The earliest Arabic magic squares

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ABSTRACT: My aim here is to shed light on the origins of magic squares in the Islamic world. This question has often been tentatively addressed, but previous studies considered only part of the evidence. The earliest Arabic texts discussing squares (al-Ṭabarī, Jābir ibn Ḥayyān, *Ghāyat al-Ḥakīm*, Brethren of Purity) are examined here together for the first time, as well as the Indian medical sources.

KEYWORDS: Magic squares, transmission, Indian medicine, al-Ţabarī, Jābir ibn Ḥayyān, *Ghāyat al-Hakīm*, Brethren of Purity, *Cyranides*

STATE OF THE ART

It is well known that magic squares enjoyed considerable success in the Islamic world as a mathematical subgenre of arithmetic, although they do not seem to have been part of the standard Arabic mathematical synthesis.¹ Insights into where mathematicians placed squares theory can be gained from inspecting the context of their first surviving testimony in Arabic mathematical texts, namely the *Commentary to Nicomachus' Arithmetic by al-Anțākī*, recently edited and translated by J. Sesiano.² The surviving text consists of a third book written as a sort of appendix to the two (non-extant) previous books which probably formed the com-

I. J. HØYRUP, In Measure, Number, and Weight. Studies in Mathematics and Culture, New York 1994, at 119.

2. J. SESIANO, Magic Squares in the Tenth Century. Two Arabic Treatises by Anţākī and Būzjānī, New York 2017. Sesiano rightly argues that the inclusion of magic squares in this commentary does not imply any awareness on the part of Islamic authors that Greek mathematicians had knowledge of them, and I remain unconvinced by Vinel's arguments. Cf. Sesiano, Magic Squares 16; N. Vinel,

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mentary on the two books of Nicomachus. This third book contains three chapters, on Euclidean definitions on number theory, magic squares, and hidden numbers. The last topic was a kind of challenge that was played out in pairs; one person thinks of a number on which he is required to perform some intricate operations, and then reveals the result to the other person, who has to guess the number as quickly as possible. The concomitance with the chapter on magic squares might then suggest that the latter were conceptualized as a similar optional complement to arithmetic - perhaps, too, for recreation, which is their main use today in the West. In India, however, there is a medical tradition of magic squares, deriving from old practices contemporary with the earliest uses of squares in the Islamic world. The magic square of three in the medical encyclopedia of al-Tabarī (Firdaws al-Hikma) predates the earliest known mathematical texts on squares (al-Antākī, al-Būzjānī, end of 10th c.) by a century and a half; although the latter obviously presuppose a considerable previous acquaintance with squares, the fact that the text of al-Tabarī is so close in time, place, and content to Indian texts using squares is practically conclusive proof that it is the origin of magic squares in the Islamic world. A. Rosu already pointed out this coincidence between the Indian testimonies and al-Tabarī, but he was undecided as to the direction of the transmission, and did not look at al-Tabarī's text.3

The context of the earliest Arabic magic squares has been interpreted in different ways, but previous studies have considered only part of the evidence. Sesiano, for example, maintains that magic squares arrived from India in the 7th c. along with chess, arguing that the first mathematical treatises on squares use chess vocabulary to describe the displacements of the numbers on the grid; he rules out an early magical use.⁴ However, it is entirely conceivable that mathematicians, equipped with chess vocabulary, would have described methods of constructing this new artifact using these terms if they were found to be useful. Sesiano does not seem to consider earlier medical use; perhaps misled by the tradition of the mathematical genre (operative since Greek Antiquity) of suppressing any extramathematical comments, he may have deduced from the mathematicians' silence that they were ignorant of medico-magical use. But he may also have confused

[«]Un carré magique pythagoricien? Jamblique précurseur des témoins arabo-byzantins», *Arch. Hist. Exact. Sci.* 59 (2005), 545-562.

^{3.} A. Roşu, «Études āyurvédiques III. Les carrés magiques dans la médecine indienne», in G. J. Meulenbeld and D. Wujastyk (eds.), *Studies on Indian Medical History*, Groningen 1987, 103-112.

^{4.} SESIANO, Magic Squares 6.

magic with astrology. As we will see, astrology did not enter the squares tradition until later, but magical medicine was present from the very first moment. For his part, probably unaware of the Indian evidence, J. Needham proposed a direct transmission from China.⁵ The Indian evidence emerged only a few years later, when Indologists Roşu and T. Hayashi brought to light the earliest known Indian squares.⁶

Hayashi described the use of a modified magic square of four in a treatise by Varāhamihira (6th c.), for the determination of the quantities of the sixteen ingredients of a perfume. Two series of numbers I to 8 were used, instead of the sixteen first numbers, probably so that the differences between the quantities of the ingredients would not be exaggerated. Varāhamihira seems to assume that his readership is familiar with these squares, since he does not present the square formally, just naming it kacchaputa. This term, originally denoting «carapace of a turtle», reveals a clear Chinese origin, since Chinese sources traditionally traced the magic square of three back to a first such square inscribed on the carapace of a turtle.7 It is striking that Varāhamihira, along with the medical authors who used the square from the 10th c. onwards, only used modified magic squares adapted to their cultural needs, and to perform functions similar to native artifacts such as *yantras* and mandalas. Indeed, the first attested medical magic square displays the even numbers 2 to 18, rather than the natural 1 to 9. There is a reason for this: namely, in this configuration they could use the number 16.16 appears in the first position in the Indian sense of the script, and is referred to by a marked term;⁸ in view of the prestige of this number in Hindu numerological culture, this cannot be a coincidence.9 We know that other diagrams had been used for the same function in earlier medical literature,¹⁰ and that in the same treatise Varāhamihira prescribes traditional yantras (meditation images) to define proportions for the construction of temples, implying that, by the time of the earliest known Indian attestations, magic squares seem to have been fully adapted to Indian culture. It is therefore

5. J. NEEDHAM, Theoretical Influences of China in Arabic Alchemy, Coimbra 1980.

6. ROşU, «Études āyurvédiques»; T. Hayashi, «Varāhamihira's pandiagonal square of the order four», *Historia Mathematica* 14 (1987), 159-166.

