and 2015b). It is worth noting that there is no hint of the Alfonsine Tables in the Tables of Barcelona.

Among the preliminary tables are those for the conversion of dates between the Julian and Hijra calendars, tables for trigonometry and spherical astronomy embedding several values of the obliquity of the ecliptic of different origins (23;33°, 23;35°, and 23;51,20°), as well as a table for the equation of time.

There are tables for both precession and trepidation. In the table for the motion of the fixed stars, the constant rate of precession is 1° in about 72 Julian years. There are two tables for trepidation, one for the mean motion of the first point of Aries and the other for the equation of access and recess of the 8th sphere. This is the same model, with a constant motion and an equation of the 8th sphere, already used in the zij *al-Muqtabis*. The motion of the apogees is presented in the same table as the mean motion of the first point of Aries and the motion of the fixed stars, and the rate of progress is equivalent to 1° in about 290 Julian years, as in the zij *al-Muqtabis*. The apogees of the Sun and the planets are sidereally fixed.

The mean motions are given at intervals of 20 years, as was the case for other Alfonsine and pre-Alfonsine sets of tables using the Julian calendar. The equations present some interesting features. In the table for the solar equation, the argument is the mean solar longitude, not mean anomaly, and the entries represent the true solar longitude. The solar equation, with a maximum of 1;52°, is obtained by finding the difference between the two quantities. Its entries derive from those of Ibn al-Kammād by applying a horizontal displacement of 79°, which is an approximate value for the longitude of the solar apogee. The lunar equations are presented in two separate tables, reflecting Ptolemy's second lunar model. There are also two separate tables for each planet, and their entries agree with those in various zijes, particularly the zij *al-Muqtabis*.

The entries in the table for the hourly velocities of the Sun and Moon, with extreme values 0;2,22°/h and 0;2,34°/h (Sun) and 0;30,12°/h and 0;35, 40°/h (Moon), mostly agree with those in the zij of al-Khwārizmī and in the zij *al-Muqtabis*.

The tables for the lunar latitude, with maximum values 5;0° and 4;29°, and the planetary latitudes are the same as those in the zij *al-Muqtabis*. This is also the case for the two tables of the components of adjusted parallax in longitude and latitude.

For syzygies and eclipses, the Tables of Barcelona follow closely the work of Ibn al-Kammād, in particular the double argument table for the time between mean and true syzygy. Also remarkable is the presence of an uncommon «table for the *samt* for solar eclipses» (Chabás 1996, p. 510) and a table for the colors of the eclipses.

The table for the excess of revolution, where the entries for one year are 6;1h and 90;15°, yielding a sidereal year of 365;15,2,30d, is unattested in the previous literature.

There is also a list of cities with their geographical coordinates, where the prime meridian is the «meridian of water», about 17;30° west of the Canary Islands, as is the case in the zij *al-Muqtabis*.

While the Latin and Catalan versions of the Tables of Barcelona do not display a list of fixed stars, such a list is found in the Hebrew manuscripts. It is the list associated with Ibn al-Kammād's *al-Muqtabis*, consisting of 30 stars the longitudes of which are obtained by adding 6;38° for precession to the stellar longitudes in Ptolemy's catalogue (Goldstein and Chabás 1996).

10. ISAAC IBN AL-ḤADIB: D. CA. 1426

Isaac Ibn al-Ḥadib (or al-Aḥdab) lived in Castile in the 1370s, left the Iberian Peninsula in about 1391 at the outbreak of anti-Jewish riots, and settled in Sicily no later than 1396, when the island was ruled by Joan I (d. 1396), King of Aragon (Steinschneider 1964, p. 168; Goldstein 1987, p. 128). Ibn al-Ḥadib's main astronomical work is a set of tables in Hebrew for conjunctions and oppositions of the Sun and the Moon, called *Oraḥ selula* (*The paved way*), including tables for computing the time and position of true syzygy, and the circumstances of solar and lunar eclipses (Goldstein and Chabás 2006 and 2019). His work, consisting of canons and tables, is preserved in more than 20 Hebrew manuscripts and it is also extant in a Greek version. In addition to his set of astronomical tables, he composed a treatise describing an equatorium, and a treatise on an astronomical instrument related to the astrolabe. In mathematics his most notable work is the *Epistle on the Number* (Wartenberg 2015).

One of Isaac Ibn al-Ḥadib's disciples was Rabbi Nissim Abū l-Faraj (early fifteenth century, Sicily). His son, Samuel ben Nissim, who converted to Christianity in the 1460s, taking the name William Raymond of Moncada and later calling himself Flavius Mithridates, compiled a set of astronomical tables in Latin, which are closely related to Ibn al-Ḥadib's zij. The tables of Mithridates are uniquely preserved in a manuscript at the Vatican. Several Greek and Muslim authorities are mentioned in the canons: Aristotle, Ptolemy, Ibn Sina, and Ibn al-Ḥātim. Of particular interest is that three other astronomers are cited: al-Battānī, Ibn al-Kammād, and Ibn al-Raqqām (see section 8 and Kennedy 1997). These are the same three astronomers mentioned by Ibn al-Ḥadib in the introduction to his *Oraḥ selula* (Vatican, BAV, MS Heb. 379, f. 2a). We are not aware of any other Latin text of the fourteenth or fifteenth centuries in which Ibn al-Raqqām's name appears.

