# The Astronomical Tables of Moses Farissol Botarel ${ }^{1}$ 

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#### Abstract

Moses Farissol Botarel (Avignon, late fifteenth century) was an astronomer who wrote in Hebrew and continued various traditions that depended on astronomy in al-Andalus which, in turn, derived in large part from the zij of al-Battānī (Raqqa, d. 929). His astronomical tables are unusual in that they combine elements from the Parisian Alfonsine Tables with elements from the tables of Levi ben Gerson (Orange, France, d. I344), Immanuel ben Jacob Bonfils (Tarascon, France, $f l$. I350), and Jacob ben David Bonjorn (Perpignan, $f l$. I360). We offer an analysis of all his tables, which are restricted to the Sun and the Moon. Of particular interest is his effort to make his tables more "userfriendly", a common trend in the late Middle Ages, while maintaining the models and parameters of his predecessors.


Keywords: Moses ben Abraham of Nîmes, 8-year cycle, solar eclipse i478, user-friendly computation, double argument tables

## Introduction

Moses Farissol Botarel was an astronomer who lived in Avignon, France, in the late fifteenth century. ${ }^{2}$ Nothing is known about his life except that he was a student of Moses ben Abraham of Nîmes, who translated the Parisian Alfonsine Tables from Latin into Hebrew (1460). Farissol Botarel's astronomical works, all in Hebrew, include commentaries on the Paris Tables (epoch I368, an adaptation of the Oxford Tables for epoch 1348$)^{3}$ and on the tables of Levi ben Gerson (I288-I344);4 canons for the planetary tables of Immanuel ben Jacob Bonfils of

[^0]Tarascon (epoch 1340); ${ }^{5}$ a calendrical work; ${ }^{6}$ and his own set of astronomical tables, called Nofet ṣufim ("drippings of a honeycomb" [Ps. I9: I I], uniquely preserved in Munich, Bayerische Staatsbibliothek, MS Heb. 343, 92a-Io3b), which is the subject of this paper. NoFeT is also an acronym used by Farissol Botarel to refer to himself at the beginning of texts he composed: $N(e$ 'um) $F$ (arissol) T(almid) [a discourse (by) Farissol, (the) disciple]. ${ }^{7}$

The scientific traditions of al-Andalus were transmitted to Christians who used Latin as their learned language and to Jews who generally used Hebrew as their learned language. With respect to astronomical tables the most important works in Latin in the late Middle Ages were the Toledan Tables and the Parisian Alfonsine Tables, both of which were greatly indebted to Arabic sources. In Hebrew no set of tables had a dominant role; rather, a large variety of such sets of tables were produced in Hebrew, beginning with the tables of Abraham Bar Hiyya in twelfth-century Spain, and continuing in Spain, Portugal, southern France, and Italy: they too were indebted to Arabic sources. ${ }^{8}$ At the end of the fifteenth century the key figure was Abraham Zacut of Salamanca, Spain, but there were others including Moses Farissol Botarel who maintained this tradition. Although each set of tables in Hebrew has its unique aspects, there are basic principles that go back to al-Andalus and to earlier Islamic sources, notably al-Battān̄̄’s zij. ${ }^{9}$

In the preface to his astronomical tables, Farissol Botarel referred to a solar eclipse that took place on July 29, I478. He severely criticized some unnamed
5. Munich, MS Heb. 3I, IO7a-I 13, 3I9a-345b; Steinschneider I895, p. I6. Bonfils compiled two sets of tables: see n. I3, below.
6. Munich, MS Heb. 249, where the year is given as $1464 / 65$; Steinschneider I895, p. I2 I.
7. See, e.g., Oxford, Bodleian Library, MS Reggio I4, 44a:I, at the beginning of Farissol Botarel's comments on the Tables of Paris. Cf. Steinschneider I899, p. 45. Presumably, this means he identified himself as a disciple of the astronomer, Moses ben Abraham of Nîmes, whom he names as his teacher: see n . I4, below.
8. For surveys of the Hebrew astronomical tradition, see Goldstein 1985a and 20II. In Portugal Judah Ben Verga (fifteenth century) composed astronomical tables in Hebrew: see Goldstein 200I; in Italy Mordecai Finzi (fifteenth century) devoted a great deal of attention to astronomical tables in his Hebrew writings: see Langermann 1988.
9. For a survey of zijes, see King and Samsó (with a contribution by Goldstein) 2001. On Zacut, see Chabás and Goldstein 2000. On al-Battānī (d. 929), see Nallino I899-I907. For the influence of the zij of Ibn al-Kammād (early twelfth century, Córdoba) on Hebrew astronomical traditions in Spain, see Chabás and Goldstein 20I 5. For references to al-Battānī in medieval Hebrew astronomical texts see, e.g., Millás I959, pp. I4-I7, and Goldstein I985b, pp. 94 and IO3.
astronomers who predicted that it would be total (in Avignon), whereas according to the Alfonsine Tables, the tables of Levi ben Gerson, and others, its magnitude should be 8 or $8 \frac{1}{2}$ digits. ${ }^{10}$ However, there is no indication in this discussion of any scientific observation of this eclipse in Avignon (see Appendix). According to the NASA website on historical solar eclipses, this eclipse had a magnitude of about I0.25 digits ( $\approx 0.856 \times \mathrm{I} 2$ ) in Avignon. ${ }^{11}$ Moreover, according to the NASA website, this eclipse was total in Salamanca, as indicated by Abraham Zacut in a list of eclipses: ${ }^{12}$ see Figures I and 2.


Figure i: The partial solar eclipse of July 29, I478 at Avignon.

Io. According to the medieval convention, a solar eclipse is measured in digits where 12 digits is equal to the solar diameter; an eclipse of I2 digits is total. The modern convention is to consider a total solar eclipse as having magnitude I , whereas partial eclipses have a magnitude less than I (measured on the solar diameter considered as I).

I I. http://eclipse.gsfc.nasa.gov/SEsearch/SEsearchmap.php?Ecl=14780729.
I2. Chabás and Goldstein 2000, p. 67. The eclipse magnitude in the Hebrew MSS of Zacut is I2 digits, i.e., total, whereas in the Latin MS its value is incorrectly given as $4 ; 30$ digits.


Figure 2: The total solar eclipse of July 29, I478 at Salamanca.

Farissol Botarel's tables are unusual in that they combine elements from the Parisian Alfonsine Tables with elements from the tables of Levi ben Gerson (I288-I344) [henceforth, Levi], Immanuel ben Jacob Bonfils ( $f l$ l I350) [henceforth, Bonfils], and Jacob ben David Bonjorn of Perpignan (fl. I360) [henceforth, Bonjorn]. ${ }^{13}$ As far as we can determine, the only other example of a late medieval astronomer who depended on both the Alfonsine tradition in Latin on the one hand, and tables composed in Hebrew on the other, was Abraham Zacut (i452I 5I 5), whose Almanach perpetuum was printed in Leiria, Portugal in I496, with the canons in Latin in some copies and in Castilian in others.