7. HAYASHI, «Varāhamihira's pandiagonal square» 161.

8. Roşu, «Études āyurvédiques» 97.

9. D. G. WHITE, The Alchemical Body. Siddha Traditions in Medieval India, Chicago/London 1996, 36-44.

10. A. Roşu, «*Mantra* and *yantra* in Indian medicine and alchemy», *Ancient Science of Life* 8.1 (1988), 20-24.

likely that magic squares had been known for a very long time, even if they had not been widely used. They probably reached India together with other Taoist imports between the fourth and the sixth centuries C.E., a period of massive cultural exchange with China.¹¹

It would be wrong not to mention S. Cammann, who, long before the evidence from the earliest Indian attestations was found, had already proposed that India was an intermediary for the transmission of the squares from China to the Islamic world.¹² Interestingly, he supported his claim by arguing that the square of three in the epistle on geometry of the Brethren of Purity derived from applying a well-known Chinese construction method to a natural square filled with numbers I to 9 in the natural order and in the Indian sense of the script. In fact, even if we just look at the squares that appear in all principal manuscripts (of order 3, 4 and 5),¹³ they do not seem to be taken from a mathematical treatise. Close inspection of those of order 4 and 5 reveals that they are both obtained from the respective natural squares through displacements (the most primitive method), but that the original natural squares were filled beginning from different angles. It seems, then, that the Brethren had basic knowledge of how to construct squares, and that they wrote their examples ad-hoc and did not copy them from a systematic work.14 In fact, some of the larger squares are built using try-and-failure15 rather than with the general methods developed in the mathematical treatises.¹⁶ In

II. WHITE, The Alchemical body 62.

12. S. CAMMANN, «Islamic and Indian magic squares. Part I», *History of Religions* 8.3 (1969), 181-209, at 190-191.

13. See the new edition by N. EL-BIZRI, *Epistles of the Brethren of Purity. On Arithmetic and Geometry. An Arabic Critical Edition and English Translation of Epistles 1 & 2*, Oxford 2012, VE--VEY / 156-158.

14. HAYASHI, «Varāhamihira's pandiagonal square» 164 posited a transmission link between the square in Varāhamihira and Islamic magic squares, because the original magic square from which Varāhamihira's was modified was probably the same as the most pervasive Arabic magic square of four, which is the one that appears in the Brethren. However, they were more probably independent developments, since the method is primitive and natural (see, e.g., the explanation of the method in al-Būzjānī).

15. CAMMANN, «Islamic» 191-194.

16. There is a second square of three in the epistle of the Brethren, written in *abjad* numerals and displaying a different position. However, since it is not referred to in the text (unlike the other squares) and because it would merely repeat the square already pictured, I take it to be a later addition illustrating the text. Cf. EL-BIZRI, *On Arithmetic and Geometry* VET / 159. CAMMANN, «Islamic

any case, the appearance of the square of three in the Brethren's epistle cannot be taken as evidence of its origins, because they probably did not copy it from any text. Instead, we should look at the square of this order in Jābir ibn Ḥayyān and al-Ṭabarī, which both display the same disposition of numbers.¹⁷ Coincidentally, it turns out that it can be obtained in the way suggested by Cammann:

THE ADAPTATION OF INDIAN MEDICAL SQUARES IN AL-TABARI

Thanks to the work of Roşu we know that magic squares were prescribed for easing labor by a series of medical authors from ca. 900 C.E. onwards.¹⁸ The first of these physicians, Vrnda, describes a ritual including the contemplation of the doubled square of three (with the number 16 in the first cell) among other treatments for easing delivery, such as drinking herbal preparations or the application of herbal pastes or roots to different parts of the body including the vagina. In particular, according to him the woman must look at the square while she drinks water previously incantated by the recitation of a mantra from the classical physician Suśruta. Later authors quoted this treatment with slight variations, such as Cakrapāṇi in the mid-11th c. C.E., who added the standard magic square of three (without doubling). Cakrapāṇi's 15th c. commentator Śivadāsasena states that after Vrnda they all prescribed this ritual, in which the magic squares were drafted on a consecrated vase.¹⁹

and Indian magic squares» 190 indicates that the square in Jābir was written in abjad numerals, but this is obviously a mistake; he was probably thinking of the abjad square in the Brethren.

^{17.} There is only one possible configuration of the square of three but, counting rotations and symmetries, it can offer eight different appearances.

^{18.} Roşu, «Études āyurvédiques» 96.

^{19.} Roşu, «Études āyurvédiques» 98.