A few users of Ibn al-Ḥadib's zij are known. Isaac ben Elijah ha-Cohen of Syracuse (fl. 1438) compiled a set of tables based on Ibn al-Ḥadib's, and Abraham Gascon (Cairo, mid-16th century) wrote a commentary on Ibn al-Ḥadib's zij and provided a worked example for the solar eclipse of August 1542, based on it. *Oraḥ selula* was also mentioned by other astronomers, notably Mordecai Finzi and Abraham Zacut.

The first four tables in this zij list the corrections to the solar and lunar positions at syzygy, given in units of time, as a function of the solar and lunar anomalies, respectively. Similar tables have entries given in units of arc. The entries in the corresponding tables are related by the values 0;2,28°/h and 0;32, 56°/h, which represent the hourly mean motions of the Sun and the Moon, respectively. Of special interest are the columns for the difference in time and the difference in longitude between mean and true syzygy. In these tables, the solar equation was taken from al-Battānī's zij whereas the lunar equation was taken from the Parisian Alfonsine Tables, a mixing of sources, which is unusual. However, the clever procedure introduced by Ibn al-Ḥadib to compute these two differences is unprecedented (Goldstein and Chabás 2019; for other approaches to this problem, see Chabás and Goldstein 1997). Moreover, we are aware of just three astronomers — Ibn al-Ḥadib, Levi ben Gerson, and Nicholaus de Heybech — who applied a table of lunar velocity based on Ptolemy's second lunar model to constructing their respective tables for the time from mean to true syzygy (Chabás and Goldstein 1992). The tables for lunar eclipses agree with the corresponding tables in Wing 4 of the *Six wings* by Immanuel Bonfils of Tarascon, displaying the digits of a lunar eclipse; the half-duration of immersion; and the half-duration of totality.

There are also tables for multiplication, the length of half-daylight for various latitudes, and colors of eclipses, mentioning Ibn al-Kammād, as well as tables for various geographical latitudes displaying the longitudinal and latitudinal components of the adjusted parallax that are almost identical with those in Ptolemy's *Handy Tables* and in al-Battânî's zij.

The tables yielding mean positions and times for conjunctions are arranged according to the Hebrew calendar with its 19-year cycle, beginning with *molad tohu*, the conjunction of Tishri, year 1 of the Era of Creation. The time of this conjunction is Monday (day 2) at 5h 204 *ḥalaqim* (where 1h = 1080 *ḥalaqim*) counting from sunset, at a location whose distance from the western extremity, according to Ibn al-Ḥadib, is 75;43,45° (Goldstein and Chabás 2006, p. 160). This location is certainly meant to be Jersualem; for the longitude of Jerusalem in some medieval Hebrew sources, see Goldstein and Chabás 2017a, p. 73.

The mean motion tables for the Sun and the Moon are arranged for conjunctions and consist of 3 sub-tables. They display the solar anomaly, the lunar anomaly, and the argument of lunar latitude for 13 consecutive months, 19 consecutive years, and groups of 19-year cycles, respectively.

Other tables with the same structure give the excess over an integer number of weeks and the mean longitude of the two luminaries at mean conjunction. For the lunar mean motion in longitude, we deduce a value of 13;10,34,52,40,30°/d, which is to be compared with the value in Ptolemy's *Almagest* (13;10,34, 58,33,30,30°/d) and those in the zij of al-Khwārizmī, the zij of Ibn al-Kammād, and the Toledan Tables (13;10,34, 52, 46°/d). For the solar mean motion in longitude, we derive a value of 0;59,8,11,20°/d. The difference between the solar mean motion in longitude and the motion of the solar anomaly is the proper motion of the solar apogee: 0;0,0,2,4°/d (or 1° in about 286 Julian years). Other medieval values for the proper motion of the solar apogee are close to this amount, e.g., for Ibn al-Kammād it is 1° in about 290 Julian years (Chabás and Goldstein 1994, p. 28).

As is the case with several other sets of tables in Hebrew compiled in the late Middle Ages (e.g., The *Six wings* by Immanuel Bonfils, and the tables by Jacob ben David Bonjorn, both of which are cited at the beginning of Ibn al-Ḥadib's canons: Vatican, BAV, MS Heb. 379, f. 1b), there is nothing in *Oraḥ selula* (or in Mithridates's tables) concerning planetary motion or astrology. The astronomical tradition on which these tables are based ultimately derives (for the most part) from Ptolemy's *Almagest*. Ibn al-Ḥadib's goal was to produce tables that are user-friendly without changing the underlying theory.