In Farissol Botarel's set of tables there are 6 chapters of canons (ff. 92a-94b) followed by I5 tables (ff. 95a-Io3b): they only concern the Sun and the Moon; there are no planetary or astrological tables, except for Table I5 (without, however, any discussion of its role in astrology). As is usually the case in this genre, the canons give instructions on the use of the tables, but not on their underlying parameters or how they were compiled. In fact, the underlying models for solar

[^1]and lunar motion are Ptolemaic and, as we shall see, the parameters are taken from different medieval sources, notably, the Parisian Alfonsine Tables, the tables of Levi, and the tables of Bonfils. In his commentary on the Paris Tables, Farissol Botarel indicated his awareness that these tables are based on the Alfonsine Tables that were translated into Hebrew by "my teacher, Master Moses of Nîmes". ${ }^{14}$ The Paris Tables with epoch I368 are a version of the Oxford Tables ascribed to Batecomb with epoch I348, which were revised for the meridian of Paris, Lyon, and Avignon, instead of for the meridian of Oxford: the translation of the Paris Tables into Hebrew was due to Solomon ben Davin of Rodez, a student of Bonfils. ${ }^{15}$ It seems then that Farissol Botarel depended almost entirely on astronomical material in Hebrew produced or translated by Jewish scholars in the fourteenth and fifteenth centuries. By contrast, virtually all contemporary Christian astronomers depended on the Alfonsine Tables and, to a lesser extent, on the earlier Toledan Tables. ${ }^{16}$

In his tables Farissol Botarel used the Julian calendar, and signs of $30^{\circ}$ (a twelvefold division of a circle) rather than signs of $60^{\circ}$ (a sixfold division of a circle), as in the standard version of the Parisian Alfonsine Tables. The stated epoch of the tables is March 29, I48I, the date on which the first mean conjunction of the Sun and Moon took place in the year that began on March I, I48I. ${ }^{17}$ With some exceptions (noted below), the tables were not copied from previous authors. The mean motion tables take advantage of an 8-year cycle that had previously been used by Levi; the underlying parameters for these mean motions come from Levi's tables but the radices to which they apply are much later than those of Levi.

There follows a brief summary of the 15 tables. Table I is due to Farissol Botarel; Tables 2 to 4 are mostly related to Levi; Tables 5 and 6 derive from Bonfils's Six Wings; Table 7 is similar to a table in Bonfils's planetary tables for I340; Ta-

[^2]bles 8,9, and io are related to tables in Bonfils's Six Wings; Table II is related to a table compiled by Bonjorn; Table 12 displays mean motions for times between syzygies, and a set of corrections to the lunar anomaly based on a column for corrections in the Ptolemaic tradition; Table I3 displays lunar corrections that are based on the corresponding corrections in the Parisian Alfonsine Tables; Table I4 is based on the solar corrections in the Parisian Alfonsine Tables; and Table I5 is based on a relatively crude value for the solar velocity (the stated purpose of this table is to aid in the determination of the moment when the Sun enters a zodiacal sign). Here is a list of the 15 tables with the folio numbers on which they appear in Munich, MS Heb. 343 .
(1) Table of Julian years since the epoch, I48I (95a)
(2) Table of radices of the solar and lunar mean motions at syzygy for 8 years, I48I to I488, for Avignon (95b)
(3) Table of radices of the solar and lunar mean motions in cycles of 8 years, extending Table 2 (95b)
(4) Tables of syzygies within a year, to be used together with Tables 2 and 3 (96a)
(5) Table for the first correction to mean syzygy due to the solar anomaly (96b)
(6) Table for the second correction to mean syzygy due to the lunar anomaly (97a)
(7) Table for the solar correction (97b)
(8) Table for the length of half-daylight for Avignon (97b)
(9) Table for adjusted parallax for Avignon (98a)
(io) Table for solar eclipses (99a)
(ir) Table for lunar eclipses (99a)
(12) Table for mean motions of the Sun and Moon and the true lunar anomaly between syzygies, at intervals of I hour (99b-Iorb)
(I3) Table for the lunar corrections (IO2a-b)
(I4) Table for the true positions of the Sun (io3a)
(15) Table for the hourly motion of the Sun (IO3b)

The Tables

| No. <br> of the <br> cycles | The Radices and the Years |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | I | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| I | [I]48I | [ 1 ]482 | [I]483 | [I]484 | [I]485 | [I]486 | [1]487 | [I]488 |
| 2 | 489 | 490 | 49 I | 492 | 493 | 494 | 495 | 496 |
| 3 | 497 | 498 | 499 | 500 | 501 | 502 | 503 | 504 |
| 4 | 505 | 506 | 507 | 508 | 509 | 510 | 511 | 512 |
| 5 | 513 | 514 | 515 | 516 | 517 | 518 | 519 | 520 |
| ... | ... | ... | ... | ... | ... | ... | $\ldots$ | ... |
| 17 | 609 | 610 | 611 | 612 | 613 | 614 | 615 | 6 I 6 |
| 18 | 617 | 6 I 8 | 619 | 620 | 62 I | 622 | 623 | 624 |
| 19 | 625 | 626 | 627 | 628 | 629 | 630 | 63 I | 632 |
|  |  |  | bissextile |  |  |  | bissextile |  |

Table i. "This table indicates the way to enter the tables of nofet ṣufim [drippings of a honeycomb] in single years [shanim peshutot] and in cycles that have elapsed since the day established as its radix which is year I48I of the Incarnation".

## Comment:

This table for Julian years is arranged in cycles of 8 years, beginning with I48I. It is noted that the third and seventh years in this cycle are bissextile (i.e., a leap day added at the end of February). For Farissol Botarel the year begins on March I and ends on February 28 or 29. Hence February 29, I483 (epoch March I) is February 29, I484 (epoch January I). ${ }^{18}$
18. In the "standard" Julian calendar the year begins on January I; this convention was called stylus communis. But there were other "styles" in the Middle Ages for the beginning of the year. Of interest to us is that for Farissol Botarel and Levi ben Gerson the year begins on March I: to avoid confusion we add in parentheses "epoch March I" where appropriate, and sometimes for a date in the "standard" Julian calendar, we add in parentheses "epoch January I". This issue only affects dates in January and February. For various medieval conventions on calendrical "styles", see O. Pedersen 1983, p. 62. We are grateful to José Luis Mancha for bringing this reference to our attention.

| radices | day, month | [weekday, and time of] mean [syz.] | mean <br> position <br> of the <br> luminaries | solar anomaly | lunar anomaly | arg. of <br> the <br> [lunar] <br> node |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| year I, <br> conj. | 29 Mar. | 5(d)20;14h | os I6; $4 \mathrm{I}^{\circ}$ | 9s 16;57 ${ }^{\circ}$ | I IS $9 ; 23^{\circ}$ | 9S $29 ; 3 \mathrm{I}^{\text {o }}$ |
| year 2, opp. | 4 Mar. | 2(d) $10 ; 4 \mathrm{I}$ | IIS 21;34 | 8s 21;40 | 3s 6;25 | 3s 22;14 |
| $\begin{aligned} & \text { year } 3 \text { (b), } \\ & \text { opp. } \end{aligned}$ | 23 Mar. | I(d) 8; 13 | os 9;52 | 9S IO; 4 | 2SII;54 | 2S 0;57 |
| year 4, <br> conj. | 26 Mar. | 6(d) II; 24 | os I3;12 | 9S 13;54 | 7s 4;36 | I IS 24;20 |
| year 5, <br> opp. | I Mar. | 3(d) 1;50 | IIS 18;23 | 8s I8;37 | I IS I; 30 | 5s 17; 3 |
| year 6,opp. | 19 Mar. | I(d) 23;23 | os 7;20 | 9S 7;3I | IOS 7; 7 | 6s 25;43 |
| year 7 (b), conj. | 24 Mar. | 7(d) 2;32 | os 10;26 | 9S 10;50 | 2S 29;49 | IS 19; 3 |
| year 8 , opp. | 27 Mar. | 5(d) 5;44 | os 14;26 | 9S 14;4I | 7S 22;32 | 8s 12;32 |

Table 2. "The first table is the table of radices of the mean conjunctions and oppositions for the beginning of the solar year for [each of the] eight years after I480 of the Incarnation, here in the city of Avignon whose distance from the eastern extremity is $146 ; 30^{\circ}$ ".

