Ibn Yūnus' Report on Early Islamic Observations for Determining the Rate of Precession of Equinoxes

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Abstract: The present article deals with Ibn Yūnus' report about an astronomical parameter, so-called "precession of equinoxes". In ancient astronomy the cause of precession was justified by different methods. The rate of precession was important for updating stars and planetary tables. Precession was discovered by Hipparchus and examined by Greek astronomers later. It apparently influenced the early generation of Muslim astronomers to repeat new observations for about three centuries. We will deal with the observational accuracy of Greek and Muslim astronomers in measuring the ecliptic longitude of Regulus, which was being employed to obtain the rate of Precession.

Keywords: history of observational and computational astronomy, Greek Astronomy, Astronomy of the Islamic Period, Ibn Yūnus, Hipparchus, Ptolemy, precession of equinoxes, rate of precession, Regulus, Armillary sphere.

Introduction

Nowadays there is wide agreement that Muslim astronomers did not merely adopt the astronomical parameters of Greeks and employed them throughout history. Since the early stages of astronomy in the Islamic Period, Muslim astronomers carried out several translations of Ptolemy's *Almagest*, endeavoured to conceive the essential Ptolemaic astronomical techniques, and began to examine the authenticity and precision of various astronomical (theoretical and observational) parameters in Ptolemy's *Almagest* (composed in the beginning of 2^{nd}c. A. D.). One of such tasks, concerns the motion and rate of precession of the equinoxes. In modern astronomy, this concept is justified by the spatial circular motion of the earth's polar axis around its ecliptic poles, one rotation of which takes 25,770 years.[1]

1. Evans, pp. 245-247.

Astronomical Basis in Greek Astronomy

Ptolemy refers to Hipparchus' book, "*On the length of the year*", in which he introduces the length of the tropical year which exceeds 365^d by less than $1/4^d$.[2] However, Ptolemy elsewhere in *Almagest*, quotes Hipparchus' more accurate value for the tropical year as: $(365^d + 1/4^d) - 1/300^d = 365; 14,48^d = (365^d + 5; 55,12^h)$.[3] Neugebauer believed that the latter value was obtained through the combination of the length of tropical year and precession (i.e., 1° per 100 years).[4] However, he assumed that the sidereal solar year, based on the fixed stars, is longer than the tropical one by $(365+1/4)^d + 1/144 = 365^d + 6; 10^h$.[5] Ptolemy acknowledged that Hipparchus had known that the length of the tropical year is less than that of the sidereal one, due to precession.[6] So Hipparchus obtained the difference at about 11^m, although his assumptions on the length of the tropical year and the value of its decrease due to precession, were not very accurate.[7] The modern value of this deficiency is about 20^m. According to Swerdlow's statement, Hipparchus must have known the length of the sidereal and tropical years, perhaps from Babylonian astronomy[8] although the latter hypothesis was severely criticized by Neugebauer.[9] In ancient astronomy, the cause of precession was justified by different methods. Hipparchus interpreted it as the westward motion of the equinoxes and solstices,[10] whereas Ptolemy assumed it to be a slow rotation of the sphere of the fixed stars around the ecliptic pole from west to the east. In this model, the point of the vernal equinox is displaced westwards, i.e. to the rear side of the sequence of the zodiac signs, and because of the slow motion of precession, the ecliptic longitude of stars increases continuously, but very slowly.[11]

2. Ptolemy, p. 131.
3. Ibid., p.139
4. Neugebauer, 1949, pp. 92-94; Swerdlow, p. 300.
5. Swerdlow, p. 300
6. Ptolemy, pp. 131, 139
7. Swerdlow, p. 294
8. Ibid., p. 305.
9. See Neugebauer, 1950, pp. 1-8; Neugebauer (1975 vol.1, p. 293), hesitantly propounds the possibility of that Hippachus adopted the length of the sidereal year from Babylonian astronomy.
10. According to Ptolemy, pp. 327, 329, Hipparchus had written a book titled: *On the displacement of the solsticial and equinoctial points*.
11. Ptolemy, pp. 327, 329.

