Al-Bīrūnī: The plate of the eclipses

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Abstract: This paper focuses on an extract from the treatise of al-Bīrūnī (973-1048 AD) *Comprehension of the possible ways for the construction of the astrolabe*, where "the plate of the eclipses" is described. This is a device that can be attached on the back side of the astrolabe. It consists of a plate, engraved on both sides, and a grid that can be attached to either side of the plate and can rotate upon it. Given the date of the lunar month, one can find the time of the moon rising and the phase of the moon, using the front side of the plate. Knowing the latitude of the moon at the opposition, one can determine whether there will be a lunar eclipse or not, using the back side of the plate, and can also estimate the magnitude, start time and duration of the eclipse. The results are approximate.

Keywords: astrolabe, lunar eclipse, al-Bīrūnī, *falak al-jawzahar*, Nasṭūlus, lunar phases.

Introduction

In the last three chapters of the treatise of al-Bīrūnī *Comprehension of the possible ways for the construction of the astrolabe*,[1] he describes in detail the construction of three devices that can be attached on the back side of the astrolabe. All of them are related to the moon.

The first one, called "the receptacle of the moon",[2] is a mechanism with gears that displays the positions of sun and moon in the Zodiac, the day of the week, the date of the lunar month and the phase of the moon. Four different models are presented, all of them giving approximate results, as al-Bīrūnī mentions.

1. There is an Arabic edition of this work: Al-Bīrūnī, *Istī'āb al-wujūh al-mumkina fī ṣan'at al-āsṭurlāb* (ed. al-Ḥosaynī), Mujam'a al-buḥūth al-islāmiyya, Meshhed, 1422 H (2001).

2. The corresponding text of al-Bīrūnī has been edited and translated into English by Donald R. Hill, "Al-Bīrūnī's Mechanical Calendar" in *Annals of Science*, 42 (1985), pp. 139-163. There is also an older description of this instrument in Eilhard Wiedemann, "Ein Instrument, das die Bewegung von Sonne und Mond darstellt, nach al-Bīrūnī", in *Der Islam*, 1913, vol.4, issue 1, pp. 5-13.

The second one, called "the plate of the eclipses", is described by authors of the 9th and 10th centuries AD.[3] It is a plate equipped with a grid that can be attached to either side of the plate and can rotate upon it. This instrument can illustrate the phase of the moon and tell the time of the moon's rising, if the date of the lunar month is known. It can also help in predicting lunar eclipses, if the latitude of the moon at the opposition is known, and provide additional information concerning the magnitude, start time and duration of the eclipse. The results are again approximate.

The third one is an instrument designed for determining the lunar crescent's visibility. The instrument comprises a plate and a grid. On the plate, the arcs of equal ecliptic longitude and the circles of equal ecliptic latitude from –5 to +5 degrees are depicted through a stereographic projection. The plate can be used for all geographic latitudes. The grid contains one or more arms, each constructed for a certain latitude. Each arm is confined by the horizon and the almucantar of depression -10° corresponding to that latitude. Al-Bīrūnī uses references of previous authors, according to which the crescent is visible when the difference of altitude / depression between the moon and sun at the moment of the moon's setting is equal to or greater than 10°.

Here the second device will be examined.

The present study includes:

(1) The Arabic text edited from 3 manuscripts:
A (ا): Istanbul, Topkapi Saray Ahmet III 3505 / 7, 661 H, ff. 213r-216v.
T (ت): Tunis, al-Zaytūna 5540, 614 H, ff. 52v-54v.
M (م): Teheran, Majlis al-Shūra 1926/1, 11th century H, pp. 124-129.
The following manuscripts have been also taken in consideration:
B (ب): London, British Library Or. 5593, 8th century H, ff. 84v-87v.
S (س): Philadelphia, Schoenberg Collection, LJS 478, 625H/1228AD, ff. 63r-65v.
L (ل): Leiden Or. 123/2, 676 H, ff. 20r-22v.
N (ن): Teheran, Majlis al-Shūra 6516/1, 11th century H, ff. 65v-69r.[4]
Q (ق): Cairo, Dār al-kutub K 8528, 1014 H, ff. 70v-73v.

3. A short reference to this instrument appears in Wiedemann, "Ein Instrument, das die Bewegung von Sonne und Mond darstellt, nach al-Bīrūnī", in *Der Islam*, p. 13.

4. The manuscript is not foliated.

The Arabic text is also compared with the edition by al-Ḥosaynī.

(2) Translation of the above text with additional figures

(3) Commentary on the text, attached to the end of the translation

(4) An animation through a PowerPoint show, where the use of the device is demonstrated. It can be downloaded from http://tinyurl.com/plate-of-eclipses

The transliteration of the Arabic letters that correspond to points on the drawings is according to the following pattern:

ا	A
ب	B
ج	C
ح	H
د	D
ر	R
ز	Z
س	S
ص	W
ط	J
ع	O
ف	F
ق	Q
ك	K
ل	L
م	M
ن	N
هـ	E
ي	I

عمل الصفيحة الكسوفيّة

ا/213ا، ت/52ب، م/124 وأمّا الصفيحة الكسوفيّة فقد اعتنى بها نسطولس[5] الأسطرلابي[6] والحسين[7] بن[8] محمّد الآدمي، وتمم أمرها عطارد[9] بن محمّد[10] الحاسب، ولذلك آثرت حكاية ما اجتهد[11] فيه منها[12] عطارد. ولأنّ هذه الصفيحة ذات وجهين ومقترنة بشبكة شبه[13] العنكبوت، فإنّه يمكن أن يزاد[14] على ظهر أمّ[15] الأسطرلاب[16] حجرة[17]، تحوي هذه الصفيحة بشبكتها[18]، ويمكن أن يجعلا معًا في عداد صفائح[19] الأسطرلاب[20] موضوعة فيه تحتها إلى أن يحتاج إليها، فيخرج إلى ظاهرها، وذلك موكول إلى اختيار[21] المختار له[22].

فلتكن الصفيحة هي التي تحيط بها دائرة ابجد على مركز هـ وقطرا[23] اجـ بد متقاطعان[24] عند

5. «نسطولس» في ت م س ق بدون نقطة على «ن»، في ب: «بسطولس»، في ن: «لاسطولس»؛ الحسيني: «نسطورس»
6. «الأسطرلابي» في ا ت س ل ق: «الأصطرلابي»؛ الحسيني: «الأصطرلابي»
7. «الحسين» في م ب ل ن: «الحسن»؛ الحسيني: «الحسن»
8. «بن» محذوفة في ن
9. «عطارد» في ل: «عخارد»
10. «بن» (بدون النقطين) زائدة في ن
11. «اجتهد» في ا وم: «احتهد»
12. «منها» محذوفة في ا والحسيني
13. «شبه» في م: «شيه»
14. «يزاد» في ت: «يدار»، في م غير واضح
15. «أم» محذوفة في م
16. «الأسطرلاب» في ا وت: «الأصطرلاب»؛ الحسيني يستخدم «الأصطرلاب» دائما
17. «حجرة» في م: «حجر»
18. «بشبكتها» في م: «بشيكها»؛ الحسيني: «بشبكها»
19. «صفائح» في م: «الصفائح»
20. «الأسطرلاب» في ا وت: «الأصطرلاب»
21. «اختيار» في ا: «اختياد»، في ت وق: «احتياج»
22. «له» محذوفة في م ون
23. «وقطرا» في م: «وقطر»
24. «متقاطعان» في م: «يتقاطعان»

المركز على زوايا قائمة، ونفرض[25] كلّ واحدة من قوسي ار، بح[26] سدس الدائرة. ونصل[27] هـر، هـح[28] ونصل أيضًا جـر[29] يقطع هح على نقطة ط، وهب على كـ. ثمّ ندير على مركز كـ دائرة تماسّ خطّ هـر[30] على نقطة ص، وعلى مركز ط دائرة بمقدار المخطوطة على مركز كـ تماسّ جه فتتماسّ[31] الدائرتان على نقطة ي.

ونفرز هـم مساويًا لـهكـ[32] وندير على مركز م دائرة سلف[33] مساوية للدائرة المخطوطة على مركز كـ. ثمّ ندير على مركز هـ وببعد هـس قوسًا تماسّ[34] الدائرتين المخطوطتين على

ا/213ب مركزي ط، م من جهة مركز // الصفيحة، وهي قوس نس. ونخطّ أيضًا على مركز هـ وببعد هـف قوسًا تماسّ تينك الدائرتين[35] من جهة محيط الصفيحة، وهي قوس فع. وكذلك نخطّ على[36] مركز هـ وببعد هـص قوس صقل. ثمّ نقسم كلّ واحدة من قوسي صق، قل باثني عشر قسمًا متساوية، ونكتب أعدادها فوقها

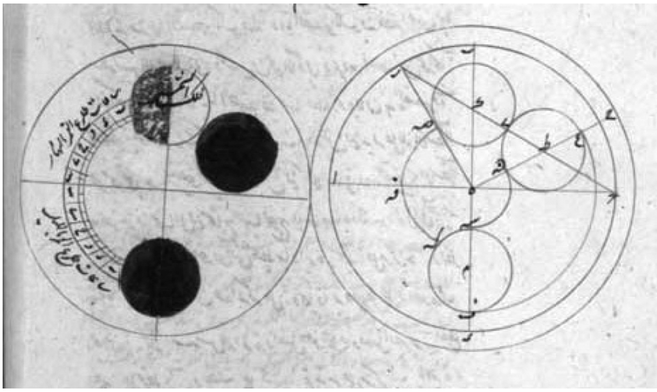
ت/53ا فيما بينها[37] وبين قسي مخطوطة حولها على مركز هـ[38] كما // جرى به الرسم في

م/125 أجزاء الحجرة // وأجزاء الارتفاع وما أشبهها[39].

ونسوّد دائرتي ط، م بالسيمسختج[40] ونفضض[41] الفضاء الذي بينهما إلى لدن قوسي[42] نس، عـف.

25. «ونفرض» في م: «ويفرض»
26. «ار، بح»: الحسيني «از، ى ج»
27. «ونصل» في ت: «ويصل»
28. «هر، هح» في م: «هز، هجـ»؛ الحسيني «هز، هج»
29. «جر»: الحسيني «ج ز»
30. «هر»: الحسيني «هز»
31. «مركز كـ تماسّ جه فتتماسّ» في م: «المركز الذي هو كـ تماسّ رجه فيماسّ»
32. «لـهكـ» في م: «هكـ»
33. «م دائرة سلف» في م: «هـ دائرة سلف»، في ا وت والحسيني: « م دائرة»
34. «تماسّ» في ت وم: «يماسّ»
35. «تينك الدائرتين» في م: «تلك الدائرة»
36. «على» محذوفة في ت
37. «بينها» في م والحسيني: «بينهما»
38. «هـ» محذوفة في م
39. «أشبهها» في م والحسيني: «أشبههما»
40. "بالسيمسختج" الحسيني: «بالسيمسنحتج»
41. «ونفضض» في ا: «ونقضض»؛ الحسيني: «ونقصّص»
42. «قوسي» في م: «قسي»

ونكتب[43] فوق أقسام[44] صق ساعات طلوع القمر بالنهار، وفوق أقسام قل[45] ساعات طلوع القمر بالليل. ولنسمّ[46] دائرة كـ فلك الشمس ونكتبه[47] فيها. وهذه صورتها وصورة المفروغ منها[48].



م / ص: 125

ا/ 214أ ثمّ نهيئ للشبكة صفيحة مساوية لمقدار التي[49] عملناها، ونزيد// في غلظها ليقوى بذلك ويمتن. ولتكن الدائرة التي[50] على محيطها[51] دائرة ابجد مساوية لدائرة ابجد على الصفيحة الأولى، ونعيد فيها الدائرتين المخطوطتين على مركزي كـ م، ونخرج خطّي هصر[52]، هط مماسّين لدائرة[53] كـ. ونفرض كلّ واحدة من قوسي رح[54]، طي مساوية لثلث قوس بر[55] ونخرج هي، هح. ثمّ

43. «ونكتب» في م: «ويكتب»
44. «أقسام» في م: «الأقسام»
45. «قل» في م: «صق» مصحّح تحته كـ «قل»
46. «ولنسم» في ت: «ولتسم»
47. «ونكتبه» في م: «ويكتبه»
48. «فافهم تصب إن شاء الله تعالي» زائدة في م
49. «لمقدار التي» في ت: «المقدار للتي»
50. «الدائرة التي» في ت: «التي»، في م: «الدائرة»
51. «محيطها» في م: «محيط»
52. «هصر» في ا: «هصن»؛ الحسيني: «هص»
53. «لدائرة» في ت: «الدائرة»
54. «رح» في م: «زحـ»؛ الحسيني: «زح»
55. «بر» في ت: «س»؛ الحسيني: «ب ز»

ندير على مركز هـ وببعد وتر نصف[56] قوس رح[57] دائرة؛ هي للفلس الذي ينتظم به الشبكة في القطب. ثمّ نفرج البركار[58] بقدر وتر عشرة أجزاء من أجزاء دائرة أبجد، إذا جزئت بثلاثمائة وستّين جزءا وندير على مركز هـ وبقدر تلك الفتحة[59] قوسًا، فيكون تقاطعها مع خطّ هر[60]؛ هو مري البعد الأوّل، وتقاطعها مع خطّ هط[61]؛ هو مري[62] البعد الثاني. ونجعل هف

م/126 من خطّ هجـ[63] // بقدر فتحة وتر خمسة عشر جزءا، وتكون[64] نقطة ف مري عرض القمر.

ونفرز ال بقدر فتحة وتر[65] خمسة أجزاء، وندير على مركز هـ وببعد هل نصف دائرة، يكون طوقًا مع نصف دائرة ابجـ[66] ليتعلق منه[67] ما يحتاج إلى تعليقة من المريات وغيرها. ونفرز ح ع بقدر[68] فتحة وتر سبعة أجزاء، وندير على مركز هـ وببعد هع قوس عن إلى خطّ هط، فتكون[69] نقطة ع مري ساعات نصف زمن الكسوف والمكث، ونقطة ن مري عدد أصابع الكسوف المعدّلة[70]. ونفرز رس[71] بقدر

ت/53ب فتحة وتر // أحد عشر جزءا، فتكون نقطة س مري ساعات ابتداء الكسوف.

ا/214ب ثمّ نأخذ من نقطة ص، التي هي نقطة التماس، // قدر ثلاثة أجزاء من أجزاء الدائرة على خطّ صه، وندير على مركز هـ وببعد الموضع الذي انتهينا إليه قوسًا تبتدئ[72] من دائرة كـ إلى جهة ا، حتى تنتهى[73] إلى قطر اهجـ. ونضم البركار بقدر ثلاثة أجزاء أيضًا،

56. «نصف» محذوفة في م
57. «رح» في م: «زحـ»؛ الحسيني: «زح»
58. «البركار» في ت: «بالبركار»
59. «الفتحة» في ت: «الصفيحة»
60. «هر» في م والحسيني: «هز»
61. «هط» في ا: «هر»
62. «مري» الحسيني: «المري»
63. «هجـ» في ا وم: «هي»
64. «وتكون» في ت: «فتكون»
65. «وتر» محذوفة في م
66. «ابجـ» في م: «اجـ»
67. «منه» في م: «منها»
68. «بقدر» في ا: «بقد»
69. «فتكون» في م: «فيكون»
70. «المعدّلة» في م: «والمعدّلة»
71. «رس» الحسيني: «زس»
72. «تبتدئ» في م: «يبتدئ»
73. «تنتهى» في م: «ينتهى»

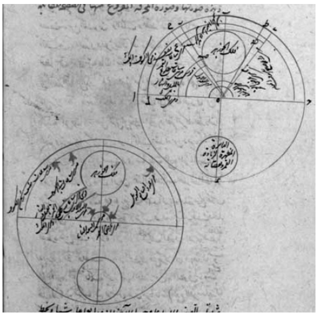
وندير على مركز هـ قوسًا كذلك مبتدئة[74] من دائرة كـ ومنتهية[75] إلى قطر اهجـ وهي[76] قوس مري ساعات طلوع القمر بالليل والنهار، ومري المكث لأنّ هذين المريين[77] متحدان. فلذلك[78] نعمله عند التقاء هذه[79] القوس مع دائرة كـ غير خارج عنه شيئًا[80] كثيرًا، ونعلّقه من القوس.

ثمّ نخرق نصف دائرة ابجـ ونترك فيه نصف فلس[81] القطب متعلّقا بقطره من نصف دائرة ادجـ ونترك الطوق الذي يحيط به نصف دائرة ابجـ ونصف الدائرة المخطوطة على مركز هـ وببعد هل. ونكتب على دائرة كـ[82] «فلك الجوزهر»، ونعلّقه من هذا الطوق بأقرب المواضع منه، وكذلك[83] نترك فيه قوس مري ساعات طلوع القمر، متعلّقة من فلك الجوزهر ومن نصف قطر اهـ[84]. ونعلّق[85] مري البعد الأوّل ومري البعد الثاني من الفلس، ومري ف[86] الذي هو لعرض القمر من نصف قطر هجـ. ونعلق[87] مريات ع، س، ن من الطوق على أحسن هيئة وتقدير نقدر عليه. ثمّ نخرق دائرة م[88]، فيكون لإظهار زيادة النور في جرم[89] القمر ونقصانه.