## Comments:

(I) In Col. I. the code (b) indicates that the year is bissextile, that is, day 29 is added to February. The code (d) in Col. 3 refers to the weekday, where I(d) = Sunday, etc. The time of day is counted from noon.
(2) In the title we are told that the geographical longitude of Avignon is $146 ; 30^{\circ}$ from the eastern extremity. This is the same as the longitude of Tarascon according to Bonfils (canons to his planetary tables for I340: Munich, MS Heb. 386, I 5a). In fact, the distance from Tarascon to Avignon is about 24 km and they differ in geographical longitude by $0 ; 9^{\circ} .{ }^{19}$
19. Times Atlas of the World 1971, pp. 25, 239. In most lists of geographical coordinates compiled in the Islamic world longitude is measured from the western extremity: see Kennedy and
(3) The entries in Table 2, year I, cols. 2 and 3, give the date, the weekday, and the time for the first mean conjunction in that year: Mar. 29, I48I, day 5 (Thursday), at 20; 14 h. This precise time and date can be derived from Levi's tables. Levi's Table 36.I is for I32 I, and it displays a conjunction on Mar. 28 , day 7 (Sat.), at I9;I4h. ${ }^{20}$ From I32I to I48I there are I60 years or 20 cycles of 8 years. In Levi's Table 37 the entry for 20 cycles is $30 d$, day 6 (Fri.), at $13 ; 44 \mathrm{~h} .{ }^{21}$ We add 28 d I9;14h and 30 d I3;44h, and the sum is 59 d $8 ; 58 \mathrm{~h}$. This is a mean conjunction, but no longer in March. So we subtract the duration of I mean synodic month, 29d I2;44h and the result is 29 d 20; I4h, exactly as in Table 2, year I.

Analogously, the entry for time in Table 2, year 4, is Mar. 26, I484, day 6 (Friday) at II;24h. As in year I, this is a conjunction. In fact, 37 mean synodic months (or I093d) have elapsed since the conjunction listed for year I. To recompute this entry, multiply 37 times Levi's value for the length of the mean synodic month $(29 ; 31,50,7,54 d),{ }^{22}$ and add the result to the entry for year I. Hence

$$
37 \cdot 29 ; 3 \mathrm{I}, 50,7,54 \mathrm{~d}+20 ; 14 \mathrm{~h}=1092 \mathrm{~d}+15 ; 1 \mathrm{IOh}+20 ; 14 \mathrm{~h}=1093 \mathrm{~d}+11 ; 24 \mathrm{~h},
$$

in exact agreement with the text.
(4) In Table 2, year I, the entry for the mean position of the luminaries is os I6;4I ${ }^{\text {o }}$; in other words, the mean solar position for Mar. 29, I48I at 20; I4h is os $16 ; 4 I^{\circ}$. The entries in this column can also be derived from Levi's tables. In Levi's Table II the mean solar position on Feb. 28, I360 (epoch March I) is IIS I6; $15,2 I^{\circ}$ (with a variant of IIS $16 ; 15,23^{\circ}$ that does not affect the result here $),{ }^{23}$ and for I 20 years the entry is $0 ; 59, \mathrm{I} 6^{\circ}(\mathrm{I} 20=\mathrm{I} 480-\mathrm{I} 360)$. In Table I2 the entry for 29 days is $28 ; 35,2^{\circ}$ and for $20 ; 14$ h it is $0 ; 49,54^{\circ}$ (with interpolation). ${ }^{24}$ The sum of these four quantities is $16 ; 39,33^{\circ}$ (after subtract-

[^3]ing a complete rotation). The agreement is very close, but not exact: the difference between text and computation is $0 ; \mathrm{I}^{\circ}\left(=\mathrm{I} 6 ; 4 \mathrm{I}^{\circ}-\mathrm{I} 6 ; 40^{\circ}\right)$.
(5) The solar anomaly in Table 2 , year I , is $9 \mathrm{~s} 16 ; 57^{\circ}$; hence the longitude of the solar apogee is obtained by subtracting the anomaly of the Sun from its longitude: $\mathrm{I} 2 \mathrm{~S}+$ os $\mathrm{I} 6 ; 4 \mathrm{I}^{\circ}-9 \mathrm{~S} \mathrm{I} 6 ; 57^{\circ}=89 ; 44^{\circ}$. However, one expects the solar apogee at this date to be somewhat greater than $90^{\circ}$. This means that the entry for the solar anomaly in the row for year I was not derived from Levi's tables because for Levi the solar apogee was $93^{\circ}$ in I334. ${ }^{25}$ If one computes the differences between the corresponding mean solar position and solar anomaly in all 8 cases, the results are not constant (as they should be), for they range from $89 ; 18^{\circ}$ to $89 ; 54^{\circ}$, with an "average" of $89 ; 42,30^{\circ}$. This "average" is consistent with the underlying value for the solar apogee in Table I4 (see below). In Bonfils's planetary tables for I340 there is a table for precession (Munich, MS Heb. 386; see also Goldstein and Chabás 2017), with the title "Table for finding the motion of the fixed stars and the apogees and nodes of the planets according to al-Battānī by Immanuel ben Jacob". The entry for the solar apogee for 1340 is 2 s $29 ; 15,6^{\circ}$ and the entry for 1348 is $2 \mathrm{~s} 29 ; 22,25^{\circ}$. The difference in 8 years is $0 ; 7,19^{\circ}$, which agrees with Farissol Botarel's motion of the solar apogee in Table 3. In Bonfils's table a solar apogee of $89 ; 44^{\circ}$ corresponds to I372, well before the time of Farissol Botarel.
(6) In Table 2, year I, the entry for lunar anomaly is I IS 9;23 ${ }^{\circ}$. The entries in this column also seem to have been derived from Levi's tables. In Levi's Table 36.I the entry for the mean lunar anomaly at the mean conjunction on Mar. 28, I32I is os $8 ; 16^{\circ}{ }^{\circ}{ }^{26}$ and in Table 37 the entry for the mean lunar anomaly for 20 cycles is IIs $26 ; 56^{\circ} .{ }^{27}$ The sum of $8 ; 16^{\circ}$ and $356 ; 56^{\circ}$ is $365 ; 12^{\circ}$. As noted previously, we have to go back I mean synodic month. According to Levi, the mean motion in lunar anomaly in I synodic month is $25 ; 49^{\mathrm{o}} .^{28}$ Hence subtracting $25 ; 49^{\circ}$ from $365 ; 12^{\circ}$ yields $339 ; 23^{\circ}=$ I Is $9 ; 23^{\circ}$, in exact agreement with the entry in Table 2.
25. Goldstein 1974, p. 94.
26. Goldstein I974, p. 2 I8.
27. Goldstein I974, p. 226.
28. Goldstein I974, p. I34.
(7) The entry in the final column of Table 2, "argument of the [lunar] node" is what Ptolemy called the argument in lunar latitude, defined as the difference between the mean lunar motion in longitude and motion of the lunar node, or about $\mathrm{I} 3 ; 10,35^{\circ} / \mathrm{d}+0 ; 3, \mathrm{II} / \mathrm{d} \approx \mathrm{I} 3 ; \mathrm{I} 3,46^{\circ} / \mathrm{d}$. Levi does not have a table for the argument in lunar latitude, but his values for the lunar motion in longitude ( $\left.\mathrm{I} 3 ; 10,35, \mathrm{I}, 40^{\circ} / \mathrm{d}\right)^{29}$ and the motion of the lunar node $\left(-0 ; 3,10,37,39^{\circ} / \mathrm{d}\right)^{30}$ allow us to compute the basic parameter underlying his tables: I3; $33,45,39,19^{\circ} /$ d. Note that, to avoid subtractions, Levi tabulated $360-\mathrm{N}$, rather than N , where N is the longitude of the ascending node; hence, the argument of latitude at any given time is $L-N=L+(360-N)$, where $L$ is the mean longitude of the Moon.