11. Anonymous zij for 1400: early fifteenth century

Vatican, BAV, MS Heb. 384, a lengthy fifteenth-century codex, contains a number of texts, including unpublished canons to the zij by Judah ben Asher II, *Ḥuqqot šamayim* (*Ordinances of the heavens*). According to the catalogue of the Hebrew MSS in BAV, fols. 284a–384b are all assigned to Judah ben Asher II (Richler et al. 2008, p. 329). However, only fols. 284a–341b belong to him; the table on fols. 347a–359b is anonymous and not related to the treatise by Judah ben Asher II (Goldstein and Chabás 2015, p. 320: see sections 12 and 15).

This manuscript also includes an anonymous zij for 1400 uniquely preserved on folios 263a–278b that is separately paginated in Hebrew alphabetic numerals, beginning with '1' (fol. 263a) and ending with '27' (fol. 276b), generally in the upper right corner of the page. For the second page (and a few others) the page number is not visible in the microfilm copy that was available to me. The pagination indicates that this zij is distinct from the rest of the items in the codex.[6]

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[6] The digital version of this manuscript at the Vatican website is incomplete (checked July 22, 2021): https://digi.vatlib.it/search?k_f=0&k_v=ebr.384. See the description in Richler et al. 2008, pp. 327–330. Many folios in this manuscript are in a poor state of preservation (some have suffered from modern attempts to repair them).

The presentation of the mean motions in this anonymous zij is very close to that in the zij of Judah ben Verga: both display lists of radices at 28-year intervals in the Julian calendar, mean motions for each year from 1 to 28, each month from January to December, and each day from 1 to 30. This interval of 28 Julian years was also used for mean motions in some adaptations of the Toledan Tables in Latin (Chabás and Goldstein 2012, p. 54). The lists of radices here begin with 1400 whereas in the corresponding tables of Judah ben Verga some radices begin with 1384 and others with 1400. The mean motions for the five planets as well as the solar and lunar mean motions are the same in both zijes (Goldstein 2003b, p. 152). There are other similarities with the zij of Ben Verga, notably, double argument tables for planetary equations and the absence of any tables for astrological purposes. Moreover, the planetary apogees in the anonymous zij are consistently 10° lower than those of Judah ben Verga (Goldstein 2003b, p. 161).

In the table for the solar equation the argument is the mean solar longitude from 1° to 360° at intervals of 1°: the entries are in signs, degrees, and minutes; the apogee is 2s 20° (close to the value in the Tables of Barcelona); and the maximum equation is 1;52° (as in the Tables of Barcelona). The entries are slightly different from those in the Tables of Barcelona because of the difference in the apogees (Goldstein 2003b, p. 165).

The table for Saturn's equation is entitled: «Table for correcting the position of Saturn and its latitude, entering with its mean position and its anomaly, yielding the true position in the 8th sphere, apogee 8s 2°» (Goldstein 2003b, p. 167). The columns are headed with the mean longitude from 2° to 352° at intervals of 10°, and the vertical arguments are the corrected anomaly from 0° to 350° at intervals of 10°. The entries are given in signs, degrees, and minutes. Judah ben Verga's corresponding table is arranged differently with entirely different entries (see Section 13). There are also two columns for the latitude of Saturn preceding the column for the vertical arguments of corrected anomaly, and the entries correspond to the entries in *Almagest* XIII.5.

Although this anonymous zij is closer to the zij of Judah ben Verga than to any other, their relationship has not been determined. It is not even clear if the author of the anonymous zij lived before or after Judah ben Verga, i.e., the direction of influence, if any, cannot be established at this time. Since the anonymous author is consistent in using 1400 as his epoch, it is likely that he lived in the early fifteenth century.

12. Daily lunar positions in a cycle of 11,325 days: mid-fifteenth century

One of the most difficult tables to analyze in the entire corpus of Hebrew astronomical tables was an anonymous list of 11,325 successive daily positions of the Moon, preserved in Vatican, BAV, MS Heb. 384, 347a–359b (on this manuscript, see section 11). In most cases the entry has degrees (less than 30) and minutes, but only has a blank space for the zodiacal sign on all pages but for the first two. The table has no title, the columns have no headings, and it was not clear how the starting date could be determined. For many years it seemed unlikely that a solution would be found. Among the data originally available was that each column has 42 entries with a running total at the bottom of each column. These running totals were written in decimal notation, using the first nine letters of the Hebrew alphabet and a symbol for zero, to be read from left to right, rather than the usual way of reading Hebrew from right to left. Hence, at the bottom of the first column is DB (בד) for 42, and at the bottom of the second column is HD (דח) for 84, and below the last entry in the final column is AAGBE (הבגאא) for 11,325. This cycle is equal to 31 Julian years and 2 days.