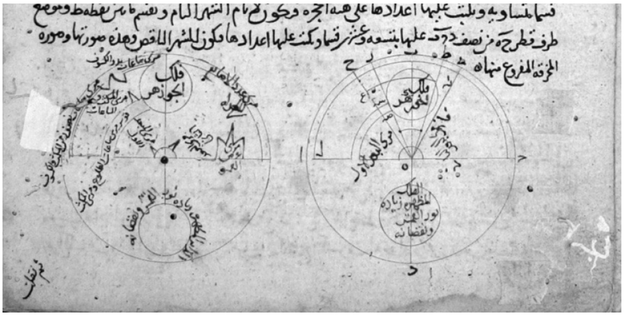
ومن الصناع من يزيد في عرض الصفيحة، ويخرج قطري رهـ[90]، طه بتمامهما، ماسين لدائرتي كـ، م. ثمّ يقسّم ما بين نقطة ر[91] و// موقع[92] طرف قطر طه في نصف م/127

74. «كذلك مبتدئة» في م: «كك مبتداة»، في ت: «أيضاً كذلك مبتدئة»
75. «ومنتهية» في م: «ومنتهأ»
76. «وهي» في م: «وهو»
77. «هذين المريين» في ت: «هذان المريان»
78. «فلذلك» الحسيني: «فكذلك»
79. «هذه» في ا وم والحسيني: «هذا»
80. «شيئا» في ا: «سًا»
81. «فلس» في ا: «فليس»
82. «ك» في ا، م، ب، ق، ل، ن: «حـ»، في ت، س: «ح»؛ الحسيني: «جـ»
83. «وكذلك» في م: «ولذلك»
84. «اهـ» في ا وم والحسيني: «دهـ»
85. «ونعلق» في م: «وتعلق»
86. «ف» في م: «ن»
87. «ونعلق» في ت: «ويعلق»
88. «م» في ا، ت، ب، ق، ل: «ل»، وفي س: «رك»
89. «وتقدير ... جرم» محذوف في م وفي مكانه «وتقديره»
90. «رهـ» في ا، ت، ب، س، ق، ل: «ح هـ»، في م: «حـ هـ»، في ن: «هـ ح»؛ الحسيني: «ح هـ»
91. «ر» في ا، ت، ب، س، ق، ل، ن والحسيني: «ح»، في م: «حـ»
92. «وموقع» في م: «وموضع»

ا/215ا <دائرة> باد بثلاثين قسمًا متساوية، ويكتب عليها // أعدادها على هيئة الحجرة؛ فتكون[93] لأيام الشهر التامّ. ويقسّم ما بين نقطة ط وموقع[94] طرف قطر رهـ[95] من نصف <دائرة> دجب[96] عليها بتسعة وعشرين قسمًا، ويكتب عليها أعدادها؛ فيكون للشهر الناقص. وهذه صورتها وصورة المخرقة المفروغ منها[97]. //



م / ص: 127



ت / و: 53ب

ت/54ا ثمّ نقلب الصفيحة الأولى على وجهها[98] الآخر الذي لم نعمل عليه شيئًا[99]، ونخطّ على مركزها دائرة مسامتة لدائرة ابجد في الوجه الآخر ومساوية لها[100]. ولتكن دائرة ابجد ونخطّ فيها قطري اهجـ، بهـد بالمسطرة المثناة لتطابق الأقطار[101] في الوجهين. ونخطّ فيها دائرة كـ على هيئة ما خططناها في الوجه الآخر، وفي الشبكة أيضًا،

م/128 ونسمّيها «فلك القمر». وندير فيها // مدارات[102] المريات التي في الشبكة، بأن نأخذ بالبركار[103] ما بين المركز وذلك المري ونديرٍ في هذه الصفيحة على مركز هـ وببعد

93. «فتكون» في م: «فيكون»
94. «وموقع» في م وت: «وموضع»
95. «رهـ» في ا، ت، ب، ق، ل، ن: «ح هـ»، في م: «جـ هـ»، في س: «هـ ح»؛ الحسيني: «جـ هـ»
96. «دجـ ب» في م: «دجـ ل»
97. «في الصفيحة المقابلة لهذه» زائدة في م
98. «وجهها» الحسيني: «وجهما»
99. «شيئًا» يبدو أن تحذف في ت
100. «لها» محذوفة في م
101. «الأقطار» في م والحسيني: «الأقدار»
102. «مدارات» في ا، ب، س، ل والحسيني: «دائرات»، في م، ن: «دوائر»
103. «بالبركار» في ت: «البركار»

ا/215ب تلك // الفتحة دائرة، فيكون مدار ذلك المري. ونخطّ أيضًا على مركز هـ وببعد[104] هكـ دائرة تقطع[105] من دائرة «فلك القمر» قوس[106] ركح[107] ونقسمها[108] باثني عشر قسمًا متساوية. ونخطّ على مركز هـ في فلك القمر قوسًا من دائرة، أعظم من المارّة على نقطة كـ لتقع[109] فيما بينها وبين المقسومة خطوط الأقسام الاثني عشر، وقوسًا أخرى فوقها من دائرة أعظم، حتى يكتب أعداد الأقسام بالجمّل فيما بينها[110] وبين التي تحتها، كما جرى الرسم[111] في[112] أقسام الحجرة وغيرها.

ثمّ نأخذ قوس ح ط[113] بقدر ثلث قوسي ركح،[114] ونركب الشبكة على هذا الوجه من الصفيحة تركيبًا ينطبق به قطر الشبكة على قطري الصفيحة ويستر[115] «فلك الجوزهر» كلّ[116] «فلك القمر». ونعلّم على موقع كلّ مري هو[117] في اليسار عن فلك الجوزهر، أعني إلى[118] جهة نقطة ا، من مداره[119] علامة أولى. ثمّ ندير الشبكة حتى توافي[120] حرف الجوزهر الأيمن الذي كان مطابقًا لنقطة ر[121]، نقطة ط، فحينئذٍ نعلّم على مواقع تلك المريات بعينها من مداراتها[122] في الصفيحة[123] علامة[124] ثانية، فيصير لنا طرفا كلّ قوس قطعها مري من تلك المريات معلومًا.

104. «تلك الفتحة ... وببعد» محذوفة في ت
105. «باثني عشر قسمًا» زائدة في م
106. «القمر قوس» محذوفة في ا
107. «ركح» في م: «زكج»، في ن: «ركحـ»
108. «ونقسمها» في ت: «ويقسمها»؛ الحسيني: «وتقسمها»
109. «لتقع» في ا: «ليقع»، في م: «لنقع»؛ الحسيني: «لتقطع»
110. «بينها» في م: «بينهما»
111. «به» زائدة في ا وت
112. «أجزاء» زائدة في م
113. «ح ط» في م: «حـ ط»
114. «ثلث قوسي ركح» في ت، ل: «ثلث قوس ركح»، في م: «ثلث قوس كحـ»، في ن: «ثلث قوس كح»، في ا: «ثلث قوسي زكح»؛ الحسيني: «زكـ ، كـ ح».
115. «ويستر» في م: «ويسير»
116. «كلّ» في م: «على»
117. «هو» الحسيني: «وهو»
118. «إلى» في م: «في»
119. «مداره» في ت: «مداراه»، في م: «مدار اه»؛ الحسيني: «مدار - اهـ-»
120. «توافي» في م: «تواقي»
121. «ر» في م: «ز»
122. «فحينئذٍ ... مداراتها» محذوف في م
123. «الجوزهر الأيمن الذي» زائدة في م
124. «علامة» في ت: «علامات»

فندير على مركز هـ[125] فوق كلّ قوس من تلك القسي، أخرى شبيهة[126] بها، حتى تقع[127] فيما بينهما خطوط الأقسام التي بها تقسم، ونكتب الأعداد فيما بين خطوط تلك الأقسام.

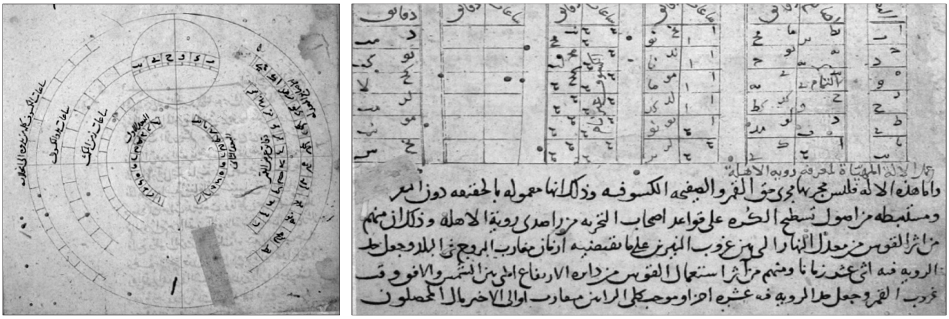
ثمّ نقسم ما رسمه مري البعد الأوّل باثني عشر قسمًا متساوية، ونضع مري البعد الأوّل[128] على كلّ واحد من تلك الأقسام، ونعلّم عند كلّ موضع[129] على موقع[130] سائر المريات المتياسرة من فلك الجوزهر، حتى نأتي[131] على الأقسام الاثني عشر، فينقسم كلّ واحد ممّا رسمه[132] تلك المريات باثني عشر قسما[133]، ونكتب في
أقسام[134] // قوس المري «البعد الأوّل» أعدادها بالنظم الطبيعي من واحد إلى اثني عشر، ا/216
مبتدئة من أسفل القوس، أعني طرفها الذي يلي[135] نقطة د من الصفيحة. ونكتب أيضًا في أقسام كلّ قوس ما يخصّها[136] في الجدول. ونبتدى في كلّ قوس
عند[137] الطرف الأسفل بما هو محاذٍ // لها في الجدول، حتى يمتلئ أقسام جميع القسي التي م/129
رسمتها[138] المريات.

فإذا فرغنا من هذا النصف، نعيد الشبكة إلى موضعها، حتى يستر[139] فلك الجوزهر[140] فلك القمر. ونقطع قوس رم[141] مساوية لقوس ح ط، ونفعل[142] بمواقع

125. «هـ» ليس في م
126. «شبيهة» في ا: «شبية»
127. «تقع» الحسيني: «يقع»
128. «باثني عشر ... الأوّل» محذوف في ا
129. «موضع» في ا وت «وضع»
130. «موقع» في ت «موضع»
131. «نأتي» في م ب ل: «ناتى»، وفي ت «نابى»، وفي ق: «يأتى»، ؛ الحسيني: «تأتي»
132. «رسمه» في م «تتمر»؟ غير واضح
133. «قسمًا» محذوف في ا وت
134. «أقسام» في م والحسيني: «الأقسام»
135. «من» زائدة في ت
136. «ما يخصّها» في ت: «يحصلها»
137. «عند» في م: «من عند»
138. «رسمتها» في ت: «رسمها»
139. «يستر» في ت وم: «يصير»
140. «هو» زائدة في م
141. «رم» في ا: «زم»
142. «ونفعل» في م: «ويفعل»

المريات المتيامنة عن فلك الجوزهر ما فعلنا بمتياسرها[143] عنه، ونثبت[144] في أقسام ت/54ب القسي التي نرسمها[145] بحركاتها // ما لها في الجدول، أعني أنّ[146] كلّ ما يحاذي[147] مبدأ عدد[148] البعد في الجدول نجعله في الطرف المحاذي لمبدأ[149] أعداد البعد في الصفيحة من[150] القوس التي رسمها[151] مريه. ثمّ نسود فلك القمر ما خلا مواضع الأقسام والأعداد منه. ونركب الشبكة على الصفيحة ونسلكهما[152] في قطب الأسطرلاب[153]، ونشدّهما [154]بفرس، إن شاء الله تعالى. [155]

وهذه صورتها وصورة المفروغ منها والجدول الذي فيه تلك المقادير المحسوبة:[156]



ت / و: 54ب-55أ

143. «بمتياسرها» في ا: «بمتياسرتها»، في ت: «بمياسرها»

144. «ونثبت» في م: «ونكتب»

145. «نرسمها» في ا: «يرسمها»

146. «أنّ» محذوفة في م

147. «يحاذي» في م: «حاذا»، في ت: «حاذى»

148. «عدد» الحسيني: «أعدد»

149. «لمبدأ» في م: «لمبتدا»

150. «من» غير واضح في م

151. «رسمها» في م: «ترسمهما»، في ا: «برسمها»

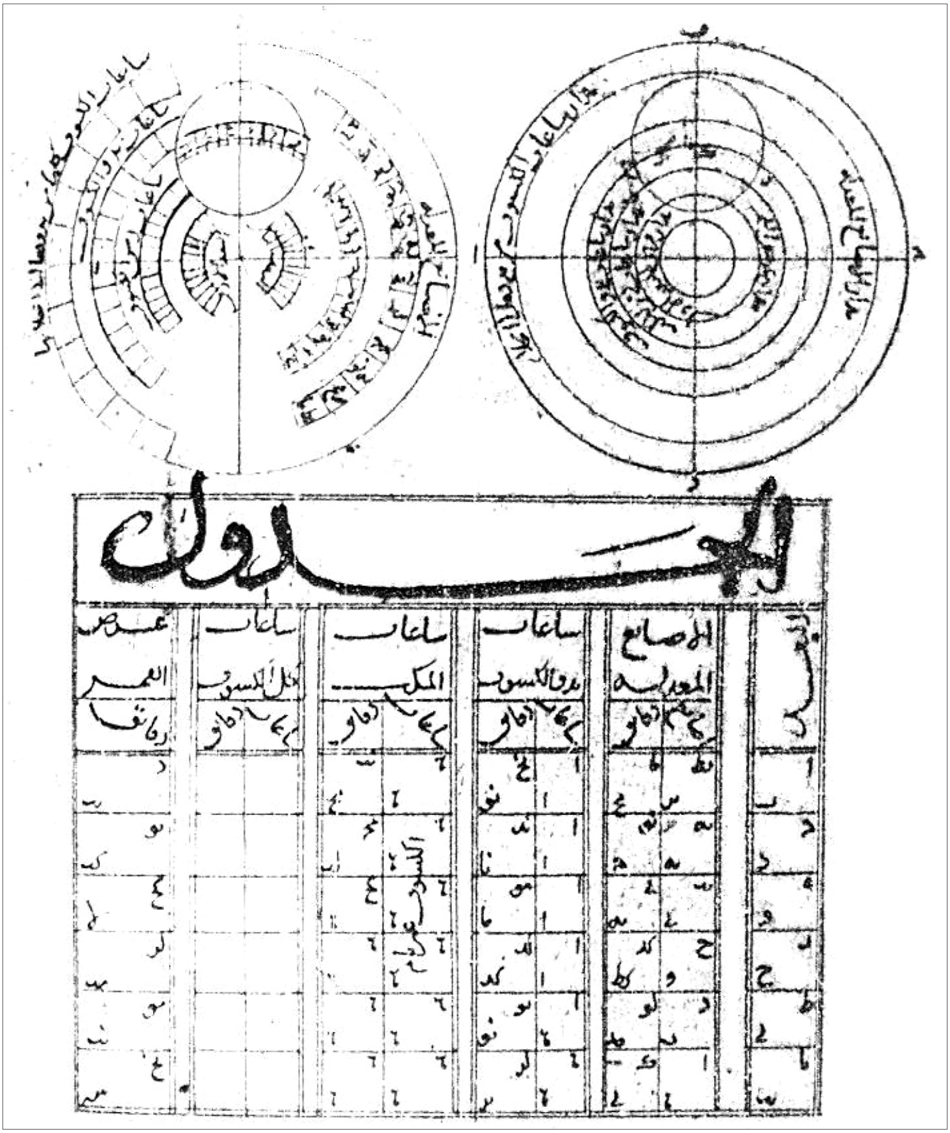
152. «ونسلكهما» في م: «ونشكلها»؛ الحسيني: «ونسلكها»

153. «الأسطرلاب» في ا وت: «الأصطرلاب»

154. «ونشدّهما» في ا وت: «ونسدهما»، في م: «ونشدها»

155. «إن شاء الله تعالى» ليس في م وت

156. «وصورة ... المحسوبة» محذوف في م ن س، «المحسوبة» في ا ب ل والحسيني: «المحسوسة» وفي ت: «محسوبة»، «والله أعلم» زائدة في ا.



ا / و: 216ب

البعد[3]	الأصابع المعدّلة		ساعات بدو الكسوف[2]		ساعات المكث		ساعات كل الكسوف[1]		عرض القمر
	أصابع	دقائق	ساعات	دقائق	ساعات	دقائق	ساعات	دقائق	دقائق
ا	يط[4]	ما	ا	نح		نب			د
ب	يز	مح	ا*+	نو		مح			يب
جـ	يه	نو	ا*+#	ند		مجـ			يو
د	يد[5]	جـ	ا*+#	نا		لب			كب
هـ	يب[8]	ي	ا*+#	مو		كح[7]			كح[6]
و	ي[10]	يه	ا*+#	ما	الكسوف غير تامّ[9]				لا
ز	ح	كد	ا*+#	لد					لز[11]
ح	و	كط[12]	ا+	كد					مب
ط	د	لو	ا	يو					مو[13]
ي	ب	مد		نو					نب
يا	ا	كـ		لز					نح
يب		ي		يز					س

جدول 1

1. في ب: «ساعات الانجلاء»، في ل: «ساعات كلى الكسوف»، في س محذوف
2. في م: «ساعات البدو»؛ هذا العنوان مكتوب تحت كلمة «الجدول» في س، وإن العنوان «بدو الكسوف في ساعات المكث » مكتوب في الأعمدة 4-9 من الجدول
3. «البعد»: في م: «العدد»، وفي ل مسح بتلطيخ
4. «يط» في س: مط
5. «يد» في ا وت: به
6. «كح» في س: بح، في م غير واضح
7. غير واضح في م. كلّ العمود فارغ في ب.
8. «يب» في ت: مد
9. مكتوب في عمود الساعات: في م بجانب أقدار الكسوف التامّ، في ا مكتوب من البعد د إلى ح، في ت مكتوب من البعد جـ إلى ي، في س مكتوب من البعد جـ إلى هـ ، في ب من البعد ب إلى ي، في ل محذوف.
10. الكلمة «التمام» مكتوب بجانب «ي» في ت، وبين «يب» و «ي» في ب وس
11. «لز» في س: لو
* محذوف في م / + محذوف في ب / # محذوف في ل
12. «كط» في س: عط
13. «مو» في س: بو

Translation

Construction of the plate of the eclipses

As for the plate of the eclipses, Nasṭūlus[157] al-Asṭurlābī and al-Ḥusayn[158] ibn Muḥammad al-Ādamī were concerned with it and ʻUṭārid ibn Muḥammad al-Ḥāsib completed its instruction. For this I preferred among them to describe what ʻUṭārid has elaborated on. Since this plate has two sides and is connected with a grid like the spider, it is possible that a limb be added on the back side of the mater of the astrolabe, enclosing this plate with its grid. They can be kept[1159] together amongst the plates of the astrolabe, situated somewhere beneath them, until needed; they then come out to its exterior and this is recommended for the chosen procedure.[160]

Let the plate be the one surrounded by the circle ABCD (figure 1), on center E, and AC, BD be two diameters that intersect each other at the center at right angles; we presume that each of the arcs AR, BH is 1/6 of the circle. We draw (the straight lines) ER, EH[161] and we also draw (the straight line) CR that intersects EH at the point J and EB at K. Then on center K we describe a circle tangential to the line ER at the point W, and on center J a circle the same size as the one drawn on center K, which touches CE; the two circles touch each other at the point I.