With Levi's parameters, in 99 mean synodic months of 29;31,50,7,54d the argument of latitude would increase by $99 \cdot 29 ; 31,50,7,54 \mathrm{~d}$. $\mathrm{I} 3 ; \mathrm{I} 3,45,39, \mathrm{I} 9^{\circ} / \mathrm{d}=\mathrm{I} 56 ; 22,46^{\circ} \approx \mathrm{I} 56 ; 23^{\circ}$. This agrees exactly with the entry in Table 3, cycle I (see below). Clearly, Farissol Botarel used Levi's tables to compute this entry in Table 3. However, although the mean motion for this column of Table 2 agrees with Levi's parameters, Farissol Botarel did not use any of Levi's values as the radix for these entries. To demonstrate this we consider the entry for the argument of the lunar node, $9 \mathrm{~s} 29 ; 3 \mathrm{I}^{\circ}$ $\left(=299 ; 3 \mathrm{I}^{\circ}\right)$, for Mar. 29, I48I at $20 ; \mathrm{I} 4 \mathrm{~h}$, and compare it to the value derived from Levi's values for Feb. 28, I302 (epoch January I) noon:
mean lunar longitude: I IS 20;36,58 ${ }^{\circ}$
mean longitude of the lunar node: is $28 ; 38,6^{\circ}$
sum (mean argument of lunar latitude $)=$ Is I9; $15,4^{\circ}\left(=49 ; 15,4^{\circ}\right)$
The difference in days is 65409 to which must be added 20 ; 14 h . Thus the number of days is $65409 ; 50,35 \mathrm{~d}$, that is, the number of days from Feb. 28, I302, noon, to Mar. 29, I48I at 20;14h. The increment in the mean argument of lunar latitude is found by multiplying the appropriate quantities:

$$
65409 ; 50,35 \mathrm{~d} \cdot 13 ; 13,45,39,19^{\circ} / \mathrm{d}=249 ; 37^{\circ} .
$$

29. Goldstein 1974, p. 107.
30. Levi's Astronomy, ch. 70; see Goldstein I974, p. 107.

Adding this result to Levi's value (taken as the initial point), $49 ; 15^{\circ}$, yields $298 ; 52^{\circ}$, whereas the text has $299 ; 31^{\circ}$. This suggests that values in Levi's tables did not serve as the radix for the computation that underlies this entry, even though the mean motion of the argument of lunar latitude depends on Levi's parameters.

The entry in Table 2, year 4, agrees exactly with recomputation based on the entry for year $I$. As indicated above concerning the recomputation of the time of syzygy in year 4 , there are 37 mean synodic months between the conjunction listed for year I and the conjunction listed for year 4 . Thus

$$
37 \cdot 29 ; 31,50,7,54 \mathrm{~d} \cdot 13 ; 13,45,39,19^{\circ} / \mathrm{d}=54 ; 49^{\circ} .
$$

We then add to this result the entry for year $\mathrm{I}, 299 ; 3 \mathrm{I}^{\circ}$, and the sum is $354 ; 20^{\circ}=$ I IS $24 ; 20^{\circ}$, in exact agreement with the entry in Table 2 year 4, I IS $24 ; 20^{\circ}$. This computation shows that the readings for the entries in Table 2 for years I and 4 are secure.

| mean conj. and opp. |  |  | mean <br> [position] <br> of the <br> luminaries | solar anomaly | lunar anomaly | arg. of the [lunar] node |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of cycles | days of <br> the <br> month | weekday, hour, min. |  |  |  |  |
| I | I | 4(d) I2;4Ih | os I; $34^{\circ}$ | Os 1;27 ${ }^{\circ}$ | Is $5 ; 5 \mathrm{I}^{\circ}$ | 5s 6;23 ${ }^{\circ}$ |
| 2 | 3 | 2(d) 1;22 | 3; 8 | 2;54 | 2S 11; 42 | IOS 12;46 |
| $\ldots$ | ... | $\ldots$ | $\ldots$ | ... | ... |  |
| 10 | 15 | 3(d) 6;5I | os 15;40 | 14;30 | I I S 28;30 | 4s 3;50 |
| 20 | 30 | 6(d) 13;42 | Is $\mathrm{I} ; 20$ | os 29; о | I Is 27; 0 | 8s 7;40 |
| 30 | 45 | 2(d) 20;33 | Is 17; O | Is 13;30 | I I IS 25;30 | os 11;30 |
| 40 | 61 | 6(d) 3;24 | 2S 2;40 | Is 28; 0 | I Is 24; 0 | 4S 15;20 |
| For the motion [?] in a quarter [of a synodic month] of the Moon; add this to the mean conj. or opp. |  |  |  |  |  |  |
| $\cdots$ | 7 | o(d) 9; I I | os 7;16,30 | os 7;16,30 | 3s 6;27,30 | 3s 7;40 |

Table 3. "The second table for cycles of 8 years of mean conjunctions and oppositions, and the [various] kinds of mean [motions]".