The points of contact with al-Ṭabarī are numerous and strikingly accurate. First of all, it must be borne in mind that this Persian physician undertook a synthesis of Syriac, Greek, Persian and Indian materials in his *Firdaws al-Ḥikma*, quoting from the classical Ayurvedic physicians as well as from unidentified Indian authors. Arabic translations of Indian medical texts were readily available in his time, mainly coming from Gundishapur in southern Persia, an important medical hub for both Christian Nestorian and Indian physicians.²⁰ In his chapter on easing labor, al-Ṭabarī introduces the square ritual as something magical (*šay'un 'ajībun*) ascribing the prescription to his father Sahl Rabbān.²¹

تأخذ خزفتين من كوز او جرة جديدة لم يصبّها الماء وتصور عليها شكلاً مثل هذا وتكتب فيها حساباً كيف ما حسبت طولاً وعرضاً او من زاويةٍ الى زاويةٍ كان خمسة عشر وتكتب حولها آيتين من الزبود وتؤتى بهما المرأة حتى تنظر الى ما فيهما من الكتابة نعماً ثم تضعهما تحت قدميها

You take two pieces of ceramics, from a new jug or jar in which no water has been poured, and draw upon them a figure like this, using numbers so that they add up to fifteen horizontally or vertically or from corner to corner. Write two lines [of the psalms] of David surrounding the figure, and bring them to the woman, so that she can see the writing on them well. Then place them under her feet.

Together with this, al-Ṭabarī proposes making a paste from the mud from a nest of swallows (*min 'uši al-khaṭāṭīfi*) and rubbing with it the pubic hair and the waist, as well as placing a coriander root in the vagina. He then proceeds to give the numbers for the square and the Arabic translation of the verses of the Psalms, which are written in Syriac around the figure in the manuscripts:

هو بالسريانية وتفسيرها اخرج نفسى من المحبس لأشكر اسمك وليؤملنى ايرادك اذا انت كافينى

20. See e.g. *Encyclopaedia Iranica* IV.3, s.v. *BOKTIŠU*⁺ (L. Richter-Bernburg); H. D. ISAAKS, «Arabic medical literature», in M. J. L. YOUNG, J. D. LATHAM, and R. G. SERJEANT, *Religion, Learning and Science in the 'Abbasid Period*, Cambridge 1990, 342-363, at 343-345.

21. Text in M. Z. SIDDIQI, Firdausu'l Hikmat or Paradise of Wisdom, Berlin 1928, p. 280. The whole chapter is translated into German in A. SIGGEL, «Gynäkologie, Embryologie und Frauenhygiene aus dem "Paradies der Weisheit über die Medizin"», Quellen und Studien zur Geschichte der Naturwissenschaften und der Medizin 8 (1941/42), 216-272. They are in Syriac and their meaning is this: «Let my soul go out of the prison, so I can thank your name. May your will give me hope, as you have rewarded me».²²

Elements that are also found in the Indian ritual are the ceramic support of the square (here apparently two *ostraka*) and the need to look at it, the making of a paste to be rubbed on the pubic region, and the placing of a root of a plant in the vagina,²³ all of which appear close to one another in the Indian texts. Crucially, al-Ţabarī also prescribes drinking incantated water in the same chapter:

تأخذ حاشية ثوبٍ جديدٍ فتكتب عليها انفتح باب السماء فخرج منه سبعة من الملائكة بايديهم قضبان من نار بالله لا ذهبتم الى فلانةٍ بيت فلانةٍ حتى تخرجوا ما في بطنها حياً كان او ميتاً [...] ثم تأخذ ماءً عذباً وتجعله في جام وتغمس الخرقة فيه وتعصره وتسقيه تلك المرأة

You take the rim of a new dress and write upon it: «The door of heaven was opened. Therefrom came seven angels holding torches. By the Lord, you have not yet visited so-and-so in the house of so-and-so, in order that she gives birth to what she has in her belly, be it alive or dead [...]». Then you take fresh water, pour it in a cup and plunge the shred in there; wring it out and have the woman drink it.

Both this passage and the verses written around the magic square are remarkably similar to the mantra from Suśruta from the Indian ritual, where it is likewise prayed that divine entities should visit the house of the woman, and where the child is explicitly portrayed as imprisoned in the womb.²⁴ In my opinion, this is sufficient proof that both prescriptions in al-Ṭabarī—that of the square with paste and root, and the water-drinking ritual—were imported and adapted from contemporary Indian medical practice, probably from a written source. The Biblical verses might suggest that the adaptor was a Christian Nestorian author, perhaps al-Ṭabarī's father himself.

22. Corresponding to Psalm 142 line 8. Cf. SIGGEL «Gynäkologie» 254 n. 3.

23. The plant in Cakrapāṇi is pāthā. The plants do not coincide, but this seems natural as most of them are endemic to the Indian subcontinent in Ayurvedic texts.

24. The mantra is transcribed in P. V. SHARMA, *Chakradatta*. *Text with English Translation*, *Delhi* 1994. Cf. translation and comments of J. FILLIOZAT, *Études de démonologie indienne*, Paris 1937, at 31-34.

But let us now examine the differences. In the rituals in al-Ṭabarī, words do not act at a distance as they do in the Indian ritual, but by direct contact with the Biblical text, either together with the square and passed through the feet, or dissolved in water wrung from a cloth and directly entering the body through drinking.