We take EM equal to EK and we describe, on center M, a circle SLF equal to the circle drawn on center K. Then we describe, on center E and radius ES, an arc that touches the two circles drawn on centers J, M on the side of the center of the plate and this is the arc NS. We also draw, on the center E and radius EF, an arc that touches those two circles on the side of the circumference of the plate; this is the arc FO. We similarly draw the arc WQL on center E and radius EW. Then we divide each one of the two arcs WQ, QL into 12 equal divisions and we write their numbers above them, somewhere between them (the divisions) and the arcs drawn around them on center E, as occurs with the drawing of the degrees of the limb and the degrees of altitude and what is similar to them (وما أشبهها).

157. ʻNasṭūlusʼ in B: ʻBasṭūlusʼ, in T, M, S, Q without dot on the first letter ʻنʼ, in N: ʻLāsṭūlusʼ.
158. ʻal-Ḥusaynʼ in M, B, L, N: ʻal-Ḥasanʼ.
159. In dual form here.
160. Lit. ʻfor the choice preferredʼ.
161. ʻER, EHʼ (هر، هح) in M: ʻEZ, ECʼ (هز، هجـ).

We color the two circles J, M black with *sīmsukhtaj* [162] and we plate with silver the empty space which is between them up to (إلى لدن) the arcs NS, OF. We write "the hours of the moon rising during daytime" above the divisions of WQ, and "the hours of the moon rising during nighttime" above the divisions of QL. We call the circle K "solar disc" (فلك الشمس) and we write that on it (the plate). This is its figure and the figure of it when completed. [163]

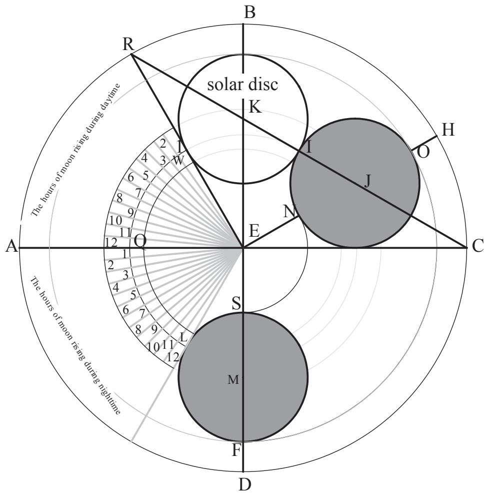


Figure 1: The front side of the plate.

Then we prepare a plate for the grid, equal to the size that we have constructed it (the first plate), and we increase its thickness, to become stronger and firmer. Let ABCD be the circle that is on its circumference (figure 2), equal to the circle ABCD on the first plate. We redraw on it the two circles drawn on the centers K, M and we draw the lines EWR,[164] EJ tangent to the circle K. We take each one of the arcs RH, JI equal to $^1/_3$ of the arc BR and we draw (the straight lines) EI, EH.

162. *Sīmsukhtaj* is a Persian compound word including *sīm* (سيم) – the old Persian word for «silver» – and *sukhtaj* (سُختَج) – the old form of the Persian verb سوخته that means «burnt». The literal translation of the compound word is «Burnt Silver», but in old Persian it was used as a metaphor that means «Pure and Fine Silver». I am thankful to Mr. Pouyan Rezvani for providing me with the information on this term.

163. In M there is the following additional sentence: Understand to get the truth / to be correct (فافهم تُصب), if the Exalted God is willing.

164. 'EWR' (هصر) in A: 'EWN' (هصن).

Then we describe a circle, on center E and radius equal to the chord of half the arc RH; this will be for the ringlet that connects the grid to the pole. Then we open the compass at the length of the chord of 10 degrees of the circle ABCD, when it is divided into 360 degrees, and we describe an arc on center E and radius equal to that opening. Let its intersection with the line ER[165] be the first pointer of "distance", and its intersection with the line EJ be the second pointer of "distance". We determine (نجعل) EF on the line EC at the length of the opening of a chord of 15 degrees; the point F will be for the pointer of the latitude of the moon.

We take AL equal to the length of the opening of the chord of 5 degrees, and we describe a semicircle on center E and radius EL; the band between it and the semicircle ABC will be for suspending from it what is necessary to be suspended among the pointers and not only those. We take HO equal to the length of the opening of the chord of 7 degrees and we draw the arc ON, on center E and radius EO, up to the line EJ; the point O will be for the pointer of the hours of half the time of the eclipse and the totality (lit. stay), and point N will be for the pointer of the number of the adapted digits of the eclipse. We take RS equal to the length of the chord of 11 degrees; point S will be for the pointer of the hours of the beginning of the eclipse.

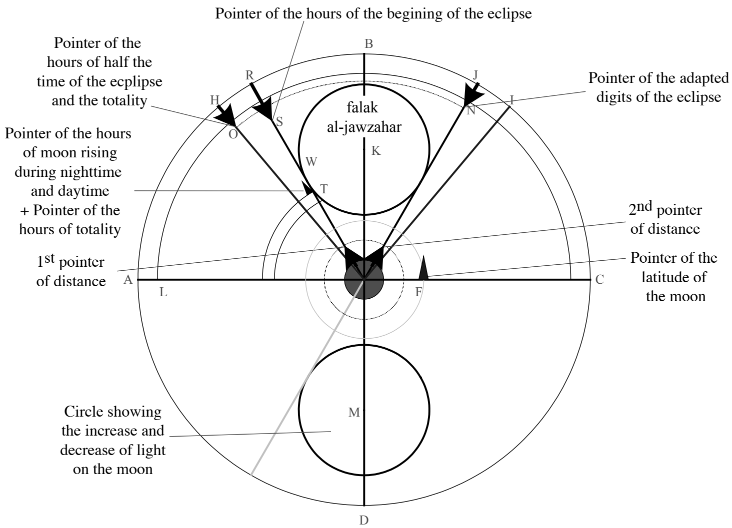


FIGURE 2: Construction of the grid.

165. 'ER' (هر) in M: 'EZ' (هز).

Then we take a length of 3 degrees of the circle from point W, which is the point of contact, on the line WE. We draw an arc, on center E and radius (equal to the distance of E from) the position where we have terminated, which starts from the circle K in the direction of A, until it arrives at the diameter AEC. We reduce the opening of the compass by the length of 3 degrees as well, and we similarly draw an arc, on center E, starting from the circle K and finishing at the diameter AEC; this will be the arc of the pointer of the hours of moon rising during nighttime and daytime and the pointer of the totality (lit. stay), because these two pointers are combined into one. For this reason we construct this pointer at the intersection of this arc with the circle K, but not so far outside of it and we suspend it from the arc.[166]

Then we cut out the semicircle ABC and we leave half the ringlet (فلس) of the pole on it, connected with its diameter with the semicircle ADC, and we leave the ring/band (الطوق) which is surrounded by the semicircle ABC and the semicircle drawn on center E and radius EL. We write on the circle K[167] *falak al-jawzahar*[168] and we suspend it (this circle) from this band at the closest place to it, and we also leave the arc of the pointer of the hours of moon rising on it, connected with the *falak al-jawzahar* and from the radius AE[169]. We suspend the first pointer of "distance" and the second pointer of "distance" from the ringlet, and the pointer F[170] that is for the latitude of the moon from the radius EC. We suspend the pointers O, S, N from the ring in the best position and estimate what we can. Then we cut out the circle M[171]; this is for the presentation of the increase and the decrease of the light on the celestial body of the moon.

One of the craftsmen increases the width of the plate and draws the whole diameters RE[172], JE as tangents to the two circles K, M (figure 3). Then he divides what is between point R[173] and the position of the endpoint of the diameter JE, in the

166. This is pointer T in figures 2, 3, 4, 7 and 9.

167. 'K' (ك) in A, M, B, Q, L, N: 'C' (حـ) in T, S: 'H' (ح). There are no circles on C or H. Probably 'ك' (K) had been misunderstood as an overlined 'حـ' by a scribe.

168. *falak al-jawzahar* (فلك الجوزهر) is a circle corresponding to a cross section of the cone of earth's shadow, at the point where the moon enters this cone.

169. 'AE' (اهـ) in A, M: 'DE' (اد).

170. 'F' (ف) in M: 'N' (ن).

171. 'M' (م) in A, T, B, Q, L: 'L' (ل), in S: 'RK' (رك), in M and N almost a line has been omitted.

172. 'RE' (ره) in A, T, B, S, Q, L: 'HE' (ح ه), in M: CE (حـ ه), in N: 'EH' (ه ح). Since ER is tangent to the circle (K, KW) and EH has no points in common with this circle, the radius ER of the circle should be extended to a diameter.

173. 'R' (ر) in A, T, B, S, Q, L, N: 'H' (ح), in M: 'C' (حـ).

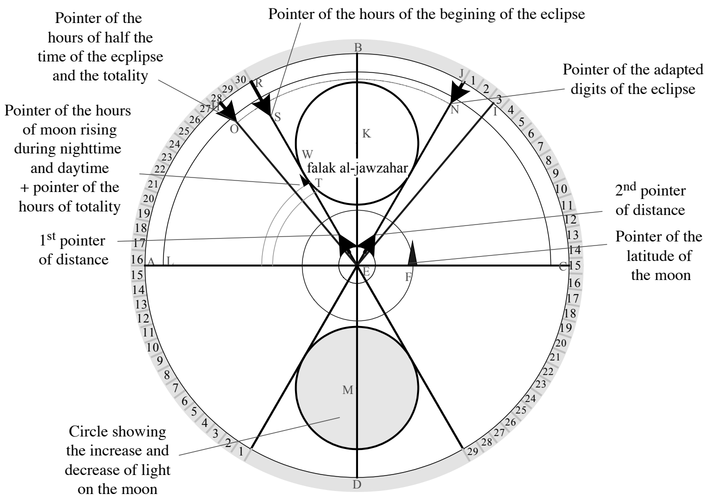


FIGURE 3: The grid with the extended rim.

semicircle BAD, into 30 equal parts and he writes their numbers on it, in the form of the limb; they are for the days of the full month. He divides what is between point J and the position of the endpoint of the diameter RE[174], in the semicircle DCB[175], into 29 parts and he writes on them their numbers; they are for the hollow month.

This is its figure and the figure of the cut out final form (figure 4).

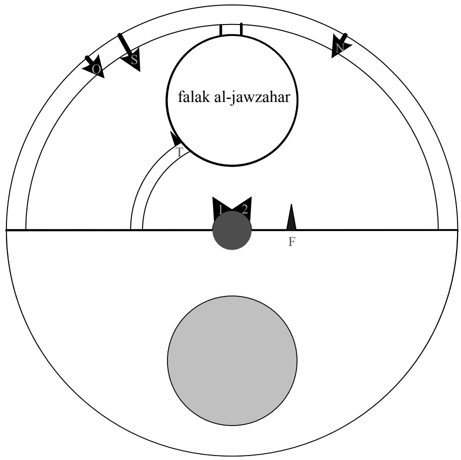


FIGURE 4 : The final form of the grid.

174. ‘RE’ (ره) in A, T, B, Q, L, N: ‘HE’ (ح ه), in M: ‘CE’ (جـ ه), in S: ‘EH’ (ه ح).
175. ‘DCB’ (دجب) in M: ‘DCL’ (دجل).

Then we reverse the first plate to its other side, on which we made nothing, and we draw, on its center, a circle exactly above (مسامتة) the circle ABCD on the other side and equal to it (figure 5). Let ABCD be the circle and we draw on it the diameters AEC, BED with a double ruler for matching the diameters on the two sides. We draw the circle K on it, in the form we have drawn it on the other side and on the grid as well, and we call it the "lunar disc" (فلك القمر). We draw on it the circular paths[176] of the pointers that are on the grid, taking with the compass the distance between the center and that pointer and we describe a circle on center E and radius equal to that opening on this plate; it will be the circular path of that pointer. We also draw a circle on center E and radius EK that cuts the arc RKH[177] from the circle of the lunar disc and we divide it into 12 equal divisions. On the lunar disc, on center E, we draw an arc of a circle greater than the circle passing through point K, so that the 12 lines of divisions will lie somewhere between it and the divided arc, and another arc of a greater circle above it, in order to write the numbers of the divisions somewhere between this arc and the arc below it, using the letters of the alphabet,[178] as it occurs in the drawing of the divisions of the limb and the like.

Then we take the arc HJ equal to the size of $^1/_3$ of two arcs RKH[179], and we mount the grid on this side of the plate in a matching assembly, where the diameter of the grid is on the two diameters of the plate,[180] and the *falak al-jawzahar* covers the whole "lunar disc". On the circular path[181] of every pointer which is to the left of *falak al-jawzahar*, namely in the direction of point A, we put the first mark (of it) at the position of the pointer. Then we rotate the grid until the right edge of *falak al-jawzahar*, which was congruent with point R[182], comes to point J, and then we put a second mark on the position of the same pointers on their

176. In A (دايرات), M (دوائر) "circles" , in T (مدرات) "circular paths".

177. 'RKH' (ركح) in M: 'ZKC' (زكج).

178. abjad numbers.

179. '$^1/_3$ of two arcs RKH' in T, L: '$^1/_3$ of arc RKH' (ثلث قوس ركح), in M: '$^1/_3$ of arc KC' (ثلث قوس كحـ), in N: '$^1/_3$ of arc KH' (ثلث قوس كح), in A: '$^1/_3$ of two 2 arcs ZKH' (ثلث قوسي زكح). In all cases only one arc is mentioned, even if dual form is used. The meaning should be '$^2/_3$ of arc RKH'. For further discussion see the Remarks, 3.

180. The "two diameters" could refer to the diameter AC drawn on the front and back side of the plate.

181. There is an additional 'AE' in M.

182. 'R' (ر) in M: 'Z' (ز).

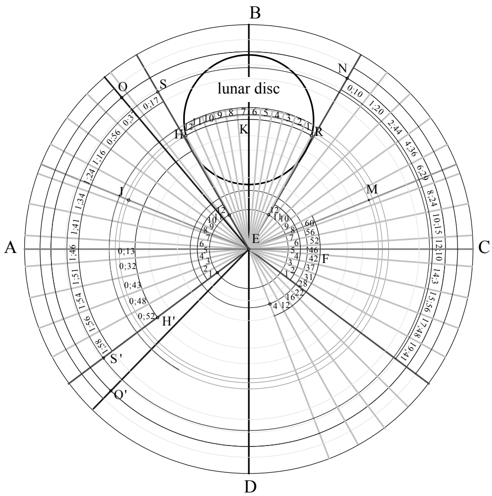


FIGURE 5: Construction of the back side of the plate.

circular paths on the plate; so we know the two endpoints[183] of each arc traced by those pointers.

We draw, on center E above each one of those arcs, another (arc) similar to it, so that the lines of the divisions that divide it lie somewhere between the two arcs, and we write the numbers somewhere between the lines of those divisions.

Then we divide the arc described by the first pointer of "distance" into 12 equal divisions and we put the first pointer of "distance" on each one of those divisions. We put a mark for each position on the spot of each one of the pointers that are to the left of *falak al-jawzahar* until we cover the 12 divisions. Each one of the arcs described by those pointers is divided into 12 divisions. We write the numbers of them on the divisions of the arc for the first pointer of "distance", in the natural order from 1 to 12 starting from the lower part of the arc, namely the endpoint that is closer to point D on the plate. We also write what is related to it on the table, on the divisions of each arc. For every arc we start from the lower endpoint with the entry that corresponds to it on the table, until the divisions of all of the arcs that are depicted by the pointers have been filled.

183. Lit. the two endpoints become known to us.

When we finish with this half, we return the grid to its place, so that *falak al-jawzahar* covers the lunar disc. We separate an arc RM[184] equal to the arc HJ and we do the same with the positions of the pointers situated to the right of *falak al-jawzahar* as we did with those to the left of it. We engrave the corresponding entries of the table onto the divisions of the arcs that we have drawn following the movement of the pointers, namely: the entries, corresponding to the first number of "distance" on the table, are placed at the endpoint (of the arc onto the plate), which is determined by the first number of "distance" on the plate; each entry is put on the arc traced by the corresponding pointer. Then we color black the lunar disc except for the places of the divisions and their numbers. We mount the grid on the plate, we pass them over the pole of the astrolabe and we secure them with the horse-wedge, if the Exalted God is willing.