Obviously the rate of precession plays an important role not only in measuring the coordinates of the "fixed" stars but also in updating the ecliptic coordinates of the sun, moon and planets. The rate value was a milestone for constructing any tabular ecliptic coordinates of the celestial bodies in *zīj*es (astronomical hand books with tables) from the Islamic Period. In his *Almagest*, Ptolemy mentions that the ecliptic longitude of Regulus (α-Leo) was measured by Hipparchus in - 128/127 A.D. According to Ptolemy, Hipparchus had previously obtained the rate of precession as 1° per century by comparing his result with those of previous Greek astronomers.[12] Ptolemy himself made a new observation to measure the longitude of Regulus in his own time on 23 Feb. 139 A.D, which led him to conclude that the rate of precession was the same as the one Hipparchus had already found (see Table)[13] Regulus was an important star for such observations on account of its being the only first magnitude star which is very near to the ecliptic; therefore, any variation in its longitude directly shows the value of the changes in position of the vernal equinoxes among stars. Finally, Ptolemy concluded that precession only affects the ecliptic longitudes of stars and it has no effect on their latitudes.[14] Moreover, finally, Ptolemy quoted several observations, concerning the rate of precession from previous astronomers who used the Pleiades, Spica (α Virgo) and β Scorpio, and almost all of them measured the rate to be 1° per century.[15] But he implicitly sets aside Timocharis' observation: the rate in 12 years as equal to $1/6° \equiv 1°$ per 72 years, since his observation was unique and contrary to the accepted rate. Ptolemy further quoted Hipparchus as stating that Timocharis' observations on precession were not trustworthy, having been made very crudely.[16]

It seems that reports by Ptolemy and his predecessors on the observation of Regulus (α-Leo) inspired early Muslim astronomers to repeat such observations from time to time in order to measure the rate of precession in their own times and, perhaps, more precisely. They needed, however, a precise value in their astronomical computations for a certain year.

12. Ibid., pp. 328, 334-338.

13. Ibid., p. 328

14. Ibid., p. 329; For some further information on Ptolemy's description of precession, see Pannekoek pp. 60-66

15. Ptolemy, pp. 333-338.

16. Ibid., pp. 329, 336. It seems that Hipparchus' faulty judgment on Timocharis' observation may have influenced Greek astronomy and, later, Islamic astronomy.

The aim of the present article is to re-compute the extant observational values of the ecliptic longitudes of Regulus, which were obtained by the first generation of astronomers of the Islamic Period, according to the years given by Egyptian astronomer Ibn Yūnus (339?-400 A.H./950?-1009), in his *al-Zīj Al-Kabīr*, where he provides us with a collection of observations of Regulus made by the earlier generations of Muslim astronomers.[17]

In Ptolemy's *Almagest*, one finds the method by which he obtained the rate of precession. According to Ptolemy's statement, he used an armillary sphere for measuring the ecliptic longitude of Regulus. We have neither information on the size of his instrument, nor on its precision.[18] It has several rings, including two specific rings for measuring the ecliptic longitude and latitude of any celestial object.

However, by modern re-computations, the value given by Hipparchus bears an error around 0;34° while Ptolemy's value was about 1; 34°. It should be noted that the adjustment of a large armillary sphere could have been a real task, since no bright star was near to the vernal equinox, and adjusting the instrument to the cardinal points must have been cumbersome; secondly, variations in temperature may have an affect on the adjustments of the metallic astronomical instrument.