## Comments:

(I) This table is also arranged for cycles of 8 years or, more precisely, for 99 mean synodic months. All but one of the parameters underlying the entries in this table were taken from Levi (for the exception, see § 3, below). For example, the time of mean conjunction or opposition can be recomputed with Levi's parameter for the mean synodic month. A cycle of 99 mean synodic months is equal to

$$
99 \cdot 29 ; 3 \mathrm{I}, 50,7,54 \mathrm{~d}=8 \mathrm{y}+\mathrm{I} ; 3 \mathrm{I}, 43 \mathrm{~d}=8 \mathrm{y}+\mathrm{Id} \mathrm{I} 2 ; 4 \mathrm{Ih},
$$

in exact agreement with the entry in Table 3, cycle 1 . To confirm the weekday, we take the number of days in 8 years $(2922 d) \bmod 7$, and the result is 3 to which must be added I complete day. Then $3+\mathrm{I}=4$, in agreement with the entry in Table 3 cycle I. From the entry for cycle io (I5d 6;5Ih) we confirm the entry for cycle I:

$$
(\mathrm{I} 5 \mathrm{~d} 6 ; 5 \mathrm{Ih}) / \mathrm{IO}=\mathrm{I} 5 ; \mathrm{I} 7,8 \mathrm{~d} / \mathrm{IO}=\mathrm{I} ; 3 \mathrm{I}, 42,48 \mathrm{~d} \approx \mathrm{I} ; 3 \mathrm{I}, 43 \mathrm{~d}=\mathrm{Id} \mathrm{I} 2 ; 4 \mathrm{Ih} .
$$

The entry for cycle 40 is 6Id $3 ; 24$ h: this derives from the entry for cycle 10 as follows:

$$
4 \cdot(\mathrm{I} 5 \mathrm{~d} 6 ; 5 \mathrm{Ih})=4 \cdot \mathrm{I} 5 ; 17,8 \mathrm{~d}=6 \mathrm{I} ; 8,3 \mathrm{Od}=6 \mathrm{Id} 3 ; 24 \mathrm{~h} .
$$

(2) Applying the same procedure for the mean positions of the luminaries, i.e., the mean position of the Sun, we again get exact agreement with Levi's parameters. With Levi's value for the solar daily mean motion, $0 ; 59,8,20^{\circ} / \mathrm{d}$, in 99 mean synodic months the mean Sun advances $99 \cdot 29 ; 31,50,7,54 \mathrm{~d} \cdot$ $0 ; 59,8,20^{\circ} / \mathrm{d}=288 \mathrm{I} ; 34^{\circ}=8 \cdot 360+\mathrm{I} ; 34^{\circ}=\mathrm{I} ; 34^{\circ}$, as in Table 3, column 3 labeled "mean position of the luminaries". All subsequent entries in this column are simple multiples of the entry for cycle $I$.
(3) The entry for cycle I for the solar anomaly os $I ; 27^{\circ}$, that is, in 99 mean synodic months the solar anomaly has advanced $8 \cdot 360^{\circ}+\mathrm{I} ; 27^{\circ}=288 \mathrm{I} ; 27^{\circ}$. Using Levi's parameter for the mean synodic month, we obtain the daily mean motion in solar anomaly:

$$
288 \mathrm{I} ; 27^{\circ} / 99 \cdot 29 ; 3 \mathrm{I}, 50,7,54 \mathrm{~d}=0 ; 59,8, \mathrm{II}, 6^{\circ} / \mathrm{d} .
$$

Now, the entry for cycle I differs from the entry for the mean position of the Sun by $0 ; 7^{\circ}\left(=1 ; 34^{\circ}-1 ; 27^{\circ}\right)$. This difference is due to a motion of precession for the solar apogee in 8 years, and corresponds to $I^{\circ}$ in about 68.5 years $\left[=\left(\mathrm{I}^{\circ} / 0 ; 7^{\circ}\right) \cdot 8 \mathrm{y}\right]$. This value for precession has not been previously
attested, but it is found in Bonfils's planetary tables for I340 (see Comments to Table 2 § 5, above). Among the medieval values for precession one finds $I^{0}$ in 66 years and $I^{0}$ in $72^{1 / 2}$ years, ${ }^{31}$ whereas Levi's value was $I^{0}$ in about 67 years. ${ }^{32}$ This is the only case in Table 3 where a parameter was not taken from Levi's tables. All subsequent entries in this column for the solar anomaly are simple multiples of the entry for cycle I .
(4) With Levi's parameters for the mean synodic month and the mean motion in lunar anomaly, ${ }^{33}$ we find that

$$
99 \cdot 29 ; 3 \mathrm{I}, 50,7,54 \mathrm{~d} \cdot \mathrm{I} 3 ; 3,53,55,56^{\circ} / \mathrm{d}=35 ; 50,47^{\circ} \approx \text { Is } 5 ; 5 \mathrm{I}^{\circ},
$$

in exact agreement with the entry in Table 3, cycle I, lunar anomaly. All subsequent entries in this column are simple multiples of the entry for cycle I.
(5) In the comments on Table 2 it was noted that the heading for this column, argument of the [lunar] node, refers to the argument of lunar latitude. The derivation from Levi's parameters has already been given there, and the agreement is exact. All subsequent entries in this column are simple multiples of the entry for cycle I.
(6) The last row of numbers is not completely aligned with those in the columns above it, and the first two words on the line above it are unclear. The entries in this row are exactly half of those in Table 4 on f. 96a (row labeled "I4 Mar."), e.g., o(d) 9 ; 1 Ih is half of $o(d) 18 ; 22$ h. This suggests that the entries in the last row are for a quarter of a mean synodic month, and that the " 7 " means about 7 days.

3I. Chabás and Goldstein 2003, pp. 256-257.
32. Goldstein I975, p. 40.
33. Goldstein 1974, p. IO7.

| mean conj. or opp. | mean <br> position <br> of the <br> luminaries | solar <br> anomaly <br> hours, min. | lunar <br> anomaly | argument <br> of the lunar <br> node |
| :--- | :--- | :--- | :--- | :--- | :--- |
| date |  |  |  |  |
| I4 Mar. | O(d) I8;22h | os I4;33 |  |  |

* Note that in this transcription 9 rows of this table in the manuscript have been omitted.

Table 4. "The third table for the months of the solar year in which mean conjunction or opposition falls".

## Comments:

(i) This table is to be used together with Tables 2 and 3 to determine the relevant data for a syzygy subsequent to the first sygyzy in a year in any cycle. The entries in Tables 2, 3, and 4 are to be added, and the results will be for a date that generally will differ from the dates listed in this table. This procedure is seemingly simple but it would have been more userfriendly to indicate the number of days since the beginning of the year rather than dates. To find data for times between syzygies Table i2 needs to be used.
(2) The entries in this table are based on the same parameters as used in Tables 2 and 3, and provide no additional information. Note that the entries for mean position of the luminaries and for solar anomaly are identical.