This is its figure and the figure of the finished (model) —figure 6— and the table that includes the calculated quantities (table 1).

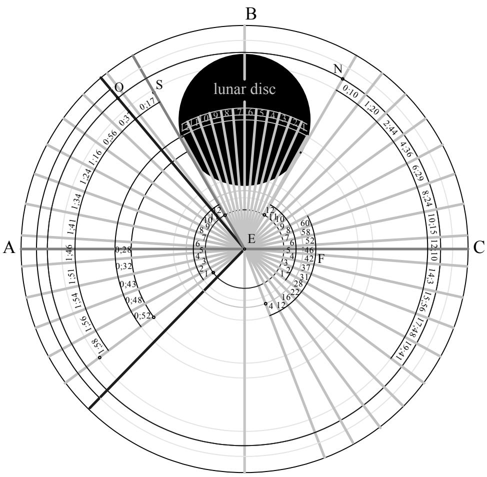


Figure 6 : The back side of the plate.

184. 'RM' (رم) in A: 'ZM' (زم).

<table>
<tr><th colspan="10">The table*</th></tr>
<tr><th rowspan="2">“Distance”</th><th colspan="2">Adapted digits[1]</th><th colspan="2">Hours of the eclipse beginning[2]</th><th colspan="2">Hours of totality[3]</th><th colspan="2">Hours of the whole eclipse[4]</th><th>Moon latitude[5]</th></tr>
<tr><th>digits</th><th>min</th><th>h</th><th>min</th><th>h</th><th>min</th><th>h</th><th>min</th><th>min</th></tr>
<tr><td>1</td><td>19</td><td>41</td><td>1</td><td>58</td><td>–</td><td>52</td><td>–</td><td>–</td><td>4</td></tr>
<tr><td>2</td><td>17</td><td>48</td><td>1</td><td>56</td><td>–</td><td>48</td><td>–</td><td>–</td><td>12</td></tr>
<tr><td>3</td><td>15</td><td>56</td><td>1</td><td>54</td><td>–</td><td>43</td><td>–</td><td>–</td><td>16</td></tr>
<tr><td>4</td><td>14</td><td>3</td><td>1</td><td>51</td><td>–</td><td>32</td><td>–</td><td>–</td><td>22</td></tr>
<tr><td>5</td><td>12</td><td>10</td><td>1</td><td>46</td><td>–</td><td>28</td><td>–</td><td>–</td><td>28</td></tr>
<tr><td>6</td><td>10</td><td>15</td><td>1</td><td>41</td><td>–</td><td rowspan="7">partial eclipse</td><td>–</td><td>–</td><td>31</td></tr>
<tr><td>7</td><td>8</td><td>24</td><td>1</td><td>34</td><td>–</td><td>–</td><td>–</td><td>37</td></tr>
<tr><td>8</td><td>6</td><td>29</td><td>1</td><td>24</td><td>–</td><td>–</td><td>–</td><td>42</td></tr>
<tr><td>9</td><td>4</td><td>36</td><td>1</td><td>16</td><td>–</td><td>–</td><td>–</td><td>46</td></tr>
<tr><td>10</td><td>2</td><td>44</td><td>0</td><td>56</td><td>–</td><td>–</td><td>–</td><td>52</td></tr>
<tr><td>11</td><td>1</td><td>20</td><td>0</td><td>37</td><td>–</td><td>–</td><td>–</td><td>58</td></tr>
<tr><td>12</td><td>0</td><td>10</td><td>0</td><td>17</td><td>–</td><td>–</td><td>–</td><td>60</td></tr>
</table>

Table 1: The table of the eclipse.

* This table is recalculated as table 6 in Remark 2.

1. Since the diameter of the falak al-jawzahar, namely the cross section of the earth's shadow cone, where the moon enters this cone, is greater than the apparent diameter of the moon, the lunar eclipse may have a magnitude greater than the 12 digits of the lunar disc (see figure 11). The entries of this column correspond to the numbers indicated by pointer N.

2. The entries of this column give the times between the beginning of the eclipse and the opposition. These entries correspond to the numbers indicated by pointer S.

3. The duration of the complete obscuration of the lunar disc, in case of a total lunar eclipse. The entries correspond to the numbers indicated by pointer T.

4. There are no entries for this column on the table in the manuscripts A, T, M, B, S, L, while the table does not exist in manuscripts N and Q. The title of this column is “Hours of emersion” in B, while in S the title is omitted. The title should have been “the hours of half the time of the eclipse and totality” and the entries should have been the numbers indicated by pointer O, according to the text.

5. Only the latitude of the moon is taken in consideration in the construction of the table. Such other important variables as the distance between the earth and the moon, hence the velocity of the moon, and the distance between the sun and the earth have been ignored. The entries correspond to the numbers indicated by pointer F.

The plate of the (lunar) eclipses – Commentary

The importance of the prediction of lunar eclipses was the incentive for the astrolabists to invent and construct a plate for the astrolabe to predict the lunar eclipses. This plate not only helps to predict the eclipses, but also to determine important values, such as the magnitude, start time and duration of an eclipse. The plate of the lunar eclipses is a double-sided plate connected with a grid that can rotate above both sides of the plate. It is stored among the other plates of the astrolabe and can be attached onto the back side of the astrolabe when in use. If someone wants to examine whether there will be a lunar eclipse using the plate, he must know the latitude of the moon at the moment of the opposition. One side of the plate is for the lunar eclipses, while the other side depicts the phases of the moon and estimates the time of moon-rising.

Al-Bīrūnī mentions that this plate has been worked out by Nasṭūlus al-Asṭurlābī, al-Ḥusayn ibn Muḥammad al-Ādamī and ʻUṭārid ibn Muḥammad al-Ḥāsib (9th -10th century).

Nasṭūlus had invented the crab astrolabe, according to al-Sijzī and al-Bīrūnī, and the hours on the alidade, according to al-Sijzī.[185] The signature of Nasṭūlus, without any dot on the first letter 'ن', has been identified on three astrolabes (in Kuwait, Cairo and London) and on a peculiar instrument that can determine the longitude of the sun, the time in seasonal hours according to the altitude of the sun and the meridian altitude of the sun throughout the year in Baghdad.[186] D. A. King refers to a manuscript in a private collection that preserves descriptions of a universal sundial and a luni-solar gear mechanism by Nasṭūlus.[187] The omission of the dot in the initial letter of Nasṭūlus' name on the astrolabes and manuscripts has caused a long debate over whether the correct name is Nasṭūlus (نسطولس) or Basṭūlus (بسطولس). It seems that the correct name is Nasṭūlus, which is another form of the

185. See Al-Bīrūnī, *Istīʻāb al-wujūh al-mumkina fī ṣanʻat al-āsṭurlāb* (ed. al-Ḥosaynī), p. 122; D. A. King, "A note on the astrolabist Nasṭūlus/Basṭūlus" in *Archives Internationales d'Histoire des Sciences* 28 (1978), pp. 115-118; and D. A. King, *In Synchrony with the Heavens – Studies in Astronomical Timekeeping and Instrumentation in Medieval Islamic Civilization*, 2 vols, Leiden: Brill, 2004-2005, v. II, pp. 471-2.

186. D. A. King, "An Instrument of Mass Calculation made by Nasṭūlus in Baghdad ca. 900" in *Suhayl* 8 (2008), 93-119. There are detailed descriptions of the astrolabes in Kuwait and Cairo in D. A. King, *In Synchrony with the Heavens*, XIIIc-3, v. II, pp. 470-484.

187. D. A. King, "An Instrument of Mass Calculation" in *Suhayl* 8 (2008), p. 98.

name Nasṭūrus, a name in use in 10th century Egypt.[188] In the eight manuscripts examined for this study, the form Nasṭūlus appears in two of them (A, L), the form Basṭūlus in one (B), in manuscript N the name is written as Lāsṭūlus, while there is no diacritic dot on the initial letter in the remaining four manuscripts (T, M, S, Q). The extract of the manuscript Mullā Fīrūz 86/3 in Mumbai (6th c. H), presented by Sezgin in GAS VI p. 288, gives indications that it contains instructions for the use of the plate of eclipses, written as "الأسطرلاب القمري" – the lunar astrolabe – and mentions that it was invented by Nasṭūlus in 280 H (893-4 AD).[189] According to the description of the manuscript, the third track "is all about the moon, its phases, and the observation of its eclipses with the lunar astrolabe".[190]

Al-Ḥusayn ibn Muḥammad al-Ādamī is mentioned by al-Nadīm as author of a book on sundials.[191] In the *Book of the Categories of Nations*, Ṣā'id al-Andalusī refers several times to al-Ḥusayn ibn Muḥammad ibn Ḥamīd, known as ibn al-Ādamī,[192] who wrote the *zīj Naẓm al-'iqd* following the *Sindhind* tradition. This work was completed by his student al-Qāsim ibn Muḥammad ibn Hishām al-Madā'inī, known as al-'Alawī, in 338H/ 949-50 AD.[193]

In the entry *Ādamī* of *the Biographical Encyclopedia of Astronomers*, Ādamī and Ibn al-Ādamī are presented as two different persons, father and son; Ādamī is noted

188. See Brieux & Maddison, "Basṭūlus or Nasṭūlus? A note on the Name of an Early Islamic Astrolabist" in *Archives Internationales d'Histoire des Sciences* 24 (1974), pp. 157-160; D. A. King, "A note on the astrolabist Nasṭūlus/Basṭūlus" in *Archives Internationales d'Histoire des Sciences* 28 (1978), pp. 115-118; D. A. King and Paul Kunitzsch, "Nasṭūlus the Astrolabist Once Again" in *Archives Internationales d'Histoire des Sciences* 33 (1983), pp. 342-343; D. A. King, "An Instrument of Mass Calculation" in *Suhayl* 8 (2008), pp. 96-99.

189. F. Sezgin, (GAS) *Geschichte des arabischen Schrifttums,* v. VI *Astronomie,* Leiden: Brill, 1978, pp. 178-179 and 288. The Arabic text of this manuscript has been edited and translated into English by Johannes Thomann, as Appendix III in "Astrolabes as Eclipse Computers: Four Early Arabic Texts on Construction and Use of the *ṣafīḥa kusūfiyya*" in *Medieval Encounters,* 2017/2 (in press).

190. Edward Rehatsek, *Catalogue Raisonné of the Arabic, Hindostani, Persian, and Turkish Mss. in the Mullā Fīrūz Library*, Bombay: Managing committee of the Mullā Fīrūz Library, 1873, pp.43-44.

191. See Gustav Flügel, *Kitāb al-Fihrist*, 2 vols, Leipzig: Vogel, 1871-72, vol. 2, p.280; the complete name is Abū 'Alī al-Ḥusayn ibn Muḥammad al-Ādamī.

192. Ṣā'id al-Andalusī, *Kitāb Ṭabaqāt al-'Umam*, edited by P. L. Cheikho. Beirut: Imprimerie Catholique, 1912, pp. 13, 49, 57–58.

193. In note 1, p. 58, Cheikho gives as correct the date 308 H that corresponds to 920-21 AD.

for his work on instruments (sundials and the disc of eclipses), while Ibn al-Ādamī wrote the *zīj Naẓm al-ʻiqd*.[194] On the other hand, D. A. King mentions that Ibn al-Ādamī compiled set of tables for vertical sundials,[195] and Sezgin presents both the *zīj Naẓm al-ʻiqd* and the book on the sundials as works of one person called Ibn al-Ādamī.[196]

ʻUṭārid ibn Muḥammad al-Ḥāsib is mentioned by al-Nadīm as the author of books on the use of the astrolabe and the armillary sphere, while Sezgin notes that his astronomical works are apparently lost.[197]

Al-Bīrūnī gives a detailed description of the construction of the plate and the grid, but he does not explain how to use it. The device can be constructed following the description. Figures 1, 6 and 4 or 3 can be used to produce a model of the plate of eclipses.

Outline of the device

On the front side of the plate (figure 1) on the lower and the right side, there are two circles painted black for the moon; the angle MÊJ is equal to 120°, where M and J are the centers of the two circles and E is the center of the plate. The space between the circles has been plated with silver. On the left side there are two successive scales for the time of the moon-rising during day or night. Each scale is included in a central angle of 60°. At the top there is a circle on center K, equal to the circles on centers J and M and tangential to the circle on center J. On circle K the title "solar disc" (فلك الشمس) is written.

On the back side of the plate (figures 5-6) exactly opposite to the "solar disc", there is a circle equal to it, on center K. This circle is painted black; the title "lunar disc" (فلك القمر) and a scale for the magnitude of the lunar eclipse are impressed on it. There are seven more scales on the plate, each one with 12 subdivisions, for the

194. F. Jamil Ragep and Marvin Bolt, "Ādamī: Abū ʻAlī al-Ḥusayn ibn Muḥammad al-Ādamī", in *The Biographical Encyclopedia of Astronomers,* New York: Springer, 2007, p.12.

195. King, D. A., "Universal Solutions in Islamic Astronomy", in *From Ancient Omens to Statistical Mechanics: Essays on the Exact Sciences Presented to Asger Aaboe*, edited by J. L. Berggren and B. R. Goldstein, Acta Historica Scientiarum Naturalium et Medicinalium, vol. 39, pp. 121–132, Copenhagen: Copenhagen University Library (1987), p.123.

196. Sezgin, GAS VI, pp. 179-180.

197. see Flügel, *Kitāb al-Fihrist*, vol.1, p.278, and Sezgin, GAS VI, p. 161.

latitude of the moon, the magnitude of the eclipse, the times of the eclipse and the "distance", namely the numbers 1-12 related to the latitude of the moon. The scales are constructed according to the entries of a table given at the end of the chapter.

The grid has seven pointers corresponding to the scales on the plates (figures 2 and 4). The lower semicircle has a circular hole, equal to the circles M and J on the front side of the plate, through which the phases of the moon can be depicted. The inner part of the upper semicircle has been cut out, leaving in place a band along the outer rim, the central ringlet, a circle related to the earth's shadow (فلك الجوزهر) –equal to the circle of the "lunar disc" on the back side of the plate– and an arc connecting this circle with the lower semicircle. Optionally, the rim of the grid can be extended and the numbers of the days for the hollow and full lunar months can be written respectively on the right and left thirds of the circumference (figure 3). In this case two extra pointers should have been added outside the grid to indicate the day of the lunar month. These pointers could be attached to the plate or to an outer rim surrounding the device, as depicted in figure 7. These two pointers are not mentioned by al-Bīrūnī.

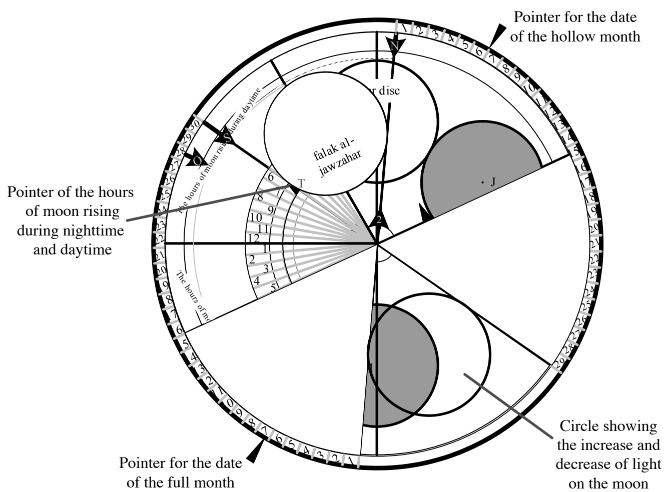


FIGURE 7: Use of the front side of the plate.

Use of the device

1. Determining the phase of the moon and the time of moon-rising

We mount the grid on the front side of the plate. By rotating the grid over the plate, we obtain for each day of the month the corresponding phase of the moon (through the hole of the grid) and the approximate time of the moon-rising in seasonal (unequal) hours after sunrise or sunset, given by the pointer T. In the case where the days of the lunar month have been added to the rim of the grid, we rotate the grid, so that the pointer of the hollow or the full month reaches the number of the day of our choice (figure 7), then we obtain the corresponding phase of the moon and the time of the moon-rising. In the results obtained by the above procedure, the anomaly of the moon's motion is not taken into consideration.

2. Determining the lunar eclipses

We mount the grid on the back side of the plate. We rotate the grid clockwise (figure 8), so that the pointer F falls onto the number corresponding to the latitude of the moon (up to 60 minutes) and we obtain the magnitude of the eclipse in digits and minutes by the pointer N, while the pointer 2 shows the "distance". The "distance" expressed by the numbers 1-12 is related to the latitude of the moon.

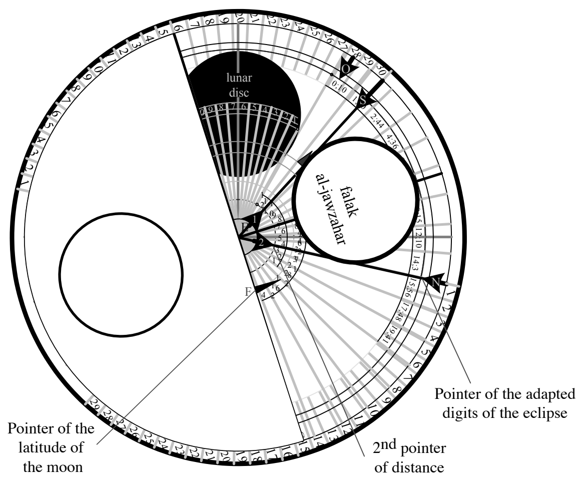


Figure 8: Use of the back side of the plate, turning the grid clockwise.