The mean error in observations from the Islamic Period is about 0; 18° (see Table). It is difficult to justify the improvement in accuracy of the Muslim observations; although, part of the explanation may lie in the fact that they probably conducted their observations in groups of competent astronomers who worked in observatories with larger and more accurate instruments (see below).[19]

Underlying conditions for re-computing the records

As already mentioned, the displacement of the equinoxes in a short period of time, (e.g. one or two years), is negligible. Muslim observers were aware of this fact and apparently believed that there is no necessity to give the precise date of

17. Ibn Yūnus, *Al-Ḥākimī Zīj*, ed. P. Caussin, reprinted by F. Sezgin in: *Islamic mathematics and astronomy*, vol. 24, Frankfurt 1997, pp. 197-209; for the original text, see Ibn Yūnus, *Al-Ḥākimī Zīj*, ms. OR. 143, Leiden Library, pp. 106-108.

18. Ptolemy, pp. 217-219.

19. For instance, see observations of 3 and 6 in the table.

their observations for each given year.[20] The best period of time for the observation of Regulus was between the 15th March and the 1st April at that time, and extensively between the 1st January and the 1st June. Islamic astronomers have recorded the years of their observations using both Yazdgerdī and Hijrī Calendars. Some observations, however, have only been recorded using one of the above-mentioned calendars.

We need to establish some standards for re-computing the given values, although the errors in results do not exceed 50″ per year. As stated earlier, Ibn Yūnus' reports do not contain any information on the dates of observations; however, the span of time leads to the reasonable conclusion that Regulus can only be observed for five months every year. Before re-computing the values of longitudes based on observations of Regulus, the following two conditions should be considered:

(1) The above-mentioned extensive window of time, for the visibility of Regulus, should necessarily be taken into account in all cases. If a discrepancy is observed for the selected date, the relevant item should not be included.
(2) As a matter of fact, the day selected should coincide with the years given by Ibn Yūnus for each report (Yazdgerdī and Lunar Hijrī Calendars), even though some cases have been only recorded in one calendar.

I have employed the trial and error method in my search for a criterion corresponding with the years given for every report. Interestingly, I noticed that the beginning day of Yazdgerdī Calendar (1 Farvardīn of YC) is a good choice, because it coincides with the given years based on Hijrī Calendar. Moreover, except for one case,[21] all dates correspond to the month of April, which coincides with the above mentioned span of time for observation. It should be kept in mind that setting of dates based on the lunar year would have not been a good choice, because it is out of sync with the solar year.

The JD (Julian Day number) of each observation was added for two reasons: the precise date and the facility of conversion into the Christian Calendar.Ibn Yūnus' reports concerning observers and observations of Regulus are listed in the

20. The motion of precession takes place in retrograde direction, in respect to the zodiacal sequence. For modern theories on precession, see: N. Capitaine *et al.*, pp. 567-586, J.L. Hilton *et al.*, pp. 351-367.

21. Based on this criterion, Ibn A'lam's observation only falls in March.

table. The longitudes of Regulus were re-computed using the *Star List* Software developed by Raymond Mercier.[22]

Observers	*Date*	*JD*	*Regulus*			
			Long.		*Lat.*	
			Obs.	*Comp.*	*Obs. comp*	
Hipparchus	-128/127, Jan. 1	1674672	119;50°	120;23,52°		
Ptolemy	139, Feb. 23	1771879	122;30	124;3,31.68		
Mumtaḥan obs.	195Y/211H/ 29 Ap. 826	2022873	133;30	133;32,49		
Ḥabash al-Ḥāsib (1)	198Y/214H/28 Ap. 829	2023968	133;0	133;35,11	0;15′	(-10;9')
Ḥabash from Mumtaḥan	198Y/214H/28 Ap. 829	2023968	133;9	133;35,11		
Damascus obs.	201 Y/217H/27Ap. 832	2025063	133; 15	133;37,50		
Ḥabash (2)	200Y/216H/28 Ap .831	2024698	133;1,47	133;36,55		
Ḥabash (3)	201Y/217H/27 Ap. 832	2025063	133;11	133;37,50		
Banū Mūsā (1)	209 Y/225H/25 Ap. 840	2027983	133;49,40	133;44,51		
Banū Mūsā (2)	216Y/232H/24Ap. 847	2030538	133;50,15	133;50,9.36		
Banū Mūsā (3)	219 Y/235H/23Ap. 850	2031633	133;27	133;52,45		
Māhānī	230 Y/ 247H/20 Ap. 861	2035648	134;6	134;2,6.8		
ʻAbd'l-Malik Sanad, Jawharī	201Y/217H/ 27 Ap.832	2025063	133;42,10[23]	133;37,50	0;10′	(- 0;15,14.6′)
Samarqandī	234 Y/251H /19 Ap. 865	2037108	133;40	134;5,7.54		
Banū Amājūr (1)	286Y/304H/ 6 Ap. 917	2056088	134;32	134;48,39		
Banū Amājūr (2)	288Y/306H / 6 Ap. 919	2056818	134;17	134;50,7.44		
Ibn Aʻlam	345Y/ 365H/ 22 Mar 976	2077623	135;6	135;37,26		