Signs of the solar anomaly

| degrees | add <br> os <br> (h) | subtract <br> IS <br> (h) | ... | subtract <br> 5 s <br> (h) | add <br> 6 s <br> (h) | ... | add 9 s <br> (h) | ... | add I IS <br> (h) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0;16 | I;53 | $\ldots$ | I;44 | 0;16 | $\cdots$ | 4;26 | $\cdots$ | 2;25 |
| ... | subtract | ... | $\cdots$ | $\ldots$ | ... | $\cdots$ | ... | $\cdots$ | $\ldots$ |
| 6 | 0; I I | 2;16 | ... | I; 2 I | 0;4I | $\ldots$ | 4;25 | ... | 2; 2 |
| ... | ... | ... | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\cdots$ | $\ldots$ |
| I2 | 0;38 | 2;36 | $\ldots$ | 0;57 | I; 5 | $\cdots$ | 4;22 | $\ldots$ | 1;37 |
| $\ldots$ | ... | ... | $\cdots$ | ... | ... | $\cdots$ | ... | $\ldots$ | ... |
| 18 | I; 5 | 2;54 | ... | 0;33 | I;29 | ... | 4;16 | $\ldots$ | I; IO |
| ... | $\cdots$ | $\cdots$ | $\ldots$ | $\cdots$ | ... | $\cdots$ | $\ldots$ | $\cdots$ | ... |
| 24 | I;29 | 3;10 | $\cdots$ | 0; 9 | I;53 | $\ldots$ | 4;8 | $\cdots$ | 0;44 |
| $\cdots$ | $\ldots$ | $\cdots$ | $\cdots$ | add | $\ldots$ | $\cdots$ | $\cdots$ | $\cdots$ | ... |
| 30 | 1;53 | 3;24 | $\ldots$ | 0; 16 | 2;16 | $\ldots$ | 3;56 | $\ldots$ | 0; 16 |

Table 5. "First correction of the mean conjunctions and oppositions with respect to the solar anomaly".

Comments:
(I) The maximum is $4 ; 26 \mathrm{~h}$ at $8 \mathrm{~s} 30^{\circ}$ to $9 \mathrm{~s} 3^{\circ}$ and the minimum is $-3 ; 54 \mathrm{~h}$ at 2 s $25^{\circ}$ to $3 \mathrm{~s} \mathrm{I}^{\mathrm{o}}$.
(2) Each entry in Table 5 is equal to the corresponding entry in Bonfils's Six Wings, Wing 2, col. I (labeled lunar anomaly $0^{\circ}$ ), less 24 h. Let B be Bonfils's entry, and F be Farissol Botarel's entry. Then $B-F=24 h$. For example, for argument $0^{\circ}, F=0 ; 16 h$ and $B=24 ; I 6 h ;$ hence $B-F=24 h$. For argument $36^{\circ}\left(=\right.$ Is $\left.6^{\circ}\right), \mathrm{F}=-2$; I 6 h and $\mathrm{B}=2 \mathrm{I} ; 44 \mathrm{~h}$; hence $\mathrm{B}-\mathrm{F}=24 \mathrm{~h}$, as before. This means that Farissol Botarel used Bonfils's entries and shifted them vertically.
(3) Table 5 has entries at $\mathrm{I}^{\mathrm{O}}$-intervals of solar anomaly whereas in the corresponding table Bonfils has entries at $6^{\circ}-$ intervals.
(4) Bonfils's Wing 2 is a double argument table, where the variables are the mean solar anomaly and the mean lunar anomaly. ${ }^{34}$ In Table 5 Farissol Botarel has only reproduced one column of Bonfils's table, which indicates that he did not fully understand what Bonfils had done. Let $\bar{x}$ be the mean solar anomaly Table 5 displays the difference in time between mean and true syzygy due to the mean solar anomaly, $\mathrm{c}(\bar{x})$, corresponding to Bonfils's $\mathrm{c}(\bar{x}$, o): see comments to Table 6, § 4.

| degrees | add os <br> (h) | add Is <br> (h) | add <br> 2 S <br> (h) | add <br> 3 s <br> (h) | subtract 8s <br> (h) | subtract 9s <br> (h) | subtract IIS <br> (h) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0;16 | 5;13 | 8;49 | IO; 7 | 8;14 | 9;35 | 4;4I |
| ... |  |  |  |  | ... |  | ... |
| 6 | I;18 | 6; 5 | 9; 8 | 10; 3 | 8;42 | 9;32 | 3;40 |
| ... |  |  |  |  | ... |  | ... |
| 12 | 2;20 | 6;53 | 9;39 | 9;53 | 9; 5 | 9;23 | 2;48 |
| ... |  |  |  |  | ... |  | ... |
| 18 | 3;20 | 7;37 | 9;54 | 9;37 | 9;2 I | 9; 7 | 1;48 |
| ... |  |  |  |  | ... |  | ... |
| 24 | 4;18 | 8;15 | IO; 4 | 9;14 | 9;31 | 8;45 | 0;47 |
|  |  |  |  |  |  |  | add |
| 30 | 5;13 | 8;49 | 10; 7 | 8;46 | 9;35 | 8;17 | 0;16 |

Table 6. "Second correction of the mean conjunctions and oppositions with respect to the lunar anomaly".

## Comments:

(I) The maximum is $10 ; 7 \mathrm{~h}$ at $3 \mathrm{~S}^{\circ}{ }^{\circ}$ and $3 \mathrm{~s} \mathrm{I}^{\circ}$, and the minimum is $-9 ; 35 \mathrm{~h}$ at 9 s $0^{\circ}$ and $9 \mathrm{~S} \mathrm{I}^{\mathrm{o}}$.
(2) Each entry in Table 6 is equal to the corresponding entry in Bonfils's Six Wings, Wing 2, row I (labeled solar anomaly $0^{\circ}$ ), less 24 h. Let B be Bonfils's entry, and F be Farissol Botarel's entry. Then B $-F=24 \mathrm{~h}$. For example, for argument $6^{\circ}\left(=0\right.$ os $\left.6^{\circ}\right) \mathrm{F}=\mathrm{I} ; 18 \mathrm{~h}$ and $\mathrm{B}=25 ; 18 \mathrm{~h}$; hence $\mathrm{B}-\mathrm{F}=24 \mathrm{~h}$. For argument $276^{\circ}\left(=9 s 6^{\circ}\right) F=-9 ; 32 h$ and $B=14 ; 28 h$; hence $B-F=24 h$,

[^4]as before. As was the case for Table 5, this means that Farissol Botarel used Bonfils's entries and shifted them vertically.
(3) Table 6 has entries at $\mathrm{I}^{\mathrm{o}}$-intervals of lunar anomaly whereas in the corresponding table Bonfils has entries at $6^{\circ}$ intervals.
(4) Let $\bar{\alpha}$ be the mean lunar anomaly. Table 6 displays the difference between mean and true syzygy due to the mean lunar anomaly, $\mathrm{c}(\bar{\alpha})$, corresponding to Bonfils's $c(o, \bar{\alpha})$. Whereas in Table 5 Farissol Botarel only displayed one column from Bonfils's Wing 2, in Table 6 he only displayed one row from Bonfils's Wing 2. Farissol Botarel believed that $\mathrm{c}(-\bar{\chi}, \bar{\alpha})=\mathrm{c}(\bar{x}, 0)+\mathrm{c}(0, \bar{\alpha})$, which is not correct. ${ }^{35}$

Degrees of solar anomaly

| subtract |  |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{o}[\mathrm{s}]$ | $\mathrm{I}[\mathrm{s}]$ | $2[\mathrm{~s}]$ |
| add |  |  |  |
|  | $6[\mathrm{~s}]$ | $7[\mathrm{~s}]$ | $8[\mathrm{~s}]$ |
| degrees |  |  |  |
| 0 | $0 ; 0^{\circ}$ | $\mathrm{I} ; \mathrm{o}^{\circ}$ | $\mathrm{I} ; 43$ |
| I | $0 ; 2$ | $\mathrm{I} ; 2$ | $\mathrm{I} ; 45$ |
| 2 | $0 ; 4$ | $\mathrm{I} ; 3$ | $\mathrm{I} ; 46$ |
| 3 | $0 ; 6$ | $\mathrm{I} ; 5$ | $\mathrm{I} ; 47$ |
| 4 | $0 ; 8$ | $\mathrm{I} ; 7$ | $\mathrm{I} ; 48$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| IO | $0 ; 20$ | $\mathrm{I} ; 18$ | $\mathrm{I} ; 52$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| 20 | $0 ; 4 \mathrm{I}$ | $\mathrm{I} ; 4 \mathrm{I}$ | $\mathrm{I} ; 57$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| 29 | $0 ; 58$ | $\mathrm{I} ; 43$ | $\mathrm{I} ; 59$ |

Table 7. "Table for the corrections of the luminaries and the argument of lunar latitude [tannin; lit. node]".