Then we rotate the grid counterclockwise (figure 9), so that the pointer 1 arrives at the number shown by the pointer 2 as the "distance". At this position:

- the pointer S shows the time of the beginning of the eclipse in hours and minutes before the opposition,
- the pointer T shows the duration of totality in hours and minutes, if the eclipse is total,
- the pointer O shows "half the (period of) time of the eclipse and totality", but the corresponding entries have been omitted on the table,[198]
- the part of the black lunar disc not covered by the circle of the grid *falak al-jawzahar* shows the part of the lunar disc occulted by the earth's shadow. The number (1-12) on the "lunar disc", closer to the right edge of *falak al-jawzahar*, indicates the magnitude of the lunar eclipse. When the eclipse is total, the *falak al-jawzahar* does not cover any part of the "lunar disc".

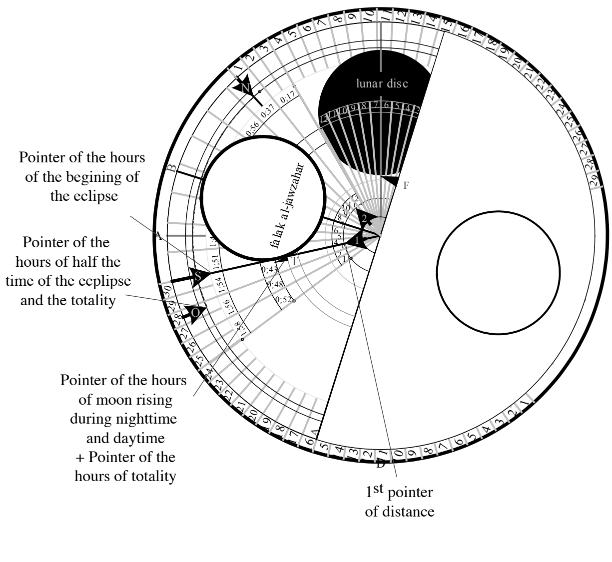


FIGURE 9: Use of the back side of the plate, turning the grid counterclockwise.

198. Pointer S shows the time between the beginning of the eclipse and the opposition, which is close to half the time of the eclipse (and totality) that pointer O should show. Probably this is the reason for which there are no entries in table 1 for the graduation of the scale corresponding to pointer O. In this table, there is a column with title "Hours of the whole eclipse", but without any entries, while in manuscript B the title of the same column is "Hours of the emersion". Since there

Comparison with the lunar astrolabe by Nasṭūlus

As previously mentioned, manuscript Mullā Fīrūz 86/3 (ff. 44v-45v) contains instructions for the use of the plate of eclipses invented by Nasṭūlus.[199] Although the text does not contain a detailed description of the instrument, one can deduce the arrangement of the various elements on the plate and discern the differences between the initial form invented by Nasṭūlus and the one described by al-Bīrūnī. A possible reconstruction of the instrument by Nasṭūlus is presented in figure 10;[200] this figure can be used to produce a model of the instrument. The entries of table 1 were used for the graduation of the scales.

The first part of the instructions deals with the phases of the moon and the time of moon rising, and the second with the study of the lunar eclipses. It seems that both operations are conducted on the same side of the plate, thus the plate designed by Nasṭūlus contains, on only one side of the plate, most of the elements presented for both sides of the plate by al-Bīrūnī.

The phases of the moon are depicted using either the difference in longitude $\Delta\ell$ between the sun and the moon divided by 3, or the day of the Arabic month multiplied by 4; both methods give equivalent results between 0° and 120°. The index of the moon attached to *falak al-jawzahar* on the grid is directed to the scale on the limb, starting from the first point of Capricorn that corresponds to 0°, to the respective degree up to 120°.[201] At this position, the circular opening on the grid,

are no entries in this column, the exact content cannot be verified. For the difference between the time of the opposition and the mid-eclipse, see Claudii Ptolemaei, *Opera quae extant omnia*, vol. 1,1 (ed.) J.L. Heiberg: VI.7, Leipzig: Teubner, 1898 – 1903, pp. 504-5 and G. J. Toomer, *Ptolemy's Almagest*, Princeton: Princeton University Press, 1998, pp.296-298.

199. For the comparison I have used the edition and translation of MS Mullā Fīrūz 86/3 in Appendix III of Johannes Thomann, "Astrolabes as Eclipse Computers: Four Early Arabic Texts on Construction and Use of the *ṣafīḥa kusūfiyya*" and the manuscript itself. I am grateful to Dr Johannes Thomann for sharing his article before its publication and providing me microfilm scans of the manuscript.

200. The positions of the scales for the hours of moon rising, the hours of totality and emersion, and the degrees of distance between the moon and the node are mentioned to be on the inner part of the plate; the exact positions are not explicitly described. For the reconstruction, the positions of the scales were chosen taking in consideration the functionality of the instrument.

201. The scale 0°-120° on the limb is the counterpart of the scales of the lunar month days depicted on the grid according to the description of al-Bīrūnī. The scale on the limb corresponds to a full month of 30 days, while those on the grid cover both the hollow and full month.

where "circle showing the increase and decrease of the moon" is written, depicts the phase of the moon, while the index of the hours on the grid indicates the time of moon rising during daytime, at the first half of the lunar month, or nighttime at its second half. The hours of day and night are inscribed on the inner part of the plate.

As for the lunar eclipse, first the time of the opposition t_o must be determined using a *zīj*. According to the text, a lunar eclipse can occur when the ecliptic latitude of the moon is not greater than 64 minutes, or when the distance between the "degree of the opposition" and the closest node (تنين / lit. dragon) is less than 12°.[202] The two approaches are almost equivalent; moon's ecliptic latitude of 64 minutes corresponds to a distance of 12;17° between the "degree of opposition" and the closest node. When a lunar eclipse occurs, the time of the opposition is taken as the time of the middle of the eclipse. For determining the magnitude and duration of the eclipse, the index of the eclipse should be directed to the corresponding degree between 1 and 12 on the scale of the "distance between the moon and node". This is situated on the inner part of the plate and is probably the counterpart of the "distance" with range between 1 and 12 in the description of the plate by al-Bīrūnī. There is no reference to a scale for the latitude of the moon.

The magnitude of the lunar eclipse is measured on the "solar disc" which is divided into 12 divisions / digits; thus the maximum magnitude to be measured is 12 digits. There is no additional scale for adapted digits. This "solar disc" is the counterpart of the lunar disc in the description by al-Bīrūnī, and this name is an indication that the elements on the two faces of the plate were condensed on only one face in this earlier version. According to the description, the part of the "solar disc" covered by *falak al-jawzahar* corresponds to the part of the moon that remains not eclipsed, while the visible part of the "solar disc" represents the eclipsed part of the moon. When these two discs are tangent, there will be a total lunar eclipse without duration, and when the discs have a distance between them the total lunar eclipse will have a duration that can be measured on the plate, within the distance between the discs. The above description shows that the pair comprising the "solar disc" and *falak*

202. Since the degree of the opposition and the ascending and descending nodes are on the ecliptic, the distance between the degree of opposition and the closest node corresponds to their difference in longitude.

al-jawzahar functions as the pair "lunar disc" and *falak al-jawzahar* in al-Bīrūnī's description, but the scale of the time of totality should be close to the "solar disc".

According to the text, the start and end time of the eclipse are calculated as t_o-t_1 and t_o+t_1 respectively, where t_o is the time of the opposition and t_1 the "time of half totality and emersion". The latter can be found via the "pointer of the hours of totality and emersion" that is attached to *falak al-jawzahar* on the corresponding scale, which is on the inner part of the plate.

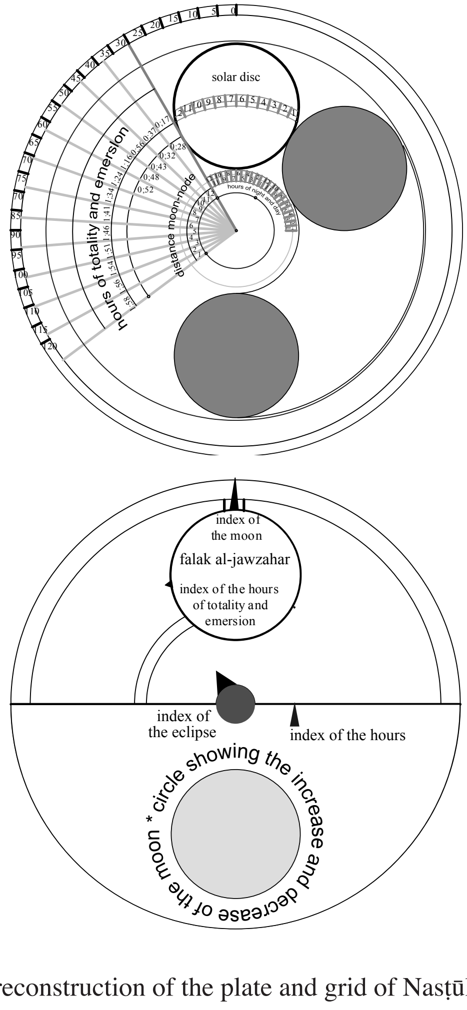


Figure 10: Possible reconstruction of the plate and grid of Nasṭūlus' lunar astrolabe.

Other instruments related to lunar eclipses from Arabic sources

In his article "Astrolabes as Eclipse Computers: Four Early Arabic Texts on Construction and Use of the *ṣafīḥa kusūfiyya*", Johannes Thomann presents another two instruments related to lunar eclipses. One of them is attributed to ʻAlī ibn ʻĪsā (fl. 830 AD) and it can provide information for both the lunar phases and the lunar eclipses. It consists of a double-sided plate and a grid, like the plate described by al-Bīrūnī, but the various elements seem to have different sizes and placements on the plate and grid. The description of the other instrument comes from an 11th century manuscript and is written by an anonymous author probably in the 10th century AD. This instrument has no moving parts and the scales drawn on it can convert the argument of latitude of the moon to ecliptic latitude and vice versa. According to the latitude of the moon, the instrument can give information on when a lunar eclipse can take place and whether it will be partial or total.

ʻAbd al-Raḥmān al-Ṣūfī (903-986 AD) presents a study on the lunar and solar eclipses using a plane astrolabe in chapters T145-147/P140-142 and T152-157/P147-153 of his first treatise on the astrolabe (version T).[203]

Some centuries later, Jamshīd Ghiyāth el-Dīn al-Kāshī (1393-1449) invented an instrument called the Plate of Zones. Among other functions, it can help to compute the times of first contact, first totality, middle of the eclipse, end of the totality and complete clearance, and also the magnitude of a lunar eclipse, when the time of the opposition, the lunar latitude at the time of the opposition and the rate at which the moon is elongating from the sun are known. This instrument consists of a plate with graduated limb, an alidade and a ruler, both of them graduated, and has been studied by E. S. Kennedy.[204]

203. Flora Vafea, "ʻAbd al-Raḥman al-Ṣūfī: Study of the lunar and solar eclipses with the astrolabe", in *Navigating across Mathematical Cultures and Times*, I.M. Vandoulakis, Liu Dun (editors), World Scientific Publishing Co, 2017 (forthcoming). Chapters T145-147 and T152-157 correspond to the numbering of chapters in the facsimile reproduction: ʻAbd al-Raḥman al-Ṣūfī, *Two books on the Use of the Astrolabe* (كتابان العمل بالأسطرلاب), Frankfurt am Main, Institute for the History of Arabic-Islamic Science, 1986, pp. 181-185 and 189-206, while chapters P140-142 and P147-153 correspond to the numbering of chapters in the edition *Kitāb al-ʻamal bil asṭurlāb by ʻAbdur Raḥman b.ʻUmar aṣ-Ṣūfī,* (كتاب العمل بالأسطرلاب), (ed. M. ʻAbdul Muʻid Khān), Hyderabad-Deccan: Osmania Oriental Publications, 1962, pp. 111-114 and 117-127.

204. E. S. Kennedy, "A fifteen century lunar eclipse computer" in *Scripta Mathematica*, Vol. 17, 1951, pp.91-97.

Remarks

1. The results obtained by this device are approximate, as al-Bīrūnī mentions at the beginning of the next chapter. Important parameters, such as the varying distance between the earth and the moon – hence the velocity of the moon – and the varying distance between the earth and the sun are not taken into consideration.

2. On the entries in table 1
The results obtained by this device are based on the pre-calculated entries of the table given at the end of the chapter (table 1).

The table provides the "distance", the adapted digits *m* of the lunar eclipse, the start time of the eclipse t_1 in hours and minutes that could be taken as half the duration of the eclipse, and half the duration of the totality t_2 in hours and minutes,[205] when the ecliptic latitude β of the moon in minutes is known. Although al-Bīrūnī deals with the detailed description of the plate's construction, he neither enters into the values presented in table 1, nor alludes to the background theory that generated this table. Below, we shall investigate the possible origins of table 1 and the background theory on which it is based.

The calculation of these magnitudes is presented by Ptolemy in the study of the solar and lunar eclipses in book VI of his *Almagest*. After the discussion on conjunctions and oppositions in chapters VI.1-4, Ptolemy defines the limits of the eclipses. When the moon is at perigee, he takes the moon's radius as R_m=0;17,40° and the shadow's radius as R_s=0;45,56°, then the greatest distance between the centers of the moon and shadow that can produce a lunar eclipse is R_m+R_s=1;03,36°. This corresponds to a distance of about α=12;12° from either of the nodes on the inclined circle of the moon,[206] and an ecliptic latitude β=1;03,19°.[207] When the moon is at apogee, the moon's radius is taken as R_m=0;15,40° and the shadow's radius as R_s=0;40,44°, so the greatest distance between the centers of the moon

205. The values in the column "Hours of totality" correspond to the values of half the duration of the total phase of the eclipse, as it will be shown below.

206. Claudii Ptolemaei, *Opera quae extant omnia*, vol. 1,1 (ed.) J.L. Heiberg: VI.5, p. 484; also G. J. Toomer, *Ptolemy's Almagest*, pp. 286-7.

207. This value is calculated using the formula sin β=sin $\alpha\cdot$sin *i*, where *i* is the inclination of the moon's orbit, taken by Ptolemy as *i*=5°. Ptolemy considers as negligible the difference between the sum R_m+R_s and the maximum lunar latitude β that can produce a lunar eclipse (see Toomer, *Ptolemy's Almagest*, pp. 296-8).

and shadow that can produce a lunar eclipse becomes R_m+R_s=0;56,24°. The corresponding distance of the moon from the closest node is α=10;48°,[208] and the ecliptic latitude of the moon is β=0;56,8.6°. The limit for moon's latitude β=60 min provided in table 1, which can produce a lunar eclipse, is between the two limits given by Ptolemy.

Then Ptolemy proceeds to the computation of the distance the moon travels during half of the eclipse and the totality in order to construct lunar eclipse tables for the positions of the moon at both the perigee and the apogee.[209] The underlying formula for half the eclipse is $s_1 = \sqrt{(R_s + R_m)^2 - \beta^2}$ (1) and for half the totality is $s_2 = \sqrt{(R_s - R_m)^2 - \beta^2}$ (2), while the ecliptic latitude β is related to the magnitude m of the eclipse (in digits) with the linear equation $\beta = R_s + R_m - \frac{m}{12}2R_m$, which can provide the magnitude as $m = 6 \cdot \frac{R_s + R_m - \beta}{R_m}$ (3). The resulting tables provide the magnitude, minutes of immersion and half totality of a lunar eclipse, when the argument of the moon's latitude, measured from the north limit of its inclined circle, is given.

The first table on p. 222 of Heiberg's edition includes the necessary correction for the various positions of the moon between perigee and apogee, while the second one gives the correspondence between the eclipse's magnitude measured in linear digits (fraction of the moon's diameter eclipsed) or area digits (fraction of the lunar disc surface eclipsed).

Formulas (1) and (2) give the arcs s_1 and s_2 traveled by the moon; to find the corresponding time, these arcs should be increased by one twelfth (for the additional motion of the sun) and then the result be divided by the hourly motion of the moon.[210]

208. Claudii Ptolemaei, *Opera quae extant omnia*, vol. 1,1 (ed.) J.L. Heiberg: VI.7, p. 502; also G. J. Toomer, *Ptolemy's Almagest*, p. 296.

209. Claudii Ptolemaei, *Opera quae extant omnia*, vol. 1,1 (ed.) J.L. Heiberg: VI.7-8, pp. 507-512, 520-522; also G. J. Toomer, *Ptolemy's Almagest*, pp. 298-302, 307-308.

210. Claudii Ptolemaei, *Opera quae extant omnia*, vol. 1,1 (ed.) J.L. Heiberg: VI.9, p. 524; also G. J. Toomer, *Ptolemy's Almagest*, pp. 306, 308.