Secondly, al-Tabarī prescribes that the pieces of ceramic be placed under the feet. The condition that they come from a new vase — in which no water has been poured²⁵—can perhaps be considered as equivalent to the Indian consecration ritual which, according to the commentator Śivadāsasena, needs to be performed on the drinking vase. The dress from which the rim is taken should also be new. The reason why the contact should be made under the feet is more difficult to fathom. Pastes are only rarely applied to the feet in Indian texts, and never specifically on the soles. One possibility would be that al-Tabarī was alluding to the *hadīth* about a would-be-warrior asking for advice to the Prophet, who responds that he should stay with his mother, «for paradise is beneath her feet»(al-janna tahta rijlayhā).²⁶ Since this piece of oral tradition was already recorded by al-Nasā'ī in the 9th c. C.E., this does not seem a far-fetched hypothesis, considering that al-Tabarī completed his work around 850 C.E. and that he used not only Biblical texts in his remedies, but also lines from the Qur'ān.²⁷ However, as far as I know, elements from the Bible and the Qur'an in other magical remedies prescribed in the Firdaws are incorporated directly in the prescriptions as some form of written text, and so this would be the only case of its kind.

We will see below that the other attestations of the square of three show a great affinity with the so-called *Cyranides*, an originally Greek medico-magical manual. As it turns out, the *Cyranides* recommends the laboring woman to hold a feather of a swallow and a root of basil for an easy delivery (I 24), quite similar to al-Ţabarī's paste (from the same bird!) and root. Elsewhere in the text, it is said that an eagle's feather must be placed under the feet of a woman suffering from a difficult labor in order to achieve a quick delivery.²⁸ Familiarity with this text is

25. Siddiqi's edition does not contain this phrase, which appears at least in MS BL Arundel Or 41 f. 135r, and which I have printed in the text above. He may have considered it to be a gloss.

27. Surah 18, line 25 is to be written on a piece of paper by a woman who feels that she is about to have a miscarriage. Cf. SIGGEL, «Gynäkologie» 258 n. 1.

28. Cyranides 3.1: ἀετοῦ δὲ πτερὸν ἐάν τις ὑπὸ τοὺς πόδας θήσῃ δυστοκούσῃ γυναικί, πάραυτα τέξεται.

^{26.} NASĀ'Ī, Jihad 6.

much more probably the reason why al-Ṭabarī places the two *ostraka* under the woman's feet.

I will now devote a few lines to what we know about Sahl Rabbān al-Ṭabarī, the father of the author of the *Firdaws* and prescriber of the square-ritual. In his biographical encyclopedia Al-Qiftī describes him as a Jew, which has been interpreted as a misunderstanding of his name Rabbān, which in fact means «senior teacher and leader», as recounted by his son in the medical encyclopedia.²⁹ Interestingly for our purposes, al-Qiftī also says that Rabbān was an astronomer or astrologer (the same word in Arabic, *munajjim*), a physician experienced in geometry and all branches of mathematics (*tabībān 'ālimān bi l-handasati wa'anwā'i al-riyādati*), and a translator of scientific texts (*halla kutubān hikmiyyatan min lughatin 'ilā lughatin*). It is doubtful, however, that his mathematical interests led him to develop methods for constructing magic squares of higher order, given the late date of the first known mathematical treatises (end of 10th c. C.E.).

Sahl Rabban is sometimes cited as being the first translator of Ptolemy's Almagest.³⁰ though this attribution relies on al-Oiftī's account, which is also probably mistaken here. Al-Qiftī³¹ explains that the astrologer Abū Ma'šar could only find «the place where the rays are cast» (matrāhu al-šu'ā'i) in al-Tabarī's translation of Ptolemy's Almagest. However, no casting of rays is discussed in the Almagest. This is a standard astrological theory, according to which planets cast rays influencing certain parts of the zodiac through aspect or vicinity. Ptolemy briefly discusses ray-casting in the Tetrabiblos (I 24), but does not specify the arcs around the planets (the orbs) affected by rays, what Abū Ma'šar was looking for. The translator al-Tabarī could have provided the customary orbs himself as a gloss. If this is the work Abū Ma'šar was referring to, then al-Ṭabarī should instead be counted among the first translators of the Tetrabiblos; this in fact makes more sense considering the difficulties entailed in the translation of the Almagest and Rabbān's early date. Actually, al-Qiftī describes the versions that did not contain that doctrine as «old» (qadīma), seemingly implying that there were many previous translations.

^{29.} S. K. HAMARNEH, Health Sciences in Early Islam II, Texas 1983, 353-358.

^{30.} O. PEDERSEN, A Survey of the Almagest, New York 2010, 16; H. Suter, Die Mathematiker und Astronomen der Araber, Leipzig 1900, 14.

^{31.} Text in J. LIPPERT, Ibn al-Qiffī's Ta'rīh al-hukamā', Leipzig 1903, 187.

Early 10th c.: Jābir ibn Ḥayyān, «Ghāyat al-Ḥakīm», Brethren of Purity

In one of the short treatises titled Book of Balances and attributed to the alchemist Jābir ibn Ḥayyān, conventionally dated to the beginning of the 10th $c.,^{32}$ we find a short presentation of the magic square of three and its medical use, for which Bālīnūs is given as a source.³³

هذه الصورة التي عددها ثلثة طولاً وعرضاً وقطرها خمسة عشر من كل جهة وبالينوس زعم انّها من عقد السحر وهي تسعة بيوت وهذه صورتها فاذا كتبت هذه الصورة على خرقتين لم يصبّهما الماء ووضعت ما تحت رجل المرأة التي قد عسر عليها ولادتها ولدت

Here is a figure whose number is three lengthwise and crosswise. Each diameter adds up to fifteen in all directions. Bālīnūs claims that this is the figure of the magic knot, and it is formed of nine cases and this is its figure. If you draw this figure on two pieces of cloth in which water has not been poured (sic) and you place them under the foot of the woman with difficulties in delivery, she will deliver.