The solution came when it was realized that the tables on folios adjacent to this table were related to it. On the last page of this list of lunar positions is a short table, with the heading in Arabic written in Hebrew characters: «Table for correcting the position of the Moon in the cycles which are prior to the principal cycle, that is the sixth cycle and those subsequent to it». Indeed, there are entries for corrections to cycles before and after the sixth cycle, where the entry for the sixth cycle is 0s 0;0°, that is, there is no correction for the sixth cycle. At this point it was necessary to identify the date of the sixth cycle, where each cycle was 11,325 days. This was solved by examining the preceding folios, 345a–346b. The heading in Arabic written in Hebrew characters for the table on fol 345a–b refers to the Hijra calendar, whereas the heading for the table on fol. 346a–b refers to the Julian calendar: «First page for the Christian years: Table for the cycles of the foreign years of the Messiah». There are five columns of 31 rows on fol. 346a: the first column is headed «first cycle» and years are listed from 1244 to 1274 (written in Hebrew characters with the usual convention of reading from right to left) with entries 11[d] to 10969[d] (to be read from left to right), emended from 10909[d]. On fol. 346b there are columns for cycles 6 to 10. The first entry in the column for cycle 6 is 1400 and next to it is 366[d], that is, the last day of 1400 is day 366, counted from the beginning of the cycle. In other words, Jan. 1, 1400 was day 1 of cycle 6. A worked example for a date in 1451 on fol. 359b confirmed this decipherment (Goldstein and Chabás 2015, pp. 328, 339–340).

Since the list of 11,325 daily positions of the Moon seemed to correspond to cycle 6, the next step was to calculate successive lunar positions from Jan. 1, 1400 to Jan. 31, 1400, using various medieval tables for lunar motion. The best fit was with the Toledan Tables: the difference was about 10;30° between entries in the list in the Vatican manuscript and those recomputed using the Toledan tables. For a detailed comparison, see Chabás and Goldstein 2020. This difference seems to mean that the entries in the Vatican list are sidereal positions whereas those recomputed with the Toledan Tables yield tropical positions: Goldstein and Chabás 2015, p. 326. As Jacob ben Makhir had done in his almanac (see section 4), here the sidereal coordinates of the Toledan Tables are used for computation, but the results are presented in tropical coordinates.

This lunar cycle of 11,325 days is also found in an Arabic zij by Ibn ʿAzūz al-Qusanṭīnī of Fez (d. 1354), in the tables of Jacob ben David Bonjorn (section 7), in the tables of Joseph Ibn Waqār (section 8), and in the tables of Abraham Zacut (section 15): see Chabás and Goldstein 2020. There is no evidence of this cycle before the fourteenth century.

13. Judah ben Verga: fl. 1455–1480

Judah ben Verga (Lisbon, fl. 1455–1480) is the only medieval astronomer known to have compiled a Hebrew zij in Portugal. In several treatises he recorded observations made in Lisbon in 1456 and 1457, and in the canons to his tables he referred to a lunar eclipse to take place in the future on Mar. 22, 1475 and a solar eclipse to take place on July 29, 1478. His tables are preserved in two manuscripts and the canons in a different manuscript; Lisbon is only mentioned once (other than in the list of cities), and he is not named as author in any of these manuscripts. Nevertheless, Judah's authorship is secure since, in the canons, an astronomical observation made in Lisbon is reported in the first person, and the same observation is recorded in a treatise by Judah on an astronomical instrument (Goldstein 2004, p. 72). In the canons, the title for this zij is *Ḥuqqot šamayim* (*Ordinances of the heavens*), which was also the title of an astronomical work by Judah ben Asher II (Goldstein 2001, p. 228; see also section 11).

The two manuscripts of Judah ben Verga's zij include a total of 30 tables of which 26 are preserved in both copies (Goldstein 2001, p. 238). This is a full set of tables: mean motions of the Sun, the Moon, and the planets; solar, lunar, and planetary equations; lunar and planetary latitudes; times of mean conjunctions; time from mean to true syzygy; parallax; eclipses; and a list of cities with their geographical coordinates, where the coordinates of Lisbon are 39;38° N, and longitude 22;54° (counted from the «meridian of water»; on this meridian, see Comes 1992–1994). The epoch of some mean motion tables is 1384 and of others it is 1400. The mean motions are given in the Julian calendar, and seem to have diverse origins: some agree with the *Almagest*, some with Levi ben Gerson, some with the Toledan Tables, and some with the Alfonsine Tables (Goldstein 2001, p. 229). Judah was an older contemporary of Abraham Zacut; although Zacut mentioned him a few times, Judah did not mention Zacut (Goldstein 2001 p. 230).