Muḥammad ibn Mūsā al-Khwārizmī (ca.780- ca.850) had also studied the solar and lunar eclipses in his *Zīj al-Sindhind*. The Arabic text of this important work is apparently lost, but the treatise is available in Latin translation, edited with commentary by Heinrich Suter.[211] As for the lunar eclipses, there are tables that provide the magnitude in digits, the minutes of immersion (*casus*) and the minutes of half the totality (*mora*) as functions of the argument of latitude (*argumentum latitudinis*), taken as the distance of the moon's center from the ascending node, when the moon is at apogee (tables 73-74) and perigee (tables 75-76). The last four columns of tables 73-75 are independent from the previous ones and serve to correct the obtained results according to the distance of the moon between apogee and perigee; they bear the title *Tabula proportionis*, while the last 5 columns of table 76 give the correspondence between linear and area digits for the eclipses. The Latin text describes how to use the tables to compute the magnitude of the eclipse, the whole duration of the eclipse and the totality. It is clear from the Latin text and Suter's commentary that the minutes of immersion and half totality in these tables are minutes of arc, not minutes of time. To convert such an entry into hours of time one should first add 1/12 of the selected entry to it and then divide the result by the velocity of the moon in minutes per hour. For example, for the position of the moon at apogee and an argument of latitude 4;30° we obtain 4 min for half the totality from table 73; then adding 1/12 of this to it, we obtain $4\,\text{min} + \frac{1}{12} \cdot 4\,\text{min} = 4;20\,\text{min}$ and dividing by the velocity of the moon at apogee, found as 30;12 min/h from table 61 (p.175), we obtain 0;08,37 h for half the totality; hence, the totality will last 17;14 min. This method is similar to that presented in the *Almagest*, but al-Khwārizmī uses different parameters for the diameters of the moon and the earth's shadow, the inclination of the moon's orbit etc.

The entries of table 1 for the start time of the eclipse and the duration of (half) of the totality are given in hours and minutes. In order to compare the

211. Suter, Heinrich, *Die astronomischen Tafeln des Muḥammed ibn Mūsā al-Khwārizmī in der Bearbeitung des Maslama ibn Aḥmed al-Madjrītī und der lateinischen Übersetzung des Adelhard von Bath*, Copenhagen: Kongelige Danske Videnskabernes Selskab, 1914. The Latin text on determining the lunar eclipses is on pp. 26-28, Suter's commentary on pp. 86-91, while the corresponding tables 73-76 are on pp. 187-190.

entries of table 1 with those of al-Khwārizmī's tables, we can follow the steps mentioned below:

(i) Conversion of the latitude β of table 1 (given in minutes) into argument of latitude α (in degrees). This can be achieved either using the formula $\sin\beta=\sin\alpha\cdot\sin i$, where i is the inclination of the moon's orbit, taken by al-Khwārizmī as i=4.5°, or using al-Khwārizmī's tables 21 and 26 (pp.132, 137) with interpolation.
(ii) Computation of the results (magnitude, half eclipse, and totality) for the arcs of latitude found in step 1, using al-Khwārizmī's tables for both perigee and apogee. The results are found with interpolation. Half the eclipse and totality are measured in minutes of arc.
(iii) Conversion of minutes of arc into hours of time by al-Khwārizmī's above mentioned method. The formulas $t_1 = \frac{13\sqrt{(R_s + R_m)^2 - \beta^2}}{12 \cdot v_m}$ (4) for the time of half the eclipse and $t_2 = \frac{13\sqrt{(R_s - R_m)^2 - \beta^2}}{12 \cdot v_m}$ (5) for half time of the totality are used.

The results of this comparison are presented in the following tables and graphs:[212]

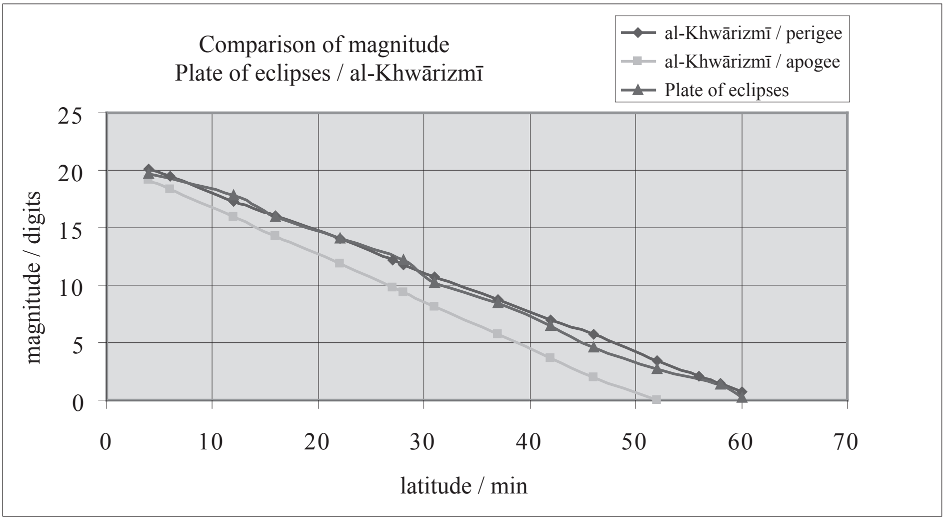
latitude	*magnitude at perigee*		*magnitude at apogee*		*table 1*	
	10-mal	60-mal	10-mal	60-mal	10-mal	60-mal
4	20.14544	20;09	19.16884	19;10	19.68333	19;41
6	19.4514	19;27	18.37064	18;22	–	–
12	17.33843	17;20	15.93679	15;56	17.8	17;48
16	15.99207	16;00	14.30517	14;18	15.93333	15;56
22	14.01795	14;01	11.84415	11;51	14.05	14;03
27	12.15859	12;10	9.817752	9;49	–	–
28	11.78847	11;47	9.412043	9;25	12.16667	12;10
31	10.75713	10;45	8.157294	8;09	10.25	10;15

(Continued)

212. Some additional values for the latitude have been inserted in the tables; these values will be useful for comparison with the recalculated table 1.

37	8.728527	8;44	5.71186	5;43	8.4	8;24
42	6.965664	6;58	3.658776	3;40	6.483333	6;29
46	5.730342	5;44	2.028143	2;02	4.6	4;36
52	3.423241	3;25	0	0;00	2.733333	2;44
58	1.407764	1;24	–	–	1.333333	1;20
60	0.684293	0;41	–	–	0.166667	0;10

Table 2: Comparison of magnitude, measured in digits, between the values of table 1 and those calculated from al-Khwārizmī's tables.



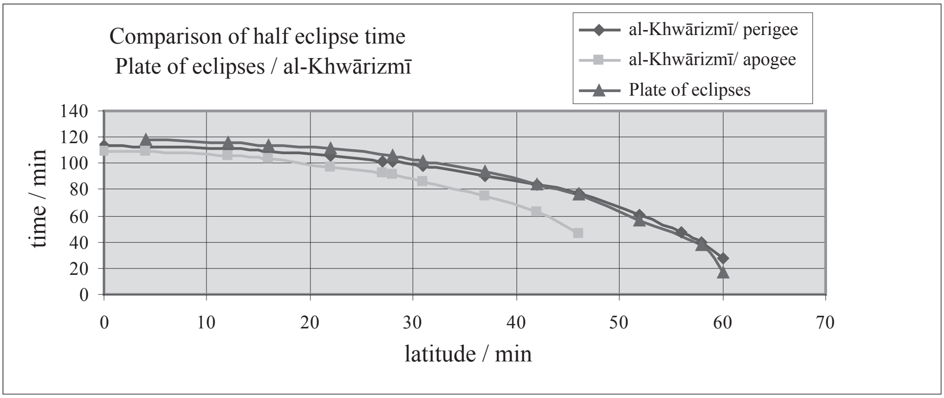
Graph 1: Comparison of magnitude between the values of table 1 and those calculated from al-Khwārizmī's tables.

latitude	*half eclipse at perigee*	*half eclipse at apogee*	*table 1*	*half totality at perigee*	*half totality at apogee*	*table 1*
0	113.021	108.979	–	50.0257	45.98786	–
4	112.7629	109.0972	118	49.39415	45.17044	52
12	110.8919	105.8799	116	44.99304	37.55843	48
16	109.1679	103.194	114	40.5749	30.52738	43
22	105.6599	96.8441	111	29.61511	5.53543	32
27	101.7859	92.71256	–	8.471969	–	–
28	100.9946	91.10401	106	1.376546	–	28

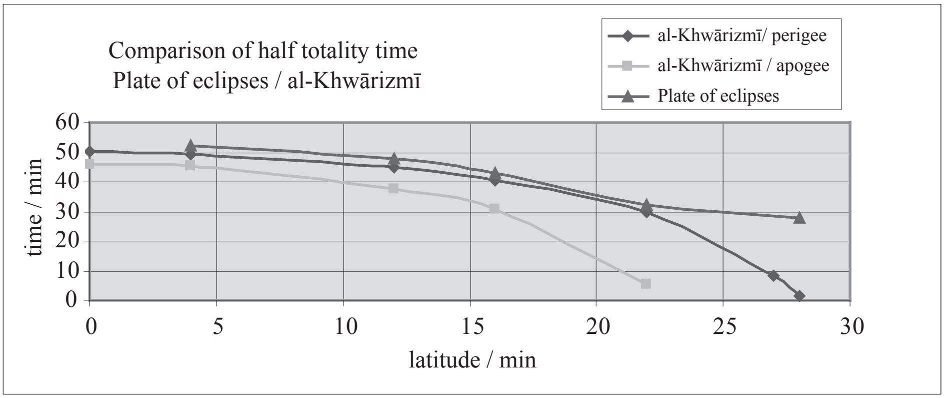
(Continued)

31	98.1699	86.33911	101	–	–	–
37	90.83749	74.67336	94	–	–	–
42	83.24119	62.60878	84	–	–	–
46	76.84627	45.88619	76	–	–	–
52	61.06701	–	56	–	–	–
58	39.61541	–	37	–	–	–
60	27.17518	–	17	–	–	–

TABLE 3: Comparison of the duration of half the eclipse and totality, measured in minutes, between the values of table 1 and those calculated from al-Khwārizmī's tables.



GRAPH 2: Comparison of the duration of half the eclipse between the values of table 1 and those calculated from al-Khwārizmī's tables.



GRAPH 3: Comparison of half duration of the totality between the values of table 1 and those calculated from al-Khwārizmī's tables.

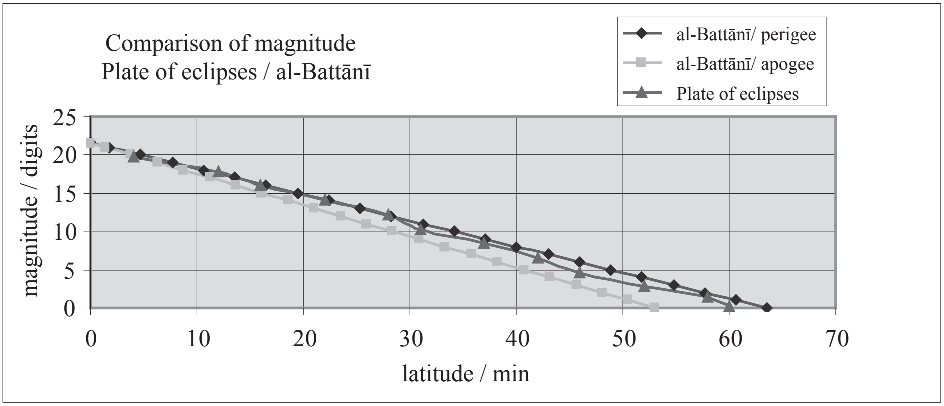
Al-Battānī, in his *Kitāb al-Zīj*, studies the lunar eclipses following the Ptolemaic tradition.[213] He describes formulas that are equivalent to the methods presented by Ptolemy to compute the magnitude, half eclipse and half totality, when the argument of latitude α (measured from the ascending node) or the ecliptic latitude β of the moon is known. The results, obtained in digits for the magnitude and minutes of arc for the moon's travel during the eclipse, are presented in two tables (vol. II p. 90) for the position of moon at apogee and perigee. For the intermediate positions of the moon, there is a table of correction (v. II p.89) similar to that presented by Ptolemy in *Almagest* VI.8. Al-Battānī, further, divides the obtained minutes of arc by the *praecessio Lunae* (سَبق) to determine the time of half the eclipse and totality. Nallino, who edited, translated into Latin and commented this work of al-Battānī, explains that *praecessio Lunae respectu Solis est motus synodicus horarius* (note 1, v. I p. 98), so the divisor should be the difference between the apparent velocity v_m of the moon and that of the sun v_s per hour.[214] In the commentary of this passage, it is noted: "*minuta morae* sunt $^1/_2$ longitudo (*bc*) *spatii* quod Luna intra umbram percurrit ab initio ad finem totalities. *Horae morae* vel *tempus morae* indicant temporis intervallum in quo Luna a *b* ad *a* pervenit, nempe dimidium tempus totalitatis" (v. I p. 275). So the formula used by al-Battānī for the computation of the time of half the eclipse is $t_1 = \frac{\sqrt{(R_s + R_m)^2 - \beta^2}}{v_m - v_s}$ (6) and for half the totality $t_2 = \frac{\sqrt{(R_s - R_m)^2 - \beta^2}}{v_m - v_s}$ (7). The corresponding magnitudes presented in table 1 are measured in hours and minutes, so these formulas could be applied for their calculation.

Since the lunar eclipse tables of al-Battānī use as an independent variant the ecliptic latitude of the moon, a comparison between the results of al-Battānī and the values of table 1 can be conducted directly using graphs. For the magnitude

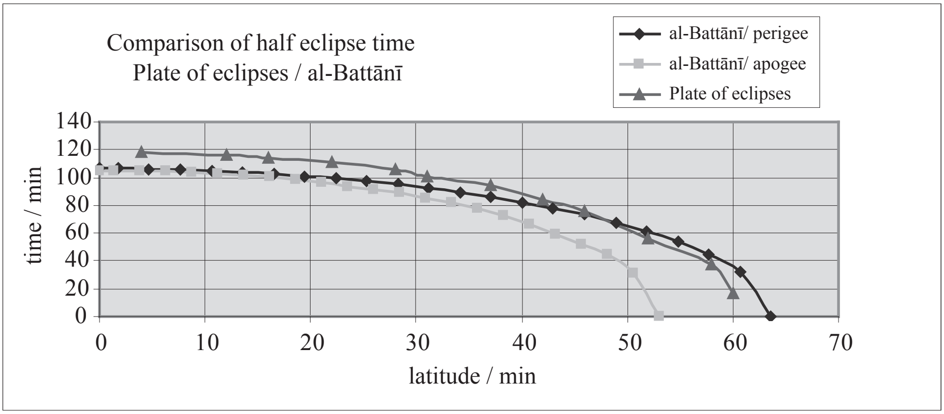
213. Al-Battānī sive Albatenii, *Opus Astronomicum,* ed. Carlo Alfonso Nallino, 3 vols. Milan: Reale Osservatorio di Brera in Milano, 1903, 1907, 1899. The study of lunar eclipses is in v.I pp.96-104 (latin translation) and pp. 275-276 (commentary), in v.II pp. 86-91 (tables) and pp.231-232 (commentary on the tables) and vol. III pp. 146-156 (Arabic text).

214. Here al-Battānī divides the arcs s_1 and s_2 by v_m-v_s for the additional motion of the sun, while Ptolemy and al-Khwārizmī increase these arcs by their twelfth and then divide them by the apparent hourly velocity v_m of the moon. The results of both methods are close to each other. For all necessary calculations I assume that the sun moves 1° in 24 hours thus v_s=2;30 min/h.

of the eclipse the values are taken directly from the tables.[215] For the half time of the totality the values of *quantitas [dimidiae] morae* of the table must be divided by $v_m - v_s$, while for the values of the time of half the eclipse the *quantitates incidentiae* and *quantitas [dimidiae] morae* should first be added and then divided by $v_m - v_s$; such a comparison is presented below in graphs 4, 5 and 6:

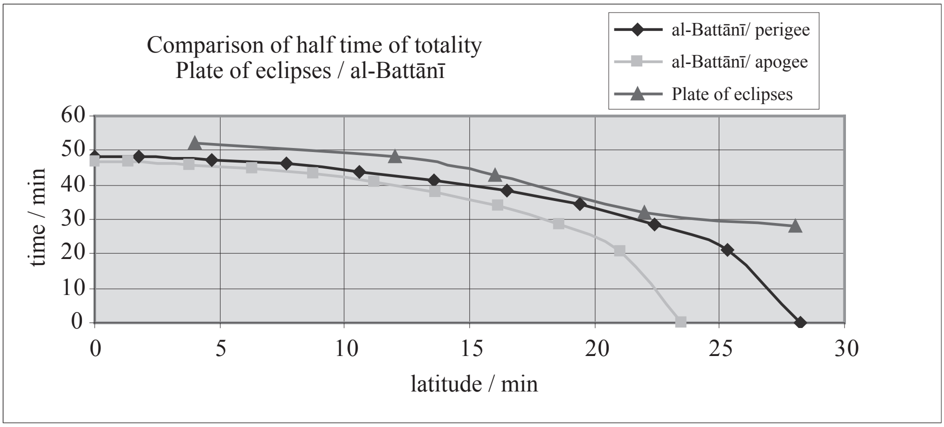


Graph 4: Comparison of the magnitude between the values of table 1 and those taken from al-Battānī's table.



Graph 5: Comparison of the duration of half the eclipse between the values of table 1 and those calculated from al-Battānī's tables.

215. Al-Battānī, *Opus Astronomicum,* ed. Nallino, v.II, p.90.



Graph 6: Comparison of half duration of the totality between the values of table 1 and those calculated from al-Battānī's tables.