It is interesting that the pieces of ceramic have been replaced here by two pieces of cloth, creating a difficulty with the condition that «no water has been poured in them» (*lam yaşubbu humā al-mā'u*). This probably has to do with the great similarity in the Arabic script of «two pieces of cloth» and «two pieces of ceramics» (خزفتين/خرقتين). As we will see, in the epistle of the Brethren of Purity the pieces of ceramic are maintained but they are said to be hung on the woman instead of placed under her feet. If this Bālīnūs was the common source, his text may not have included al-Ṭabarī's explanation that the pieces of ceramic are taken from a jar, leaving open the possibility of understanding the «two ceramics» (*khazafatain*) as two whole ceramic jars. They could not therefore have been placed under the feet, and the author of the Jābirian text would have interpreted them as being pieces of cloth instead, perhaps influenced by the magical use of a

^{32.} D. R. HILL, «The literature of Arabic Alchemy», in Young, Latham and Serjeant, *Religion, Learning and Science* 334.

^{33.} Text in the collection of sources accompanied with a French translation by M. BERTHELOT, *La chimie au Moyen Âge III. L'alchimie arabe*, Paris 1893, *NN* / 150.

menstruation cloth (*khirqa*) which he records just a few lines above, and/or because some talismans were written on cloth.³⁴

The *Ghāyat al-Ḥakīm*, recently shown to be the work of Maslama al-Qurṭubī (d.964 C.E.) and not of the later author al-Majriṭī,³⁵ and thus roughly contemporary to the Books of Balances, has a very brief and unclear mention of the square of three—perhaps showing that the mathematical definition was not understood—and its medicinal power (IV 8).

The number fifteen in three houses is the agreement for the difficulty of birth.

We see here the first instance of the later ubiquitous term indicating «magic square», wafq, primarily meaning «agreement» or «concord». However, the idea was not new, since the text of Jābir contains another term (*aqd*) also denoting the concept of fitting together, or strong bond, and one of these words (or a similar one) must have been present in their source. One could compare it with the English verb «to square» or the Spanish «cuadrar». Apart from this, we can see that both texts show an element not present in al-Ṭabarī, namely the term *bayt* («house») denoting the cells.

Let us now consider the presentation of the square of three in the Brethren's encyclopedia. For chronological purposes, it is useful to bear in mind that al-Qurtubī is thought to have been responsible for the dissemination of the encyclopedia in al-Andalus, and that he most probably copied the text during his trip to

34. Cf. above on the shred of a new dress that al-Ṭabarī prescribes to be written on, and also *Ghāyat* II 9 (*hadhihi al-ṣūratu fī khirqatin jadīdatin*). Also strange is the reading «the foot of the woman» (رجل المراة) instead of «the two feet of the woman (رجلي المراة), but this may be due to the fact that a yā' without dots after a *lām* can be difficult to distinguish.

35. M. FIERRO, «Bāţinism in al-Andalus. Maslama b. Qāsim al-Qurṭubī (d. 353/964), Author of the Rutbat al-Ḥakīm and the Ghāyat al-Ḥakīm (Picatrix)», *Studia Islamica* 84.2 (1996), 87-112. Cf. G. DE CALLATAŸ and S. MOUREAU, «Again on Maslama Ibn Qāsim al-Qurṭubī, the Ikhwān al-Ṣafā and Ibn Khaldūn: New Evidence from Two Manuscripts of Rutbat al-ḥakīm», *Al-Qanṭara* 37.2 (2016), 329-372, at 331; M. RIUS, «Ibn al-Qāsim, Maslama», *Biblioteca de al-Andalus* 4 (2006), 371-375.

the Middle East in the 930s.³⁶ This seems to be a reasonable assumption, since the first known mathematical treatises on magic squares were written toward the end of the 10th c., and, as mentioned above, the squares in the encyclopedia are constructed with primitive and not entirely systematic methods, even if we do not accept the failed squares as genuine. The text after the presentation of the squares is as follows.

وعلى هذا المثال سائر الأعداد والأشكال، إذا جمعت بينها ظهرت منها خواصِّ أخر. وأمّا منافعها والفائدة منها، فقد ذكرنا ذلك في رسالة الطلسمات والعزائم، وأوردنا طرفاً منها، ولكن نذكر منها في هذا الفصل مثالاً واحداً ليكون دلالةً على صدق ما قلنا. فنقول: إنّ من خاصّيّة هذا الشكل المتسع ومنفعته الولادة إذا كُتب على خزفتين لم يصبهما الماء وعلّقتهما على المرأة التي ضربها الطلق، وإن اتّفق أن يكون القمر في التاسع متّصلاً بربّ التاسع سهّل الولادة، او بربّ بيته من التاسع وما شاكل ذلك من المتسعات