In Judah's table for the solar equation, the argument is days, hours, and *ḥalaqim*, which are converted into degrees of solar motion. This is certainly an unusual arrangement (Goldstein 2001, pp. 244–245, 267–269). The entries are given to minutes and the maximum equation is 1;53°, which suggests that the source was the corresponding table in the zij of Ibn al-Kammād, where the maximum is 1;52,44° (Chabás and Goldstein 1994, pp. 6–10). For the equations of the outer planets, Judah provided double argument tables, where the rows are labeled with day numbers converted to the mean position of the Sun, and columns are for the center in days and then converted into degrees. The entries were computed with Ptolemy's models and, in the case of Saturn, the apogee was taken to be 252° and the eccentricity 3;25. But Judah used his own values for Saturn's mean motions rather than those of Ptolemy. In the canons there is a worked example for finding the true position of Saturn (Goldstein 2001, pp. 248–249, 270–271). The equations for the inner planets are arranged in a similar way to those for the outer planets (see Goldstein 2001, pp. 250, 271–272).

The arrangement of the table for mean conjunctions is also unusual. It is headed «Table for the days of conjunctions and the days of the distance of the Sun from its apogee, and the days of the Moon from the apogee of its epicycle, constructed for Lisbon whose longitude is 23°»; it has 3 subtables, and each subtable has 4 columns. This is the only heading where Lisbon is mentioned. The first column in subtable 1 lists years beginning with 1400 and then multiples of 28 years subsequent to it; the first column in subtable 2 lists Julian years from 1 to 28, and the first column in subtable 3 lists Julian months from January to December. Column 2 is labeled «days of the Sun», column 3 is labeled «days of conjunction», and column 4 is labeled «days of the Moon». In subtable 3, the days of conjunction refer to the excess of the days at the end of each month over multiples of a mean synodic month, and the days of the Moon refer to the excess of the days at the end of each month over multiples of an anomalistic month (Goldstein 2001, pp. 253–254, 272–275). Below these subtables are values for the mean synodic month, 29d 12;44,2,21h, which is equivalent, very nearly, to 29;31,50,8,20d, the parameter in the *Almagest*, and for the anomalistic month, 27d 13;18h, corresponding to a mean motion in anomaly of about 13;3,54°/d. For example, the entry in the column for days of conjunction for March is 1d 9;48h = 90d – 3 × (29d 12;44,3h). The values derived from this table serve as the arguments in the double argument table for the time from mean to true syzygy, that is, the 13 columns at irregular intervals from 0d to 182d (above the table), and from 183d to 365d 5;49h (below the table) are labeled «days of the Sun» which are converted to degrees of the mean Sun (the sum of 90°, for the solar apogee, and the number of days times the mean daily solar motion). The rows are labeled «days of anomaly» at intervals of 1 day from 0d to 27d 13;18h. The entries are given in hours and minutes (Goldstein 2001, pp. 255–256, 275–277).

In the canons there is a worked example for finding the time of mean conjunction on Feb. 16, 1474 at 3;18h after noon, and then the time of the corresponding true conjunction. According to Judah's canons, true conjunction took place on Feb. 15, 1474 at 22;54h after noon, that is, the time from mean to true conjunction was –4;24h. But recomputation with his tables yields –4;17h. For comparison, according to Abraham Zacut, there was a true opposition on Feb. 17, 1505 at 23;7h after noon. With the correction for 31 years of 0;27h, there was a true conjunction on Feb. 15, 1474 at 23;34h, Salamanca time. For Zacut the difference in time between Salamanca and Lisbon is 0;11h; hence, the true conjunction took place at 23;23h after noon, Lisbon time (Goldstein 2001, pp. 273, 279–280). Therefore, the difference in the time of true conjunction between Judah ben Verga's computation and that of Abraham Zacut is about half an hour (23;23h – 22;54h).

14. The impact of the Parisian Alfonsine Tables on Hebrew zijes

The Alfonsine Tables of Toledo were composed in Castilian by two Jewish astronomers, Isaac ben Sid and Judah ben Moses ha-Cohen under the patronage of King Alfonso X of Castile (reigned: 1252–1284). Only the canons survive; no copy of these tables is extant. The extensive canons indicate that the Castilian Alfonsine Tables dealt with all major problems addressed by medieval astronomers at the time and depended strongly on the work done in al-Andalus by Muslim astronomers (Chabás and Goldstein 2003). This set of tables arrived in Paris where they were recast in Latin as the Parisian Alfonsine Tables (henceforth PAT) and were widely diffused throughout Europe, beginning in the 1320s. There is no critical edition based on the manuscripts and so it is still useful to consult the *editio princeps* (Ratdolt 1483).[7]

There are five copies of PAT in Hebrew, but their relation to one another is not evident (for details, see Goldstein 2019b). The most extensive copy is preserved in Munich, MS Heb. 126, dated 1459/60, which includes almost all the tables, as well as the star list and the list of cities, that appear in the *editio princeps* in the same order. There is a brief introduction by the translator, Moses ben Abraham of Nîmes, who seems to have had access to an otherwise unknown Latin copy of PAT that was close to the manuscript used by Ratdolt some 23 years later. The earliest version of PAT in Hebrew dates from the late fourteenth century: it is preserved in Vatican, BAV, MS Heb. 381, and mentions a lunar eclipse that took place in Cyprus on August 22, 1374. A fifteenth–century copy in Milan, Biblioteca Ambrosiana, MS Heb. X-193 Sup., employs an unusual notation for entries in the tables: each sexagesimal digit is written in Hebrew characters in decimal place-value notation, to be read from left to right, although the entry as a whole is to be read from right to left (as is customary in a Hebrew text). For example, 35 is written GE (הג), that is, the first nine letters of the Hebrew alphabet are used for the digits 1, ..., 9, with a special symbol for zero (see Lévy 2003). This notation was also used for the running totals in the anonymous table for 11,325 consecutive lunar positions, but not for the positions themselves (see section 12). The two other copies PAT in Hebrew have been dated to the late fifteenth or sixteenth century.