Al-Battānī makes a further step towards accuracy: since the moon moves on its inclined circle and not on the ecliptic, the ecliptic latitude of the moon varies during the lunar eclipse. Taking the moon's latitude in the middle of the eclipse as β, let β_a be its latitude at the start time of the eclipse, β_b at the start time of totality, β_c at the end of totality, and β_d at the end of the eclipse. Then, if t_o is the time of the opposition,

the start time of the eclipse will be at time $t_a = t_o - \frac{\sqrt{(R_s + R_m)^2 - \beta_a^2 + (\beta - \beta_a)^2}}{v_m - v_s}$,

the start time of the totality at $t_b = t_o - \frac{\sqrt{(R_s - R_m)^2 - \beta_b^2 + (\beta - \beta_b)^2}}{v_m - v_s}$,

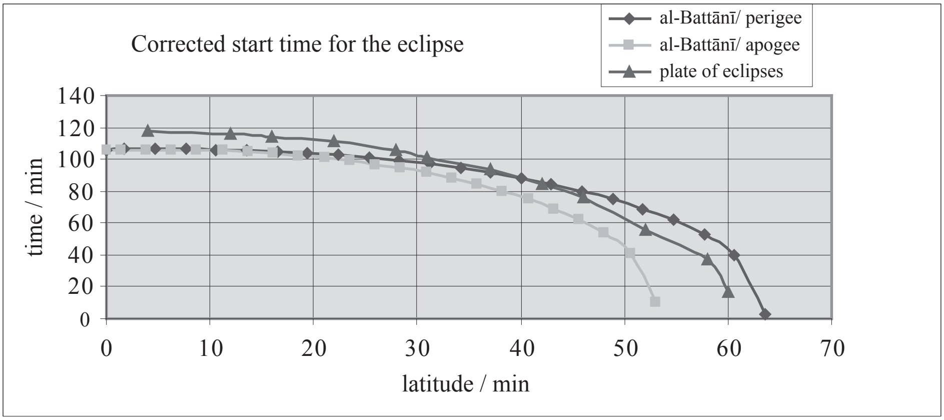
the end of the totality at $t_c = t_o + \frac{\sqrt{(R_s - R_m)^2 - \beta_c^2 + (\beta - \beta_c)^2}}{v_m - v_s}$,

and the end of the eclipse at $t_d = t_o + \frac{\sqrt{(R_s + R_m)^2 - \beta_d^2 + (\beta - \beta_d)^2}}{v_m - v_s}$. [216]

The values of the start time of the eclipse according to al-Battānī's formula $\frac{\sqrt{(R_s + R_m)^2 - \beta_a^2 + (\beta - \beta_a)^2}}{v_m - v_s}$ have been calculated and the results are compared

216. Al-Battānī, *Opus Astronomicum,* ed. Nallino, v. III, pp. 148-149 and v. I, pp. 98-99.

with the corresponding entries of table 1; the results are presented in graph 7.[217] The rest of the formulas give similar graphs.



Graph 7: Comparison of the start time of the eclipse between the values of table 1 and those calculated with al-Battānī's more accurate formula.

The comparison of the values of table 1 with the corresponding values calculated with the tables and theory of Ptolemy, al-Khwārizmī and al-Battānī leads on to the following conclusions:

(i) There is an obvious discrepancy between the values of table 1 and those calculated above for both the perigee and apogee, which cannot be explained if the moon is considered in an intermediate position.

(ii) The linear relation $m = 6 \cdot \dfrac{R_s + R_m - \beta}{R_m}$ (3) between the magnitude of the eclipse and the ecliptic latitude of the moon should result in obtaining their graphs as straight lines, but this is not the case for the graph of magnitude in table 1 (line marked with triangles in graphs 1 and 4). This fact could be ex-

217. The latitude β_a of the moon at the start of the eclipse is calculated according to al-Battānī as follows: For a given latitude β in the middle of the eclipse, the corresponding values of $s_1 = \sqrt{(R_s + R_m)^2 - \beta^2}$ and argument of latitude α are found from the tables (al-Battānī, *Opus Astronomicum* v. II pp. 90, 78-83); the argument of latitude at the start of the eclipse will be $\alpha_a = \alpha - \frac{13}{12} s_1$. The latitude β_a is found by converting the argument α_a into ecliptic latitude.

plained by assuming that either there are some copying errors in the table, or the values of the radii of the moon and earth's shadow are not taken as constants in all cases, meaning that the distance between the earth and moon had been taken varying, as if the entries of table 1 are results of observations of real eclipses.

(iii) The comparison of the time of totality from table 1 with the calculated half time of totality (table 3, graphs 3 and 6) shows that the values in table 1 refer to the half time of totality as well. The pair of values $(\beta, m) = (28, 28)$ is obviously a result of a copying error: the value β=28 min can hardly produce a total eclipse.

(iv) The graphs of the time of half eclipse corresponding to perigee and that coming from table 1 intersect each other (see graphs 2, 5 and 7). This means that different values of the parameters R_m, R_s and v_m had been used in the formulas (4) and (6) to calculate the time of half of the eclipse in table 1. This is not unusual, since the values accepted for these magnitudes by Ptolemy, al-Khwārizmī and al-Battānī are different; these values are presented below, in table 4.

	perigee			*apogee*		
	R_m	R_s	v_m	R_m	R_s	v_m
Ptolemy	17;40	45;56	–	15;40	40;44	–
al-Khwārizmī	17;17	44;16	35;40	14;38	36;27	30;12
al-Battānī	17;37,30	46;0	36;04	14;45	38;30	30;18

Table 4: Values of the parameters R_m, R_s and v_m used by Ptolemy, al-Khwārizmī and al-Battānī. The radii R_m, R_s are measured in minutes and the velocity v_m in min/h.

We proceed now to the calculation of the parameters R_m, R_s and v_m used for the entries of table 1. I assume that the formulas used for the compilation of table 1 are

$$m = 6 \cdot \frac{R_s + R_m - \beta}{R_m} \quad (3), \quad t_1 = \frac{\sqrt{(R_s + R_m)^2 - \beta^2}}{u} \quad (8) \text{ and } \quad t_2 = \frac{\sqrt{(R_s - R_m)^2 - \beta^2}}{u} \quad (9),$$

where $u = \frac{12}{13} v_m$ or $u=v_m-v_s$.[218] Since table 1 provides for the total eclipse the values of β, m, t_1 and t_2, there are 3 unknown values to be determined from the above 3 equa-

218. The former value of u corresponds to the method of Ptolemy and al-Khwārizmī, while the latter to that of al-Battānī.

tions: R_s, R_m and u. Solving the simultaneous equations produced by each row of the table corresponding to a total eclipse, we obtain the results presented in table 5:

β	t_1	t_2	m	R_m	R_s	u	R_s/R_m
min	*min*	*min*	*digits*	*min*	*min*	*deg/h*	–
4	118	52	19.68333	11.43623	30.08096	0.35020	2.63032
6	118	52	19.68333	17.15435	45.12145	0.52531	2.63032
12	116	48	17.8	19.22856	49.81616	0.58615	2.59074
16	114	43	15.93333	15.49541	41.65351	0.48126	2.68812
22	111	32	14.05	19.08674	47.60804	0.56722	2.49430
28	106	28	12.16667	-22.2545	5.12730	–	–
28	106	10	12.16667	15.21098	43.63351	0.48826	2.86855

TABLE 5: The solutions of the simultaneous equations (3), (8) and (9). Values on grey background are unacceptable, values in bold box replace the values above them to provide acceptable results.

Only the line corresponding to latitude β=16 gives as a solution a full set of three acceptable values, while, if the latitude β=4 is replaced by β=6, the resulting solution is also acceptable. For latitude β=28, only t_2=10 can give acceptable values for R_m and R_s.

For the rows of table 1 that correspond to a partial eclipse, there are only two simultaneous equations and three parameters to be determined. To proceed to the solution, I assume that the ratio $R_s/R_m=\lambda$ is constant. Since the solutions presented in table 5 are not helpful for determining the parameters, I apply this procedure to the entire table 1. The value λ=2.6 used by Ptolemy does not provide acceptable solutions.[219] In all cases examined for the latitude β=4 (and sometimes for β=5) the results for R_m, R_s and v_m are not acceptable; thus it could be replaced by β=6. Also for β=28 no total eclipse is possible, while β=27 can produce such an eclipse.[220] Only a small range of values for λ ($2.64 \leq \lambda \leq 2\ ⅔$) can give acceptable values for the pa-

219. The value λ=2.6 is mentioned in Claudii Ptolemaei, *Opera quae extant omnia*, vol. 1,1 (ed.) J.L. Heiberg: V.14, p. 421 and VI.5, p. 480; also G. J. Toomer, *Ptolemy's Almagest*, pp. 254 and 285. This value produces unacceptable solutions for the latitudes 4, 12 and 28 and does not allow a total lunar eclipse for β=27 or 28.

220. According to table 75 (Suter, *Die astronomischen Tafeln des al-Khwārizmī*, p. 189), al-Khwārizmī gives a total eclipse for β=28 (t_2=1;22,35 min) only when the moon is at perigee (see

rameters; among them λ=2.65 produces results (m, t_1, t_2) with the greatest correlation coefficients when compared with the corresponding values from table 1. The mean values of the results computed for every row of table 1 for λ=2.65 are R_m=16;30,38 min,[221] R_s =43;45,10 min and u=30;43,05 min/h; then the hourly velocity of the moon will be $v_m=u+v_s$=30;43,06+2;30=33;13,06 min/h, or $v_m=13u/12$=33;16,41 min/h. These values correspond to a position of the moon near its mean distance from the earth. The value β=58 results in m=0;49,21 and t_1=0;32, which are far from the entries of table 1, thus it has been changed to β=57. According to the values of the radii, the limit of the latitude for a lunar eclipse is $\beta \approx R_s+R_m$=60;15,48 min and the limit for a total lunar eclipse is $\beta \approx R_s-R_m$=27;14,32 min.

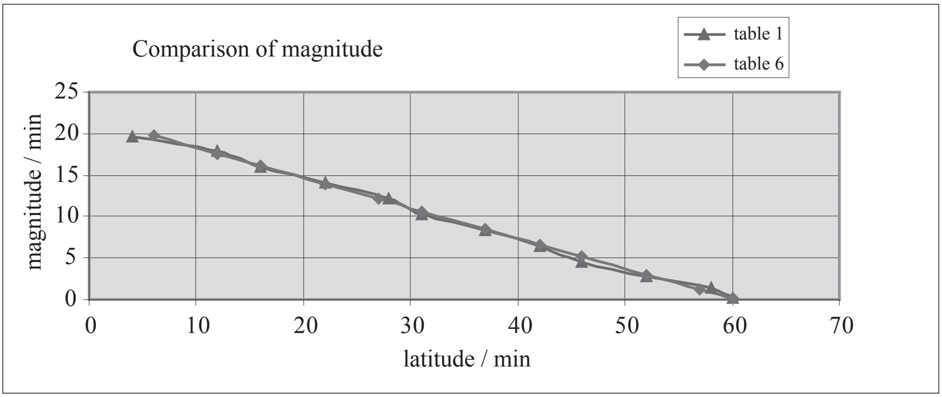
The recalculated values of the magnitudes are presented in table 6 and the comparison between the given and recalculated values is depicted in graphs 8-10.

"distance"	*magnitude m*	*half eclipse t_1*		*half totality t_2*		*latitude β*
		$v_m=13u/12$	$v_m=u+v_s$	$v_m=13u/12$	$v_m=u+v_s$	
	digits	h;m,s	h;m,s	h;m,s	h;m,s	min
1	19;43	1;57,20	1;57,07	0;52,0	0;51,54	6
2	17;32	1;55,34	1;55,21	0;47;51	0;47,46	12
3	16;05	1;53,41	1;53,29	0;43;09	0;43,04	16
4	13;54	1;49,47	1;49,35	0;31,26	0;31,23	22
5	12;05	1;45,25	1;45,14	0;07,06	0;07,05	27
6	10;38	1;41,07	1;40,56	–	–	31
7	8;27	1;33,05	1;32,55	–	–	37
8	6;38	1;24,34	1;24,25	–	–	42
9	5;11	1;16,11	1;16,03	–	–	46
10	3;0	0;59,36	0;59,29	–	–	52
11	1;11	0;38,17	0;38,13	–	–	57
12	0;6	0;11,01	0;11,0	–	–	60

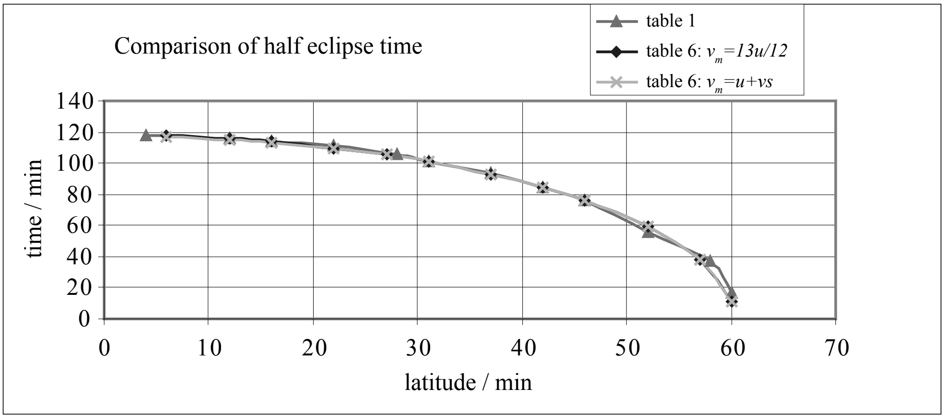
Table 6: Recalculated table of magnitudes of lunar eclipse for R_m=16;30,38 min, R_s=43;45,10 min and u=30;43,05 min/h.

table 3 above) and al-Battānī presents the value β=28;16 as the limit for a total lunar eclipse at perigee (al-Battānī, *Opus Astronomicum*, v.II p.90).

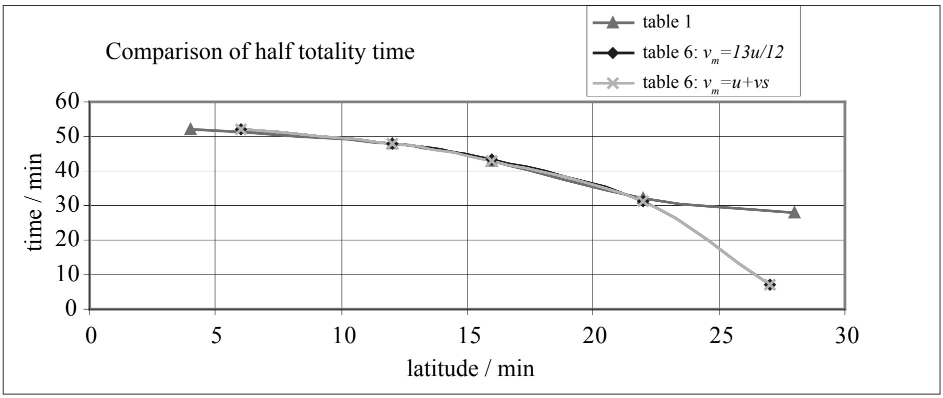
221. The least value is R_m=15;57,56 for β=31 and the greatest one R_m=17;33,39 for β=12.



GRAPH 8: Comparison between the given and recalculated values of magnitude.



GRAPH 9: Comparison between the given and recalculated values of half eclipse time.



GRAPH 10: Comparison between the given and recalculated values of half time of totality.

The "distance" mentioned by al-Bīrūnī seems to be used as an ancillary element defining the rows of table 1 and connecting the function between the right and left semicircles of the plate. In table 1, there is no unit of measurement below its name, unlike the other magnitudes included in the table. On the other hand, in the instructions for the lunar astrolabe by Nasṭūlus, the distance between the degree of opposition and the closest node, which corresponds to their difference in longitude $\Delta\ell$, is the principal element for determining the duration, the start and end time and the magnitude of the lunar eclipse. This difference in longitude is related to the ecliptic latitude β of the moon with the formula $\tan\beta = \tan i \cdot \sin \Delta\ell$, where i is the inclination of the moon's orbit, taken by Ptolemy equal to 5°. The choice of the values for the latitude in table 1 has no obvious reasoning, but it could be related either to the above mentioned distance $\Delta\ell$, or to the distance α between the center of the moon and the closest node ($\sin\beta = \sin i \cdot \sin\alpha$). The values of latitude calculated for both cases are presented in table 7; these values are compared to those given in table 1 and the corrected ones taken from table 6. The calculated values of latitude, using either the distance $\Delta\ell$ or α, do not diverge significantly. Thus, a probable reasoning that could to an extent explain the particular choice of values for the latitude in table 1 is their derivation from the distance $\Delta\ell$ or α.[222] It is remarkable that a distance of 12° produces a result between 62 and 63 min, which is greater than the corresponding value of 60 min in table 1 and less than the limit of 64 min for a lunar eclipse, as mentioned in the instructions for Nasṭūlus' instrument (β=64 min corresponds to $\Delta\ell$=12;17°).

distance	*latitude β using Δℓ*	*latitude β using α*	*latitude β table 1*	*latitude β table 6*
degrees	*minutes*	*minutes*	*minutes*	*minutes*
1	5.25	5.22	4	6
2	10.50	10.44	12	12
3	15.74	15.66	16	16
4	20.98	20.87	22	22
5	26.21	26.08	28	27
6	31.44	31.28	31	31
7	36.65	36.47	37	37
8	41.86	41.65	42	42

(Continued)

222. The step of the latitude calculated using $\Delta\ell$ or α is decreasing between 5;15 and 5;8,24 min, or 5;13,12 and 5;7,12 min respectively, and the step of the rounded values is 5 or 6 min, while the corresponding step is between 2 and 8 in table 1 and between 3 and 6 in table 6.

9	47.05	46.81	46	46
10	52.22	51.96	52	52
11	57.38	57.10	58	57
12	62.53	62.22	60	60

TABLE 7: Calculation of the latitude corresponding to the distance $\Delta\ell$ and α, and comparison with the values of tables 1 and 6.