Table: The observations of Regulus by early astronomers of the Islamic Period.

The Rate of the Precession of Equinoxes

According to a well-known quotation, Muslim astronomers, unlike Ptolemy and his predecessors, obtained a rate of the precession equal to 1° per 66 years.[24] The latter

22. The software is accessible, through: http://www. raymondm.co.uk/

23. The text gives 133; 41,10 which leads to an incorrect result.

24. Shāh Khuljī (d. 874 A.H. / 1469) in this *Explanation of the Īlkhānī Zīj* (*Tāwzīḥ-e Zīj-e Īlkhānī*, p. 29) tells us that al-Ma'mūn's observers obtained a rate of precession of 1° per 66 years

value is obviously more accurate than the one obtained by Ptolemy (see above).[25] Ibn Yūnus introduces better values for the rate of precession. I recomputed the tabular values and noticed that the rate of 1° per 66 years is not easily extracted from every couple of successive values (see below). This may be due to the proximity of time between Muslim observations. Therefore, even negligible errors could have easily affected the rate of precession. If this is the case, Muslim astronomers probably compared the ancient Greek observations to their own. It should be noted that an increase of the time span between observations results in a decrease of the observational errors in the final result. However, I identified the following way in order to justify the value of 1° per 66 years: the first Islamic observation was accomplished by the *Mumtaḥan* group in 195Y/826A.D.[26] The difference between Ptolemy's observation and that of the *Mumtaḥan* is 687 years, and the difference in the longitude of Regulus is 11°; thus 687/11 ≡ 1° in 62.45 years. Then one may consider the difference in longitudes and the period of time between Hipparchus' observation and the *Mumtaḥan*'s one in this way: 953/ 13; 40° ≡ 69.73 years per degree. The average of these two obtained values, approximately results in 1° per 66.09 years. I call this computation as the "average method".

This is only a hypothesis that has not yet been supported by historical sources. Before listing observational reports, Ibn Yūnus mentions that the "science" of observing stars and comparing their positions is a serious task. Therefore, those that oppose the endeavors of observers, when possible errors [in observations] happen, should realize this fact.[27] This issue indicates that Ibn Yūnus, and perhaps other Muslim astronomers noticed that reliance on a limited number of observations by early Muslim astronomers is an awkward decision. As Ibn Yūnus cites, from Ḥabash, that the difference between the observational value of the longitude of Regulus between the Mumtaḥan group (Baghdad) and the Damascus observers in 198 Yazdgerdī was 9' (see Table). Moreover, sometimes in a short period of time, the ecliptic longitude of Regulus increases correctly by the effect of precession while, on the contrary, it shows retrograde motion incorrectly during a different period of time (see Table). Ibn Yūnus noticed that it is impossible to rely

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and 8 months, and compared this result with that obtained in Ptolemy's time.

25. The modern precise value is 1° per 71.58 years.

26. The first group of astronomers from the Islamic Period (Aṣḥāb Al-Mumtaḥan) who worked in Baghdad at the Shammāsiya Observatory, and who revised different astronomical parameters, including the rate of precession received from Ptolemy's *Almagest*.