William Batecombe, about whom almost nothing is known, adapted PAT to the meridian of Oxford and provided radices for 1348, modifying the presentation of PAT by John of Lignères who worked in Paris (North 1977, Chabás 2019, pp. 227–236). This set is now called the Oxford Tables, although in the Middle Ages it was generally called *Tabule anglicane*. It consists of five different groups of tables (1) mean motions; (2) a table of true longitude of the Sun in a year; (3) a double argument tables for the true longitude of the Moon; (4) double argument tables for the true longitudes of each of the five planets; and (5) double argument tables for the latitudes of each of the five planets. There were two adaptations into

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[7] For a transcription of parts of the Alfonsine Tables with a set of canons based entirely on the *editio princeps*, see Poulle 1984.

Hebrew: the first version is based on the Latin text of the Tables of 1368 for Paris which were slightly updated from the Oxford Tables (Chabás and Goldstein 2018). In one copy of the first Hebrew version the translator's name is given as Solomon ben Davin of Rodez (ca. 1370, southern France), who was a student of Immanuel Bonfils (see section 6). The second Hebrew version was compiled by Mordecai Finzi (fl. 1440–1475) where the radices are given for noon, December 31, 1443, Mantua, Italy.[8] Recomputation of Finzi's radices suggests that he depended on the Oxford Tables of 1348 rather than directly on PAT (Chabás and Goldstein 2018, pp. 111–112).

There are three Hebrew zijes that use elements from PAT. The most important is Abraham Zacut's *ha-Ḥibbur ha-gadol* (*The great composition*), which is the subject of section 15. Another author who used PAT was Moses Farissol Botarel (Avignon, late fifteenth century), whose tables for the Sun and the Moon combine elements from PAT, Levi ben Gerson, Immanuel Bonfils, and Jacob ben David Bonjorn (Goldstein and Chabás 2017c, pp. 33–34). Farissol Botarel referred to a solar eclipse that took place on July 29, 1478. In his commentary on the Paris Tables of 1368, Farissol Botarel indicated that these tables were based on the Alfonsine Tables that were translated into Hebrew by «my teacher Master Moses of Nîmes» (Goldstein and Chabás 2017c, p. 33). In sum, Farissol Botarel depended entirely on previous astronomical tables available in Hebrew.

Mordecai Finzi compiled a set of astronomical tables preserved in a manuscript in his own hand (Langermann 1988, pp. 20–23). This set includes the Alfonsine solar and lunar equations, as well as the Alfonsine equations for the five planets. Finzi also took tables from Levi ben Gerson (oblique ascensions for Orange, France, solar declinations), from Abraham Bar Ḥiyya (solar and lunar eclipses, and parallax), from Jacob ben Makhir (the difference in the hourly motions of the two luminaries: see section 4). Moreover, he referred to Bonfils's tables of 1340 for the hourly velocities of the Sun and the Moon and, in the context of calculating planetary apogees, and he cited Isaac Ibn al-Ḥadib (Langermann 1988, p. 17). Finzi also has a double argument table «for dividing the elongation between the Sun and the Moon at the times of conjunction and opposition by the hourly lunar velocity, by a different method» (Langermann 1988, p. 22). In this

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[8] For an account of Finzi's scientific works, see Langermann 1988. Finzi was probably the translator from Italian into Hebrew of a description of an equatorium in Italian (Goldstein 1987, pp. 120–121).

table the headings of the columns, the velocities in elongation, range from 0;27,30°/h to 0;34°/h at intervals of 0;0,30°/h, and the vertical arguments for the elongations range from 0;12° to 7;0° at intervals of 0;12° with a final row for 7;6°. An entry in this table is the time between mean and true syzygy. This solution to the problem of finding this time interval goes back to Ibn al-Kammād and, with variants in the presentation, is found in several sets of tables in Hebrew (see Chabás and Goldstein 1994, p. 14, and Section 8).