The magnitude of a lunar eclipse can be measured on the scale 1-12 within the lunar disc on the plate of eclipses, following the tradition of Nasṭūlus' instrument. This measurement can be visually conducted, but can neither provide accurate results, nor the magnitude of a total lunar eclipse with duration. In addition, the rotation of the grid corresponding to each unit of "distance" is 8.043055° (see remark 3 below), while the step of the latitude is not constant in table 1. For this reason, a new scale has been introduced onto the right semicircle of the plate, where the pointer N[223] indicates the magnitude in adapted digits.[224] However, the values of the magnitude in table 1 are not in complete agreement with the variation of the latitude; these values have been recalculated in table 6. This fact could be explained if the entries of table 1 were considered as data coming from various observations of lunar eclipses with varying distances between earth and moon.

3. On the measure of arc HJ

In the construction of the scales on the back side of the plate (figure 5), the manuscripts present a variety of possibilities for the construction of the arcs HJ=RM. According to the manuscripts, the arc HJ could be taken as equal to:

(i) $^1/_3$ of arc KH (ثلث قوس كح) in M, N (considering 'ـحـ' that appears in M as 'ح'),
(ii) $^1/_3$ of arc RKH (ثلث قوس ركح) in T, L
(iii) "$^1/_3$ of 2 arcs RKH" (ثلث قوسي ركح) in B, S, Q, A (considering 'ز' that appears in A as 'ر'). This can be interpreted as ⅔ of arc RKH (ثلثي قوس ركح).

The measure of arc RKH is equal to that of the central angle RÊH that it subtends, which can be calculated as RÊH=60°-2HÊW, where W is the point where the tan-

223. The arc of the pointer N is closer to the rim of the plate than the scale on the lunar disc, for this reason it is longer so the pointer could provide better accuracy in reading the magnitude of the eclipse.

224. The adapted digits are linear, not area digits as those presented by Ptolemy for measuring the obscured area of the lunar disc in *Almagest* VI 7-8 (Toomer, *Ptolemy's Almagest*, pp. 302-305, 308).

gent ES touches the circle K (see also figure 13). Since HÊW=ω=1.045° approximately, [225] RÊH=60°-2.09°=57.91°. Hence the measure of arc RKH is 57.91° and that of KH=$^1/_2$·RKH is 28.955°. Each of the above cases gives the following results:

(i) If HJ=$^1/_3$·KH=$^1/_3$·28.955°=9.652°, then HH´=RH+HJ=57.91°+9.652°=67.5617°, thus each of the 12 subdivisions will be 5.630°. Assuming that the rotation of the grid is 0° when *falak al-jawzahar* coincides with the lunar disc, the "distances" 1-5, when they are indicated by the first pointer of "distance", produce the following rotations of the grid:

"distance"	*rotation*
1	67.562°
2	61.932°
3	56.302°
4	50.672°
5	45.042°

Table 8. Rotation of the grid corresponding to "distances" 1-5, if HJ = $^1/_3$·KH

When the rotation is less than 60°, the two circles -the lunar disc and *falak al-jawzahar*- intersect each other. According to the rotations presented in table 8, the "distances" 3-5 will produce a partial eclipse of the lunar disc, namely the *falak al-jawzahar* on the grid will cover a part of the black lunar disc on the back side of the plate, which is absurd, because the data of table 1 affirm that the "distances" 1–5 produce a total lunar eclipse.

(ii) If HJ=$^1/_3$·RKH=$^1/_3$·57.91°=19.303°, then HH´=RH+HJ=57.91°+19.303°=77.213°, thus each of the 12 subdivisions will be 6.434°. Following the previous assumption, the "distances" 1-5 indicated by the first pointer of "distance", will produce the following rotations of the grid:

"distance"	*rotation*
1	77.213°
2	70.779°
3	64.345°
4	57.911°
5	51.477°

Table 9. Rotation of the grid corresponding to "distances" 1-5, if HJ=$^1/_3$·RKH

225. For the calculation of angle HÊW see remark 4.

According to the rotations presented in table 9, the “distances” 4 and 5 will produce a partial eclipse of the lunar disc, since the rotations are less than 60°. This is absurd, taking in consideration the data given in table 1.

(iii) If HJ=⅔·RKH=⅔·57.91°=38.607°, then HH´=57.91°+38.607°=96.517°, thus each of the 12 subdivisions will be 8.043°. Following the previous assumption, the “distances” 1-6 indicated by the first pointer of “distance”, will produce the following rotations of the grid:

distance	*rotation*
1	96.517°
2	88.474°
3	80.431°
4	72.388°
5	64.345°
6	56.302°

TABLE 10. Rotation of the grid corresponding to distances 1-6, if HJ=⅔·RKH

In this case, the “distances” 1-5 produce a total lunar eclipse, while “distance” 6 produces a partial eclipse, and the results are consistent with the data given in table 1. Hence the correct choice is the third one: HJ=⅔·RKH.

On this side of the plate, all the arcs traversed by the pointers of the grid subtend a central angle of 96.5166°, which is equal to 1⅔ of the angle RÊH, since the two endpoints of each arc are marked while rotating the grid by the above angle.

4. On the ratio of the radius of *falak al-jawzahar* to that of the lunar disc on the plate of eclipses.

In Arabic astronomy, *falak al-jawzahar* is a circle corresponding to a cross section of the cone of earth’s shadow, at the point where the moon enters this cone.[226] The radius R_s of this circle is greater than the moon’s radius R_m, not equal as depicted on the grid of the plate of eclipses. As previously mentioned, Ptolemy con-

226. This definition for *falak al-jawzahar* is mentioned by al-Ṣūfī in chapter T153 of his first treatise on the use of the astrolabe (in MS Istanbul, Topkapi Saray, Collection Ahmet III, 3509/1); see the facsimile reproduction of the above manuscript in ‘Abd al-Raḥman al-Ṣūfī, *Two books on the Use of the Astrolabe*, p. 189, and Vafea, “‘Abd al-Raḥman al-Ṣūfī: Study of the lunar and solar eclipses with the astrolabe”, chapter T153 / P148.

sidered the ratio $\lambda = R_s/R_m$ approximately as 2.6, while the value of this ratio that can vindicate the entries of table 1 is approximately $\lambda=2.65$ (see remark 2 above).

However, the instructions for the construction of the plate reveal additional suggestions for the value of this ratio. Since the arc RKH contains 12 digits, the arc HJ=⅔ RKH contains 8 digits (figure 5). Thus the arc HJ could in theory be divided into 8 digits, enumerated from 13 to 20. On the left side of the device, when the first pointer of "distance" is placed between 1 and 5, the number (from 13 to 20) closer to the right edge of *falak al-jawzahar* on the grid would indicate the magnitude of the total eclipse. In particular, when this pointer is placed at the beginning of the scale of "distance" that corresponds to a latitude $\beta=0°$, the circumference of *falak al-jawzahar* will touch point J, indicating a magnitude of 20 digits for a central lunar eclipse. In this case the moon crosses the earth's shadow passing through its center (figure 11); that is, the center of the moon will pass through the axis of the cone of the earth's shadow. If the values $\beta=0$ and $m=20$ are used in formula (3) $m = 6 \cdot \frac{R_s + R_m - \beta}{R_m}$, we obtain $20 = 6 \cdot \frac{R_s + R_m}{R_m} \Leftrightarrow \frac{R_s + R_m}{R_m} = \frac{10}{3} \Leftrightarrow \frac{R_s}{R_m} = \frac{7}{3}$, thus the ratio of the radii will be $\lambda = R_s/R_m = 2^1/_3$.

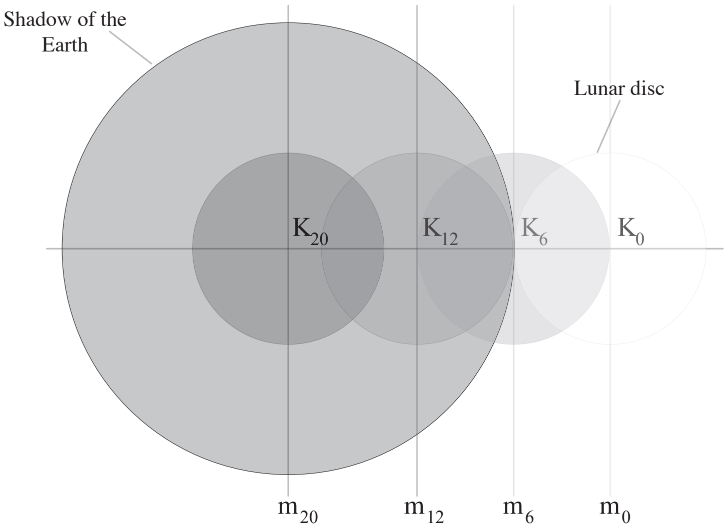


Figure 11: The relative position of the lunar disc and the earth's shadow for lunar eclipses of magnitude 0, 6, 12 and 20 digits.

In a central lunar eclipse, the right edge of *falak al-jawzahar* on the grid reaches the point J, thus the circle (K, KJ) depicts the size of the shadow of the earth in proportion to the lunar disc on the back side of the plate (figure 12). In this case, the ratio R_s/R_m could be calculated as 2.224. The calculation is as follows:

On the front side of the plate (figure 1), the arcs *AR* and *AB* are equal to $^1/_6$ and $^1/_4$ of a circle respectively, that is *AR*=60° and *AB*=90°; thus the arc *RB=AB-AR*=30° and the central angle $B\hat{E}R$=30°. The circles ABCD on the rim and the circles on center K (for the solar and lunar discs) on the front and the back side of the plate are equal, hence on the back side of the plate (figure 5) $B\hat{E}S$=30° (*ES* is tangent to the circle K). The arc *RKH* passes through the center K of the circle, intersecting the circle at points H and R, hence the point H is inside the angle $B\hat{E}S$, so $B\hat{E}S=B\hat{E}H+H\hat{E}S$. The angle $H\hat{E}S=\omega$ can be calculated as ω=1° 2′ 42″=1.045°.[227]

227. Calculation of the angle $\omega=H\hat{E}S$.

Figure 13 is taken as a fragment of figure 5. Let W be the point where the tangent ES touches the circle (K, *r*), *r=KH=KW*, and x be the angle $H\hat{K}W$.

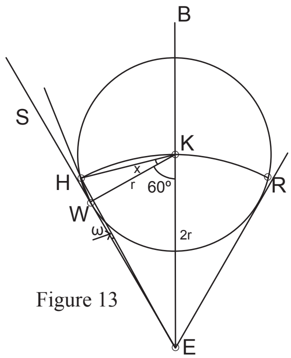


Figure 13

The triangle *KWE* is right-angled (*KW*⊥*EW*) and $W\hat{E}K$=30°, thus *KE*=2*r* and *WKE*=60°.

In the triangle *KHE* we know: *KH=r*, *EK=EH*=2*r* and the angle $E\hat{K}H = 60° + x$ (1)

We apply the law of cosines to the triangle *KHE*: $HE^2 = HK^2 + KE^2 - 2 \cdot HK \cdot KE \cdot \cos(60+x) \Rightarrow$ $(2r)^2 = r^2 + (2r)^2 - 2 \cdot r \cdot 2r \cdot \cos(60+x) \Rightarrow 4r^2 = 5r^2 - 4r^2 \cdot \cos(60+x) \Rightarrow 4r^2 \cdot \cos(60+x) = r^2 \Rightarrow$ $\cos(60+x) = 1/4 \Rightarrow 60 + x = 360k \pm arc\cos 1/4 \Rightarrow x = 360k - 60 \pm 75.5225$. For $k = 0$ and taking the sign "+" we have x=75.5225°-60° ⇒ *x*=15.5225°. (2).

The triangle *HKW* is isosceles (*KH=KW*), then $W\hat{H}K = H\hat{W}K = \frac{180° - x}{2} = 90° - \frac{x}{2}$. (3)

From (3) and (1) we have: $W\hat{H}E = W\hat{H}K - E\hat{H}K = 90° - \frac{x}{2} - (60° + x) = 30° - \frac{3x}{2}$. (4)

The angle *HWS* is formed by the chord *HW* and the tangent *EWS*, thus $H\hat{W}S = \frac{x}{2}$. (5)

Then $K\hat{E}H = B\hat{E}H = B\hat{E}S - H\hat{E}S = 30° - 1.045° = 28.955°$ and $R\hat{E}H = 2 \cdot K\hat{E}H = 2 \cdot 28.955° = 57.91°$. Since $H\hat{E}J = \frac{2}{3} R\hat{E}H = \frac{2}{3} \cdot 57.91° = 38.607°$, $K\hat{E}J = K\hat{E}H + H\hat{E}J = 28.955° + 38.607° = 67.562°$.

In a circle of radius R, the length of a chord that subtends a central angle θ is equal to $2R{\cdot}\sin{}^{\theta}/_{2}$, then $KH{=}2R{\cdot}\sin{}^{28.955}/_{2}$ and $KJ{=}2R{\cdot}\sin{}^{67.562}/_{2}$ and the ratio of the radii of the *falak al-jawzahar* and the lunar disc will be:

$$\frac{R_s}{R_m} = \frac{KJ}{KH} = \frac{2R \cdot \sin\dfrac{67.562}{2}}{2R \cdot \sin\dfrac{28.955}{2}} = \frac{\sin 33.781}{\sin 14.478} \cong 2.224$$

The above study shows that there are various inconsistent values for the ratio of the radius of *falak al-jawzahar* to that of the lunar disc used in the plate of eclipses. The implied size of *falak al-jawzahar* and the position of point J are approximate.

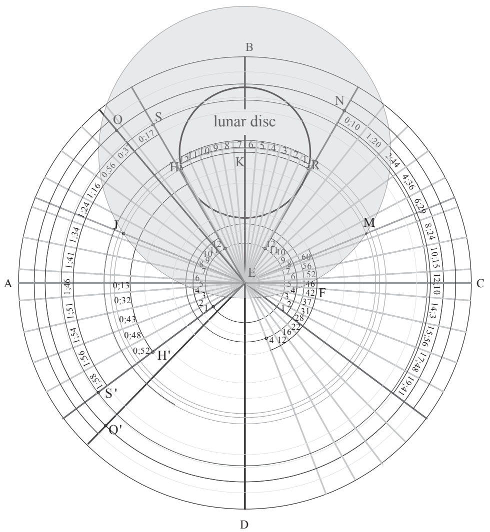


Figure 12: The earth's shadow-circle (K, KJ) placed on the back side of the plate.

In the triangle *HEW* using (4) and (5) we obtain:

$$\omega = H\hat{E}W = 180° - W\hat{H}E - H\hat{W}E = 180° - \left(30° - \frac{3x}{2}\right) - \left(90° + 90° - \frac{x}{2}\right) = 2x - 30°$$

Since x = 15.5225° (2), then ω = 1.045° = 1° 2' 42".

5. In the case where the first pointer of "distance" is placed on the interval between 1 and 4, which produces total eclipses (figure 9), the "lunar disc" is covered by the circular sector formed by the radii of the grid terminating at the 15th and 29th days of the hollow month and it is not completely visible. It is not feasible to cut out that circular sector, because this would affect the depiction of the moon phases, when the grid is placed on the front of the plate.

6. The term "كسوفيّة" (*kusūfīya*) that determines the plate in the title of the chapter "عمل الصفيحة الكسوفيّة" is related to the term "كسوف" (*kusūf*) usually used for the solar eclipse, while for the lunar eclipse the word "خسوف" (*khusūf*) is used; but this plate cannot be used for a solar eclipse. The "solar disc" drawn on the front side of the plate does not play any role. It is probably there to remind us that the absence of its light causes the phases of the moon and the eclipses.[228]

Conclusions

(1) The plate of lunar eclipses extends the range of problems that can be solved with an astrolabe. The values of the scales on the plate are consistent with the lunar eclipse theory by Ptolemy and the related works by al-Khwārizmī and al-Battānī, but different values for the radii of the moon and the earth's shadow are used.

(2) This instrument is an evolved form of the lunar astrolabe invented by Nasṭūlus: it is extended on both faces of the plate, containing additional scales for the lunar latitude and the adapted digits of magnitude, and uses the lunar disc for measuring the magnitude of a lunar eclipse instead of the solar disc. For the lunar phases the two scales for the days of the full and hollow month on the grid replace the scale of 0°-120° on the limb of Nasṭūlus' instrument.

(3) This plate can provide a view of the moon phase at any day of the lunar month plus the time of moon-rising during night or day, when the grid is mounted on front side of the plate, and can help to predict lunar eclipses, and give additional information about them, when the grid is mounted on the back side and the latitude of the moon is known. It does not deal with solar

228. The term "eclipse" comes from the Greek word "ἔκλειψις" that means "absence".

eclipses, although the term "كسوف" (*kusūf*) / solar eclipse is repeatedly used, instead of "خسوف" (*khusūf*) / lunar eclipse.

(4) The results obtained using this plate are approximate, and al-Bīrūnī clarifies this at the beginning of the next chapter. This is because important parameters, such as the distances between sun, earth and moon, are not taken in consideration. In some cases, the values for the various scales of the plate presented in table 1 diverge from the expected ones and are recalculated in table 6.

(5) The construction of this plate does not use the stereographic projection, as other plates described in the same book by al-Bīrūnī. It is a smart way to present the data of tables enriched with visual approximations of both, the phase of the moon and the lunar eclipse.

(6) The representation of the lunar eclipse on the back side of the plate is not realistic. The lunar disc is black, while the *falak al-jawzahar* has a brighter color –that of bronze. When the *falak al-jawzahar* completely covers the "lunar disc", there is no eclipse. Moreover, the size of *falak al-jawzahar*, as it is drawn on the grid, does not correspond to the size of the earth's shadow. Thus, this *falak al-jawzahar* should not be considered as a cross section of earth's shadow, but as a simple circular pointer that gives the magnitude of the eclipse.

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