27. See Ibn Yūnus, p. 197.

unconditionally on all the reports. Before completion of his discussion on the motion of equinoxes, he purposefully quotes several couples of observations from his reports to show how both 66 and 70 years can be extracted for the rate of precession:

- First couple of observations: Ḥabash (2) in 200Y and Banū Amājūr in 288Y respectively. The computation is as follows:
  For the duration of 88 years: 134;32° – 133;12° ≡ the motion is 1;20°. This corresponds to 1° in 66 years if we take into account a longitude of 133;14° in 200Y:
  134;32° – 133;12° ≡ 1;18°, giving as a result 1° in 67.6 years (which is not a good result).
- Second couple of observations: Ḥabash in 200Y and Samarqandī in 234Y. The following computation should be considered:
  For the duration of 34 years: 133;40° – 133; 11° ≡ the motion is 29', the result being 1° in 70.3 years. If 133;42°[28] – 133;11° ≡ the motion is 31' and the results is 1° in 66 years.
- Interestingly, I noticed that the motion of 1° per 70 years could have been obtained from the difference between Mumtaḥan in 198Y, (quoted from Ḥabash) and Banū Mūsā (3), in 219Y, for the duration of 21 years:
  133;27°–133; 9° ≡ the motion is 18' and it results in 1° per 70 years (see Table).

By quoting the observations of Ḥabash and Samaqandī, Ibn Yūnus wants to show that with a negligible change of 2' in the longitude of Regulus, both rates of 66 and 70 years might have been extracted. It seems that he did not eventually arrive at a definite rate for precession from these reports. However, he tentatively introduces either 1° per 66 years or 1° per 70 years without arriving at a conclusive result. Ibn Yūnus was probably aware of the above-mentioned "average method" or approximate rate of precession in advance, and he only searched for them in his reports.

By collecting several reports, comparing them and then stating that for the duration of a hundred years the motion is 1;33°[29] Ibn Yūnus inexplicably re-

28. Ibn Yūnus, p. 205, quotes the value as: 133; 41°.

29. Ibid., p. 207. The value can be read: 1;13°, 1;53° and 1;18° but the best one is: 1;33°.

jects the value of the motion rate given by Ptolemy without naming him (see above).[30] On this basis one concludes that the motion is 1° per 64.5 years.

Only two observations have measured the latitude of Regulus: Ḥabash al-Ḥasib (1) and Khālid ibn ʻAbd al-Malik Marwārūdī and his group, the former being more accurate.

According to Gonābādī, Ibn Aʻlam obtained the rate of motion as equal to 1° per 70 years, although he does not mention the second observation with which, Ibn Aʻlam obtained this result.[31] I found out that Ḥabash's observation (1) and Ibn Aʻlam's observations can lead us to this result:

For the duration of 147 years: 135;6°– 133;0° ≡ the motion is 126' and for 1°, it results in 70 years.

The Best Observations of the Longitude of Regulus from the Islamic Period

We now list the most accurate observations containing the least errors (See Table). It should be noted that some observations show values with the precision of less than 30'; however, even these records do not represent the best ones. We are able to order the most precise observations as follows: The observation of the Mumtaḥan observers in 195 Y/826 A.D. has an error of 3', while the Banū-Mūsā's observation (2) is less than 1' off and Māhānī, in 230Y/861 A.D., is 4' off (see Table).[32] It seems that the observations carried out in observatories or conducted by a group of astronomers tend to be more accurate.

The best rate of precession which can be obtained from these Muslim observations is 1° per 70.3 years (see above). However, IbnYūnus elsewhere in his *Al-Ḥākimī Zīj*, states that he observed Regulus and obtained the rate of 1° per 70.25 years.[33]

30. Ibid., p. 207.

31. Gonābādī , p. 52.

32. The astonishing accuracy of the Banū-Mūsā's (2) observation should be considered as an incidental success, since such accuracy is not seen in the Banū-Mūsā's first and third observations.