It is by this example that other numbers and forms are combined in such a way that other properties of theirs appear. We have noted some of the aspects of the benefit and interest of this in the epistle on talismans and incantations, and we mention one example in this chapter so that it might point to the truthfulness of what we stated there. We therefore say that [in the case of] the ninefold figure, its benefit is in easing birth if it is drafted on two pieces of ceramic, in which water has not been poured, and these are hung over the woman in labor, then, if the moon happens to be in the ninth [house] and is connected with the lord of the ninth, birthing is eased; or also [if] the lord of its house³⁷ is in the ninth; and similarly with [other] ninefold things.³⁸

This text represents a genuine step forward: it is the first text that introduces astrological concepts, treating the square as an actual talisman. It is hard to determine where this idea comes from. I will return to this issue below, but for now it is worth stressing that we have three roughly contemporary sources mentioning the medical use of the square of three, with very similar wording, and relatively close chronologically to what may be the very first appearance of the square in Arabic literature. The most economical hypothesis is that all three share the same

^{36.} See DE CALLATAŸ and MOUREAU, «Again on Maslama» 335; H. F. al-Hamdānī, «Rasā'il Ikhwān aṣ-Ṣafā in the literature of the Ismā'īlī Ṭaiyibī Da'wat», *Der Islam* 19 (1931), 281-300.

^{37.} The «house of the moon» is the 3rd.

^{38.} EL-BIZRI, Epistles 12E-128 / 158-159.

source. However, this source was probably not the *Firdaws*, since there the ritual was clearly explained (particularly the *ostraka* and their placement) and we would have expected it to be transmitted faithfully.

One clue to answering this question is to remember that the Jābirian text presents Bālīnūs as its source. Bālīnūs was the alleged author of a number of Arabic texts transmitting Hermetic material, especially on talismans, for which he received the epithet «master of talismans»; he seems to have been identified with the Greek Pythagorean sage, Apollonius of Tyana, who is believed to have travelled to India and gathered knowledge from the Brahmans.³⁹

Unfortunately, the known extant texts explicitly attributed to $B\bar{a}I\bar{n}u\bar{s}$ do not contain any mention of the magic squares,⁴⁰ but further indications can be found by comparing our available attestations. Looking at the context of the passages on the square of three in the J \bar{a} birian text and the $Gh\bar{a}ya$, we will discover the occurrence in both of them, only a few lines above, of the same marvellous «fact»: that a naked menstruating woman lying down facing the sky could prevent a field from being attacked by hailstorm.⁴¹ The J \bar{a} birian text also mentions that she would deter lions, adding other surprising natural laws in relation to human menstruation and animals. Similarly, chapter IV 8 of the $Gh\bar{a}ya$, which contains the reference to the square, lists curious facts of nature related to animals, plants, and stones; the author begins by revealing his mysterious source, a book found in a Syrian temple in the times of Cleopatra (*zamān Qlūbatra*).

As mentioned above, the *Cyranides* is a Roman-era Greek Hermetic text of a medico-magical nature, prescribing remedies made of animal parts. In the form that has reached us, it contains a first book divided into the 24 letters of the alphabet—each one featuring a bird, a plant, a stone, and a sea animal, sometimes describing the construction of talismans—and three more books structured in alphabetical entries, dedicated to four-legged animals (II), birds (III) and fish (IV);

39. See U. WEISSER, Das «Buch über das Geheimnis der Schöpfung» von Pseudo-Apollonios von Tyana, Berlin 1980, 10-28; F. SEZGIN, Geschichte des Arabischen Schrifttums IV, Leiden 1971, 77-91.

40. See a review of the contents of the extant texts in WEISSER, *Das Buch über das Geheimnis* 28-39.

41. *Ghāya* IV 9, *Book of Balances* \\\\ / 150. This idea reappears in the agronomical treatise attributed to Ibn Wāfid (J. SAMSÓ, *Ciencias de los antiguos en al-Andalus*, Madrid 1992, 303-4), not surprisingly given that the vast majority of the material in the text is derived from Greek and Byz-antine sources; cf. J. R. GUZMÁN ÁLVAREZ, «El compendio de agricultura atribuido a Ibn Wāfid / al-Nahrāwī: nuevas perspectivas sobre su autoría», *Anaquel de Estudios Árabes* 16 (2005), 83-124.

however, ancient and medieval references to its contents make it clear that more material was available to them than has been preserved.⁴²

The work enjoyed vast success in Islamic and European literature, as witnessed by extant Arabic, Latin, Old French and English versions,⁴³ and also by the fact that various medico-magical handbooks sought prestige by attaching the name Cyranides to their titles.⁴⁴ This is the kind of famous book on the genre «talismans and medicine» that might have attracted the attention of our three sources at roughly the same time. Pending a more systematic investigation, I have encountered three close parallels of marvellous facts in the Jabirian text and the Cyranides: on eating the flesh of vipers,⁴⁵ on wearing the tongue of a hyena,⁴⁶ and on eating crabs.⁴⁷ Furthermore, the first book of the extant Cyranides quotes a certain Harpocration, who poses as the author of the book, and who declares to have found this Hermetic knowledge written down on a pillar in a temple of Little Alexandria, near Babylon, where he asked an old Syrian man to translate it for him (I prol.). On the one hand, this story seems to match al-Qurtubi's claim to have gathered the marvellous facts he lists in a book found in a temple in the time of Cleopatra. The author of the Ghāya may have brought in the name of Cleopatra because of the famous ancient alchemist with that name, perhaps identifying her with the Egyptian queen (cf. the frequent Arabic assimilation of the astronomer Ptolemy with the King of Egypt). A similar strategy could have been followed by the author of the Jābirian text, who may well have seen in the Syrian translator the figure of the talisman-maker and author Bālīnūs, an important authority in the Jābirian corpus. Indeed, Bālīnūs frequently adopted the role of pupil, translator, or discoverer of Hermetic knowledge in occultist Arabic literature,⁴⁸ and Philostratus relates that