15. ABRAHAM ZACUT: D. 1514

Abraham ben Samuel Zacut was the most eminent astronomer in the Iberian Peninsula in the final decades of the fifteenth century. He was born in Salamanca and left Spain in 1492, arriving first in Portugal, then leaving Portugal for North Africa in 1497, and finally settling in Jerusalem where he died. His two main works on astronomical tables in Hebrew have not been edited, but both a contemporary Castilian translation as well as a modern Castilian translation of his *ha-Ḥibbur ha-gadol* have been published (Cantera 1931). Moreover, Latin and Castilian versions of the tables in the *Ḥibbur* with new canons were published in Leiria, Portugal, in 1496, entitled *Almanach perpetuum*, with radices for 1473, Salamanca (Chabás and Goldstein 2000). The *Almanach perpetuum*, in turn, was translated into Arabic twice (Samsó 2004). His other set of tables in Hebrew has radices for 1513, Jerusalem. Each set of tables will be treated separately.[9]

The *Ḥibbur*, written in 1478, consists of 19 canons together with about 65 tables, most of which are identical with those published in 1496 (Chabás and Goldstein 2000, pp. 53–55, 98). Only a few of his tables will be highlighted here. The principal tables are extensive almanacs whose entries are the true positions of the Sun, the Moon, and the five planets at intervals of a day or a few days. The entries for the daily solar positions were computed for a four-year cycle, beginning March 1, 1473 (Chabás and Goldstein 2000, pp. 55–56, 101–105). In the case of the Moon entries are for its true longitude for 11,325 days, or 31 years and 2 days, at intervals of 1 day, a cycle introduced by Jacob ben David Bonjorn ha-Poʿel (see sections 7 and 12). In the *Almanach perpetuum* there is also a table for all syzygies

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[9] For some other tables compiled by Zacut (including fragments), see Goldstein and Chabás 2018, pp. 48–62, and Goldstein 2013a.

in the cycle of 31 years and 2 days beginning in March 1478, following Jacob Bonjorn's procedure, but this table is missing in the manuscripts of the *Ḥibbur* (Chabás and Goldstein 2000, pp. 113–115). For Saturn the entries are its true longitudes (in degrees and minutes) beginning in March 1473 for days 10, 20, and the last day of the month for a period of 60 years (Chabás and Goldstein 2000, pp. 68, 131–132), For the other four planets there are similar tables. In all cases the true longitudes were derived from the Parisian Alfonsine Tables and included a correction of 0;0,27,20d for the difference in geographical longitude between Salamanca and Toledo (Chabás and Goldstein 2000, pp. 60, 110–113). There are also tables for the mean motions of center and anomaly for the planets, beginning in January 1476 for the meridian of Salamanca; the entries are again derived from the Parisian Alfonsine Tables (Chabás and Goldstein 2000, pp. 68–75, 134–137). In addition to eclipse tables and the equation of time, Zacut included a list of 32 cities with their geographical coordinates (Chabás and Goldstein 2000, pp. 64–65, 155; Cohn 1918, pp. 31–33), and a star list with 61 entries (Chabás and Goldstein 2000, pp. 68, 145–150). The longitudes in this star list differ from the corresponding longitudes in Ptolemy's star catalogue by 6;38°, as is the case in the star list by Ibn al-Kammād, suggesting that it served as a source for Zacut (possibly indirectly): on the significance of the value 6;38° in the context of Zacut's view of precession, see Goldstein and Chabás 2021. Each star has associated with it one or two planets for astrological purposes, as is the case with many other star lists (for Ibn al-Kammād's list, see Goldstein and Chabás 1996).

Of special interest are Zacut's table for the astrological houses, his table for the elongation the Sun from the lunar node, and his table for the daily progress of Mercury (Goldstein and Chabás 2000, pp. 62–63, 75, 100–101, 118–119, 145). For astrological purposes the zodiac at any given time is divided into 12 houses, beginning with the rising point of the ecliptic (or the ascendant) on the eastern horizon (North 1986). Zacut has 12 monthly subtables beginning in March, and for each day we are given (1) true solar time in hours and minutes, and (2) the longitude of 6 cusps (the first point of the astrological houses) at noon for the latitude of Salamanca in signs and degrees. By combining both sets of data, this table can be used to determine the longitudes of the cusps at any time by means of the following rule: if $H(x)$ is the hour angle of the Sun on a given date x, and λ_{I} is an entry for the longitude of the ascendant at noon, then $\lambda_{\mathrm{I}}(H(x) \pm t)$ gives the ascendant at a time t after (+) or before (–), noon of day x. In ch. 8 of the *Ḥibbur* Zacut explains the use of this table, using an example for 3;20 p.m. at Salamanca on August 12 when the ascendant was Sgr 28°. Although tables for the astrological houses are relatively common in zijes, this procedure is unprecedented. The key insight is that every point on the ecliptic from λ = 0° to λ = 360° crosses the eastern horizon in the course of a day, and so the ascendant at noon of each day of the year, which also varies from λ = 0° to λ = 360°, corresponds to the ascendant crossing the eastern horizon at some time before or after noon on a specific day.