33. Ibn Yūnus, *Al-Ḥākimī Zīj*, ms. OR. 143, (Chapter 8), p. 123, Leiden Library.

Concluding Remarks

As a whole, Muslims astronomers obtained more accurate values for the motion of equinoxes, compared to Greeks ones. The best observations from the Islamic Period apparently lead us to the rate of precession being equal to 1° per 66 or 70 years, although the value of 1° per 70 years was excellent, it could have been extracted from several reports (see Table), and it is very near the modern value (see note 25). However, it seems that the rate of 1° per 66 years was applied as a standard value in many texts (particularly in *Zīj*es from Islamic Period in the following centuries); furthermore, it was in use more frequently during the Islamic Period. One may reason that later observers mostly tended to favour the rate of 1° per 66 years because it was obtained by the Mumtaḥan group (see p. 107). However, there is no information on the second observation, with which the astronomers of Al-Ma'mūn obtained the latter result; and the table does not help us in this case. It may be assumed that the stronger reason goes back to the above-mentioned "average method" which takes advantage of a longer period of time between both Greek and Muslim observations.

Although the results of Muslim observations were more accurate than those of the Greeks, the time proximity of their observations (see above) prevented them from reaching a conclusive result. Of course, the accuracy, adjustments and size of their instruments might have skewed the results. Ḥabash reported numerous observations and carried out some of them. According to Ibn Yūnus', Ḥabash wrote two treatises entitled *Al-Arṣād bi Baghdād* and a treatise in which quoted observations were made in Damishq (*Risālat allatī yadhkuru fī-ha raṣad Dimashq*),[34] as well as *al-Zīj al-'Arabī* . On the basis of these sources, Ḥabash reports observations of the longitudes of Regulus from his contemporary astronomers. These two treatises have not yet been found, and the relation between the above-mentioned *zīj*-es and two extant copies of Ḥabash's *Zīj* [35]also remains unclear.

It seems that the enthusiasm for the re-observation of some Ptolemaic parameters, including "the motion of precession", among the early generation of Muslim

34. It seems that these two treatises dealt with the observations were carried out in the Shamāsīya Observatory in Baghdād and Qāsīyūn Observatory in Damascus respectively (both under supervision of al-Ma'mūn, the 'Abbāsid Caliph).

35. See Lorch, R., s.v. "Ḥabash-e Ḥāsib" *Encyclopaedia of the World of Islam*, vol.7, (in Persian), Tehran 1382 H.S /2003.

astronomers quickly diminished in later centuries, as only a handful of observations were conducted in the beginning of the 4th c.A.H /10th c.A.D. (See Table).

In the 7th c.A.H/13th c.A.D, Naṣīr al-Dīn al-Ṭūsī and Muḥyī al-Dīn al-Maghribī still reported the rate of precession as 1° per 70[36] and 66 years respectively.[37] Based on my research, the rate of 1° per 70 years was only used by Ibn A'lam, Naṣīr al-Dīn al-Ṭūsī and al-Kāshī [38] in their zījes, although this issue needs more investigation. It should finally be noted that the value of 1° per 70.3 years remained the best one during the medieval period until Tycho Brahe (1546-1601 A.D.) arrived at a more accurate rate for the precession of equinoxes.[39]

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36. Naṣīr al-Dīn al-Ṭūsī, fol.99v.

37. Muḥyī al-Dīn al-Maghribī, fol. 62v.; See also Gonābādī, p. 52 and Sayili, p.78.

38. See Edward Kennedy, p. 33.

39. Tycho Brahe obtained the rate of 1° per 70.588 years, which corresponds to 51″per year (Evans, p. 282). Indeed, his value was better than that of Muslim astronomers by about 0.3°. For the rate of 70.3 years, see Ḥabash (2) and Samarqandī (See above, p. 108).

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