42. David BAIN, «Some textual and lexical notes on *Cyranides», Classica et Mediaevalia* 47 (1996) 151-168, at 152.

43. Bodleian Library MS Arab. d.221 contains the first book in an Arabic translation. For the medieval Latin and Old French versions, see L. DELATTE, *Textes latins et vieux français relatifs aux Cyranides*, Liège 1942.

44. BAIN, «Some textual and lexical notes» 153. Cf. the texts included in DELATTE, *Textes latins et vieux français*.

45. Elephantiasis and other diseases are healed thereby; Cyranides 2.12; Book of Balances NA / 151.

46. To protect against dog attacks; Cyranides 2.40; Book of Balances 119 / 151.

47. A remedy for scorpion bite; Cyranides 4.28; Book of Balances \\A / 151.

48. Consider, for example, the famous story of the Tabula Smaragdina, a piece of the *Hermetica* allegedly transmitted by Bālīnūs. See also an account where he is described as a student of Hermes

Apollonius of Tyana was retained in Babylon by the king Vardanes for almost two years (1.22, 1.39); furthermore, although he was not a Syrian, he appears to have spent a long time in Antioch.⁴⁹

Now, is it conceivable that an Arabic translator/compiler of the *Cyranides* could have been active excerpting the *Firdaws* and incorporating the square after the completion of the *Firdaws* in 850, just some time before our sources were written at the beginning of the 10th c — and that his work would have enjoyed enough success to be available and attractive to the three of them? I think it is more probable that this work was done by Sahl Rabbān al-Ṭabarī, who was active at the beginning of the 9th c., bearing in mind that he was a physician interested in magical rituals and a translator of Greek scientific works, who seems to have had a penchant for adding his own supplements. The geometrical term *qutr* («diameter»), which appears in the Jābirian presentation of the square and not in the *Firdaws*, may point towards this conclusion, given Sahl Rabbān's expertise in geometry according to al-Qiftī. It is even possible, though not certain, that he is the astrologer by the name of Ṭabarī

ASTROLOGY

I will now discuss the astrological content which the Brethren assign to the square of three. After a presentation of the elements of the medico-magical ritual comparable to the other attestations, the text (quoted above) declares that birthing is easier if the moon is in the ninth house and in aspect with the lord of the ninth, or if the lord of the house of the moon (the third) is in the ninth house; and, it adds,

in K. VAN BLADEL, *The Arabic Hermes. From Pagan Sage to Prophet of Science*, Oxford 2009, 158-159, and a text where he appears as a translator of Hermes: D. PINGREE, «Al-Ţabarī on the Prayers to the Planets», *Bulletin d'études orientales* 44, 105-117, at 112.

^{49.} Phil. Vit. Apol. 1.16. Malalas (10.51) records that Apollonius built apotropaic statues in Antioch.

^{50.} An astrologer named Țabarī is cited as a source for the prayers to the planets in III 7. PIN-GREE, «Al-Țabarī on the Prayers» discovered a Latin translation of the original text, in which the author is named *abuelabec altanarani*, and attempted to prove that this was the older astrologer 'Umar ibn Farrukhān al-Țabarī. However, the argument is not very convincing (cf. BLADEL, *The Arabic Hermes* 100 n. 157) and he could also be our astrologer Rabbān al-Ṭabarī. The corrupted Latin name could have resulted from the translator's confusion between Ṭabarī and the philosopher, astronomer, and historian of religion Abū al-'Abbās Irānshahrī.

similarly for other ninefold things.⁵¹ The Brethren also mention the magic square of three in the epistle on music (ch. 14). The context is an exposition of the tetrad formed by the four seasons (e.g., spring) and its correspondence with other wellknown tetrads: the part of the zodiac traversed by the sun during that season (e.g., from Aries to Gemini), a quarter of the month (e.g., the first week), one of the four elements of nature (e.g., air), etc. In the conclusion, as examples of this sort of combination, the Brethren mention medicines and talismans.

المثال في ذلك الشكل المتّسع في تسهيل الولادة إذا كُتب فيه الأعداد التسعة في الشهر التاسع من الحمل في الساعة التاسعة من الطلق، ويكون ربّ الطالع في التاسع او ربّ التاسع في الطالع، او يكون القمر في التاسع او متّصلاً بكوكبٍ منه في التاسع، وما شاكل ذلك من المتّسعات

An example of this is the ninefold square which eases childbirth when the nine numbers are entered in the ninth month of pregnancy at the ninth hour of labor, the lord of the ascendant being in the ninth [house], or the lord of the ninth in the ascendant, or the moon in the ninth, or in aspect with a planet in the ninth from it, and similarly for other ninefold things.⁵²