[^5]
## Comments:

(I) This is a table for the solar equation: below the table are the other 6 signs that also serve as headings for the columns together with a second column for the degrees associated with these signs. The reading of the title is secure but tannin, which usually means (lunar) node, seems to mean argument of lunar latitude in this text (see Comment I to Table io, below). It is likely that Farissol Botarel intended to include a table here for the lunar latitude as a function of the argument of lunar latitude, but it is missing in this copy.
(2) The maximum of $\mathrm{I} ; 59^{\circ}$ for the solar equation is common to many zijes, and this parameter is already in al-Battān $\overline{1}$ 's table for the solar equation. ${ }^{36}$ The table for the solar equation in Bonfils's planetary tables for I340 with a maximum of $\mathrm{I} ; 59, \mathrm{IO}^{\circ}$ is preserved in Munich, MS Heb. 386, 22b-23a, where there is also a column for the solar velocity. The entries for the solar equation in both al-Battānī and Bonfils are given to seconds and, when rounded, differ from those in Table 7.

Signs of the true position of the Sun

| degrees | (h) | $\begin{gathered} \text { I } \\ \text { (h) } \end{gathered}$ | $2$ <br> (h) | $\begin{gathered} 6 \\ (\mathrm{~h}) \end{gathered}$ | $\begin{gathered} 7 \\ \text { (h) } \end{gathered}$ | $\begin{gathered} 8 \\ \text { (h) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| o | 6; о | 6;45 | 7;24 | 6; о | 5;15 | 4;36 |
| I | 6; 2 | 6;47 | 7;24 | 5;58 | 5;13 | 4;36 |
| 2 | 6; 4 | 6;47 | 7;25 | 5;57 | 5;12 | 4;35 |
| 3 | 6; 6 | 6;50 | 7;26 | 5;55 | 5;10 | 4;34 |
| 4 | 6; 8 | 6;5I | 7;27 | 5;54 | 5; 9 | 4;33 |
| 5 | 6;10 | 6;52 | 7;28 | 5;52 | 5; 8 | 4;32 |
| ... | ... | ... | ... | $\ldots$ | ... | ... |
| ıо | 6;15 | 7; 0 | 7;32 | 5;45 | 5; 0 | 4;28 |
| ... | ... | ... | $\ldots$ | ... | $\ldots$ | $\ldots$ |
| 20 | 6;31 | 7;12 | 7;38 | 5;29 | 4;48 | 4;22 |
| ... | ... | ... | ... | ... | ... | ... |
| 29 | 6;44 | 7;22 | 7;40 | 5;16 | 4;38 | 4;20 |

Table 8. "Table for finding the hours of half-daylight for this our horizon [i.e., geographical latitude]".
36. Nallino I899-I907, 2:78-83. In al-Battān̄̄'s table, the maximum is $\mathrm{I} ; 59,10^{\circ}$.

Comments:
(I) Below the table are the other signs (in this order): 5, 4, 3, II, IO, 9; there is also a second column for degrees descending from $30^{\circ}$ to $I^{\circ}$ associated with the signs below the table.
(2) Longest half-daylight of 7;40h is usually for geographical latitude $44^{\circ}$ (e.g., Avignon). See the corresponding table in Bonfils, Wing 3 (for lat. $44^{\circ}$ ), where many entries differ by a small amount, although the minimum and maximum are the same $(4 ; 20 \mathrm{~h}$ and $7 ; 40 \mathrm{~h})$. The maximum should occur at $90^{\circ}\left(=\operatorname{Cnc} 0^{\circ}\right)$ whereas here it occurs at $89^{\circ}\left(2 \mathrm{~S} 29^{\circ}\right)$ and $9 I^{\circ}\left(3 \mathrm{~S} \mathrm{I}^{\circ}\right)$. Similarly, the minimum should occur at $270^{\circ}\left(=\mathrm{Cap} 0^{\circ}\right)$ whereas here it occurs at $269^{\circ}\left(8 \mathrm{~s} 29^{\circ}\right)$ and $27 \mathrm{I}^{\circ}\left(9 \mathrm{~S} \mathrm{I}^{\circ}\right)$ : there are no entries for $90^{\circ}$ and $270^{\circ}$.

Subtable 9.I. Cancer

| Cancer 3[s] |  |  |  |
| :---: | :---: | :---: | :---: |
| - | longitude | latitude |  |
| hours of the day | $(h)$ | $s[u b t r a c t]$ | $a[d d]$ |
| $7 ; 40$ | $\mathrm{I} ; 10$ | $8 ; 23^{\circ}$ | $7 ; 7^{\circ}$ |
| 7 | I 5 | $7 ; 53$ | $6 ; 33$ |
| 6 | I 8 | $7 ; 15$ | $5 ; 53$ |
| 5 | I 5 | $6 ; 25$ | $5 ; 5$ |
| 4 | $\mathrm{I} ; 8$ | $5 ; 43$ | $4 ; 3 \mathrm{I}$ |
| 3 | $0 ; 54$ | $4 ; 58$ | $4 ; 0$ |
| 2 | 39 | $4 ; 15$ | $3 ; 33$ |
| I | $0 ; 19$ | $3 ; 22^{\mathrm{a}}$ | $3 ; 32$ |
| noon | mid-heaven | $3 ; 39$ | $3 ; 39$ |
| I | $0 ; 19$ | $3 ; 32$ | $3 ; 52$ |
| 2 | 39 | $3 ; 33$ | $4 ; 15$ |
| 3 | $0 ; 54$ | $4 ; 0$ | $4 ; 58$ |
| 4 | $\mathrm{I} ; 8$ | $4 ; 3 \mathrm{I}$ | $5 ; 43$ |
| 5 | I 5 | $5 ; 5$ | $6 ; 25$ |
| 6 | I 8 | $5 ; 53$ | $7 ; 15$ |

(Continued)

| 7 | I5 | $6 ; 33$ | $7 ; 53$ |
| :---: | :---: | :---: | :---: |
| $7 ; 40$ | I;10 | $7 ; 7$ | $8 ; 23$ |
| - | - | $s[$ ubtract $]$ | $a[d d]$ |
| Cancer $3[s]$ |  |  |  |

a. Read: 3;52. Cf. Bonfils's tables for parallax in his Six Wings: Munich, MS Heb. 386, 58b.

Table 9. "Table for parallax in longitude and latitude when the Moon is at the beginning of each sign of the zodiac for [geographical] latitude $44^{\circ "}$.