The entries in Zacut's table for the elongation of the Sun from the lunar node are the differences in longitude between the Sun and lunar ascending node for each year in a period of 56 years, and for each hour of the day, in signs, degrees, and minutes. The purpose of this table is to call attention to eclipse possibilities: when the Sun's elongation from the lunar node at syzygy is sufficiently small, it is appropriate to compute the circumstances of a lunar or solar eclipse. This type of table is very rare, for only three other examples are known, two of which are by Zacut himself: in his tables for 1513 (Goldstein and Chabás 2018, pp. 40–41) and in his *Tabule verificate* (Chabás and Goldstein 2000, p. 29–30; for Zacut's authorship of the *Tabule verificate*, see Goldstein and Chabás 2018, p. 22). The third occurrence is in the zij of Ben Verga (Goldstein 2001, pp. 241, 265). Zacut's double argument table for the daily progress of Mercury has as its arguments the equated center and the equated anomaly. It is similar to an anonymous table for the daily progress of Mercury in Vatican, BAV, MS Heb. 384, fols. 372a–374a, but the underlying algorithm has not been discovered. The entries in the Vatican manuscript are given to seconds whereas Zacut's entries are only to minutes (but not rounded from the entries in the Vatican manuscript). Still, it is likely that they have a common source that has not been identified. In the Vatican manuscript there is a complete set of double entry tables for the daily progress of all five planets and the Moon (364a–384b), but this is not the case for Zacut (Chabás and Goldstein 2012, pp. 99–101).

The tables of Zacut for 1513 for Jerusalem are extant in a few manuscripts (Goldstein and Chabás 2018, pp. 33–48). As stated at the beginning of the canons, Zacut compiled this set of tables in Jerusalem in 5273 AM (= 1513). The canons include some worked examples which are most helpful. In these tables the focus is on the Sun and the Moon, and their motions are arranged for the Hebrew calendar, i.e., the era of Creation, with 19-year cycles and months beginning with Tishri. The first table is for mean conjunctions in 19-year cycles, followed by the solar positions in complete 19-year cycles, where the underlying solar mean motion is 0;59,8,19,37,...°/d, as in the Parisian Alfonsine Tables. The next table is for finding the true position of the Sun at mean conjunction in signs, degrees, and minutes, for each year from 1 to 19 and for each Hebrew month from Tishri to

Elul, that is, there are entries for all 235 months in a 19-year cycle. The entry in this table is to be added to the appropriate entry in the previous table to find the solar position at a given conjunction in a given year. Similar tables for lunar mean motion and anomaly follow, where the underlying parameters were also taken from the Parisian Alfonsine Tables to sexagesimal thirds. The next two tables concern the elongation of the mean Sun from the lunar node (here inappropriately called «the argument of lunar latitude»), for eclipse possibilities (as noted above). Then comes a table for correcting the solar position when the Moon is at the perigee of its epicycle, in hours and minutes. This is the first of three tables for determining the time from mean to true conjunction, and all three are based on the table for this purpose by Nicolaus de Heybech that is also found in Zacut's *Tabule verificate* (Goldstein and Chabás 2018, pp. 26–27; Chabás and Goldstein 1992). A table for the equation of time follows with a maximum of 0;32h at Sco 6°–10° and a minimum of 0;0h at Aqr 12°–26°. The entries in this table agree with those in the zij of al-Battānī, the Toledan Tables, and the Parisian Alfonsine Tables, although in those tables the entries are in degrees rather than in minutes of an hour. So, for example, the maximum 0;32h = 7;54°, for 1h = 15° (Chabás and Goldstein 2012, pp. 37–41). The next two tables are for solar and lunar eclipses, followed by a table for the digits of eclipse. Finally, there is a table for the parallax components for the latitude of Jerusalem, 32°, longest daylight 14;16h, where the parallax in longitude is given both in time and in longitude, otherwise only found in Zacut's *Ḥibbur*, *Tabule verificate*, and *Almanach perpetuum* (Goldstein and Chabás 2018, pp. 29–30; Chabás and Goldstein 2000, pp. 31–32, 62, 122–124). The entries for time are related to those for longitude by the factor 0;32,56°/h, which is the hourly lunar mean motion.

Despite the fact that Zacut based the tables of 1513 on the Hebrew calendar, he still appealed to tables and parameters from the Parisian Alfonsine Tables as well as from the tables of Nicolaus de Heybech, as he had done in his previous sets of tables. In sum, Zacut was very well informed of his predecessors who compiled astronomical tables in Hebrew, and explicitly referred to Levi ben Gerson and Jacob Bonjorn, among others (Chabás and Goldstein 2000, pp. 49–52). He was equally capable of computing positions with the Parisian Alfonsine Tables: in other words, he was familiar with astronomical traditions both in Latin and in Hebrew. Zacut's astronomical works had an impact on Christian, Muslim, and Jewish communities, as evidenced by subsequent Latin editions of the *Almanach perpetuum* (and texts derived from it) published after 1496, contemporary citations in Latin, translations into Arabic, and references in Hebrew sources (Chabás and Goldstein 2000, pp. 161–171).

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