It is striking that only one of the ninefold connections given coincides in the two accounts (namely, the moon being in the ninth house). It is unusual that a maker of talismans should allow so much freedom. Furthermore, since the magic square was a relatively new device, it is unlikely that many different writers on talismans would have dealt with the subject and passed on their methods for making this particular talisman, all them being collected here. This may indicate that the Brethren improvised these astrological relations. Also, there is very little astrology in the extant version of the *Cyranides*.⁵³ We cannot rule out the possibility

51. Astrological houses go back to several Hellenistic systems for dividing the zodiac into twelve sections beginning with the ascendant, the most relevant being the four cardinal points or cardines, corresponding to 1st (ascendant), 4th (*immum caelum*), 7th (descendant), and 10th (mid-heaven). They were frequently identified with the zodiacal signs, building a one-to-one correspondence beginning with the ascendant sign and running counter-clockwise through the zodiac. Their lords should be understood as the planets governing the corresponding zodiacal signs.

52. Text in O. WRIGHT, Epistles of the Brethren of Purity. On Music. An Arabic Critical Edition and English Translation of Epistle 5, Oxford 2010 \\\-\\\/ 159.

53. BAIN, «Some textual and lexical notes» 153.

that Rabbān al-Ṭabarī, being acquainted with astrological theory, had already accompanied his ritual with astrology, but in view of the fact that he adapted the Indian ritual so closely, and since the Indian medical sources do not bring astrology into the discussion, this seems to me to be improbable.

As to the Brethren's possible motivation for adding astrological content, in chapter 9 of the epistle on music, Hindus are included in a list of human groups who have a preference for a certain number in their ordering of the world. Hindus are said to give prominence to sets of nine,⁵⁴ and indeed there are many important ninefold entities in Indian cosmology which the authors might have had in mind — even correlating with each other in the way that interested the Brethren, such as the nine heavenly bodies or the nine Nāths.55 The Brethren's insistence on the number nine in the astrological relations of the magic square may be due to their knowledge of these elements of Indian culture, to the correct presupposition that the square came from India, and to the intention of presenting the square as an object in which multiple ninefold relations were in action.⁵⁶ Given that the ninth house, or a distance of nine houses, plays an important role in both lists of astrological conditions, the Brethren could have drawn inspiration from their source, if it described the square as being divided in nine houses $(buy\bar{u}t)$ – and in fact we can be confident that it did, since the term appears in both the Jabirian account and the Ghāya.

Finally I will comment briefly on one aspect of the association of magic squares with the seven planets. The account in the encyclopedia of the Brethren is the only text where magic squares are attributed astrological significance without being associated with planets. Later astrological writers assigned the squares of orders 3 to 9 to the seven planets in a one-to-one correspondence, linking the creation of the talismans to the astrological configuration and the properties of the planets. However, it is less well known that two systems in fact existed. One of them as-

54. WRIGHT, *Epistles* \mathbb{W} / 136. The Ismā'īli sect of the Kayyāliyya is also said to have preferred sets of nine, but Shahrastani's review of their cosmology does not give much prominence to this number compared with three and five (*Book of Sects and Creeds* ch. 6.4.g).

55. The nine Nāths, originally historical Buddhist monks, were identified with other cosmological sets of nine like the nine bodily orifices; similarly, the planets were assigned nine gemstones by alchemical authors. See WHITE, *The Alchemical Body* 91 *and* 159.

56. The importance of this level of interconnectedness in the *Epistles* is attested in the fact that in just the short chapter on magic squares (26) of the epistle of geometry they cite the epistle on arithmetic, the one on talismans, and the one on music.

signed the square of three to Saturn, that of four to Jupiter, and so on in the sequence of the spheres until the moon, which received the square of nine; this is the most common, appearing in the texts of al-Zarqālī, al-Būnī, Agrippa and Paracelsus (system I). Cardano, however, uses a system that begins with the moon, which corresponds to the square of three (system II). As B. Hallum has shown, the second system also appears in the Arabic tradition, being found in several anonymous texts which in some cases present a discussion of the reasons for choosing this system and not the other.⁵⁷ Awaiting a study of these manuscripts by Hallum, I am inclined to believe that system I is older, for a number of reasons. First, in a relatively short span of time it spread more widely, which is often a mark of antiquity in this sort of texts: more often than not, authors tended to pick the more ancient system, precisely because antiquity conferred prestige. Furthermore, some accounts of system II refer to system I in order to attack it, but this does not occur the other way around. Finally, the order of the planets from Saturn to the moon is the traditional and by far the most frequently attested one in Arabic talismanic authors, so it would have seemed natural to the first authors assigning squares to planets to assign the first square (that of three) to the first planet, Saturn, and so on. However, this does not mean that no further justification was necessary. Al-Zarqālī's treatise, which is structured in sections from Saturn to the moon, begins the first section on Saturn with a brief reference to the importance and the meaning of the ninth house («voyage») as well as its relation with the moon, but this is later not specifically brought into relation with Saturn or the squares.⁵⁸ It seems then that the author attempted to incorporate to some degree the account of the Brethren, which linked the square of three with the ninth house and the moon, even though it played no role in his system of squares. This move can be understood as a non-confrontational strategy in a field in which authority was ultimately bestowed by tradition and antiquity.

^{57.} British Library Or. 5591, ff. 208v-209r; British Library 2361, f. 267r; Chester Beatty Ar. 5078, f. 113v-114r. I owe this information to B. Hallum.

^{58.} As read from British Library Add. 9599, f. 133r.