## Comments:

(i) Table 9 is based on the difference between lunar parallax and solar parallax; this kind of parallax is called "adjusted parallax", and it is only appropriate to be used in computing the conditions of solar eclipses. ${ }^{37}$ There are four subtables for parallax on 98a, and another three on 98b, each headed by a zodiacal sign. On f. 98a, subtable 9.I has the heading Cancer. There follow subtables for Leo, Virgo, Libra, which are also to be used for Gemini, Taurus, Aries, respectively. On folio 98b are subtables for Scorpio, Sagitarius, and Capricorn, of which the first two are also to be used for Pisces and Aquarius, respectively. See Table 9.2 (Virgo and Taurus) for an example. The hours listed in the first column are to be understood as before or after noon.
(2) Column I agrees with Bonfils, Wing 5; longest half-daylight of 7;40h is appropriate for Avignon (latitude $44^{\circ}$ ): cf. Table 8 , above.
(3) The entries in col. 2 are symmetric about mid-heaven and are very close to those in Bonjorn's parallax table, col. 2, for month 4, where the longest halfdaylight is $7 ; 35 \mathrm{~h} .{ }^{38}$ Bonjorn computed his tables for geographical latitude $42 ; 30^{\circ}$ (for Perpignan) rather than for $44^{\circ}$ (Avignon).
(4) Columns 3 and 4 agree with Bonfils, Wing 5, columns 9 and io labeled: "two columns for the node of the Moon" (meaning, "the two columns for the true argument of lunar latitude"), with subheadings "subtract" and "add": cf. Munich, MS Heb. 386, 58b. The entries in column 4 repeat those in column 3 in reverse order.
37. Neugebauer 1975, pp. 990-994.
38. Chabás I992, p. 244. For discussion of Bonjorn's tables for adjusted parallax and a recomputation of them, see Chabás I99I, pp. 299-303.

Subtable 9.2. Virgo and Taurus

| Virgo 5[s] |  |  |  |
| :---: | :---: | :---: | :---: |
| hours of the day | longitude | latitude |  |
|  | (h) | s[ubtract] | a[dd] |
| 6;45h | I;37 | 4; $5^{\circ}$ | 2;33 ${ }^{\circ}$ |
| 6 | 37 | 42 | 56 |
| 5 | 33 | 27 | $57^{\text {a }}$ |
| 4 | 26 | 23 | 2;55 |
| 3 | I;14 | 20 | 3; 2 |
| 2 | 0;55 | 24 | 3;24 |
| I | 39 | 4;52 | 4;10 |
| noon | 20 | 5;17 | 4;55 |
| I | o; 3 | 5;56 | 5;54 |
| 1;6 | mid-heaven | 5;59 | 5;59 |
| 2 | 0;16 | 6;26 | 6;42 |
| 3 | 29 | 7;19 | 7;49 |
| 4 | 43 | 8; I | 8;4I |
| 5 | 46 | 23 | 9; I I |
| 6 | 47 | 8;46 | 9;30 |
| 6;45 | 0;44 | 9; о | 9;48 |
| - | - | a[dd] | s[ubtract] |
| hours of the day | longitude |  |  |
| Taurus I[s] |  |  |  |

a. Bonfils, Six Wings (Munich, MS Heb. 386, 59b), reads: 5 I.

Comments:
(I) Reading down, the entries are for Virgo; reading up the entries are for Taurus.
(2) Column I agrees with Bonfils, Six Wings: Munich MS Heb. 386, 59b.
(3) The entries in col. 2 are very close to those in Bonjorn's parallax table, col. 2, for month 6 , where the length of half-daylight is $6 ; 44 \mathrm{~h} .{ }^{39}$
39. Chabás 1992 , p. 245.
(4) Columns 3 and 4 agree with Bonfils's Six Wings, but for one entry. These entries also agree with those for Taurus in the Six Wings (with some variants): Munich, MS Heb. 386, 57b.

| argument of lunar latitude for [an eclipse of] the Sun |  | digits of the eclipsed body | half- <br> duration <br> of the <br> eclipse (') |
| :---: | :---: | :---: | :---: |
| O [S] | $5[\mathrm{~S}]$ |  |  |
| 6 | I I |  |  |
| $7 ; 0^{\circ}$ | 23; $0^{\circ}$ | 0 | 0 |
| 6;30 | 23;30 | 0 | 0 |
| 6; 0 | 24; 0 | 0 | 0 |
| 5;30 | 24;30 | I | 23 |
| 5; 0 | 25; 0 | 2 | 3 I |
| 4;30 | 25;30 | 3 | 37 |
| 4; 0 | 26; 0 | 4 | 43 |
| ... | $\ldots$ | ... | ... |
| I; O | 29; 0 | IO | 56 |
| 0;30 | 29;30 | I I | 57 |
| 0; 0 | 0; 0 | I 2 | 58 |

Table io. "Table for solar eclipses".
Comments:
(I) The heading for the first two columns is tannin shemesh (lit. the node of the Sun). In this text the word for "node" means the argument of lunar latitude and, in this instance, it refers to the conditions for a solar eclipse. The expression Hoq ha-tannin (lit. argument of the node) occurs in Bonfils's Six Wings, Wing I, meaning the argument of lunar latitude, and tannin le-Hamma (lit. the node for the Sun) occurs in Wing 6, with the meaning of argument of lunar latitude for the conditions of a solar eclipse: Munich, MS Heb. 343, 25b-26a.
(2) Table Io is similar to Bonfils, Six Wings, Wing 6, where the two columns of the argument are the same as here. However, Bonfils has a set of columns that depend on the corrected lunar anomaly, whereas Table io does not display columns for different values of lunar anomaly. The entries in columns 3 and 4 in Table io do not correspond exactly to the entries in any of the columns in Bonfils's Wing 6.
(3) Col. 3 displays the digits of a solar eclipse for the argument of lunar latitude in cols. I and 2. These digits of eclipse are measured on the solar diameter, set equal to I2 digits; hence a total solar eclipse has magnitude 12 digits.

| argument of lunar latitude for [an eclipse of] the Moon |  | digits of the eclipsed body | half-duration of the eclipse <br> (h) | half-duration of totality <br> (') |
| :---: | :---: | :---: | :---: | :---: |
| O [S] | 5 [S] |  |  |  |
| 6 | I I |  |  |  |
| I2; $0^{\circ}$ | I8; $0^{\circ}$ | 0 | 0; 0 | 0 |
| I I; 30 | 18;30 | I | 0;32 | ... |
| II; O | I9; 0 | 2 | 0;44 | ... |
| 10;30 | 19;30 | 3 | 0;55 | ... |
| IO; 0 | 20; 0 | 4 | I; I | ... |
| 9;30 | 20;30 | 5 | I; 8 | $\ldots$ |
| 9; 0 | 2I; 0 | 6 | I; I4 | ... |
| 8;30 | 21;30 | 7 | I; 18 | $\ldots$ |
| 8; 0 | 22; 0 | 8 | I;23 | ... |
| $\ldots$ | $\ldots$ | $\ldots$ | .. | $\ldots$ |
| 6; 0 | 24; 0 | I 2 | I;37 | 0 |
| 5;30 | 24;30 | I3 | I;40 | 22 |
| 5; 0 | 25; 0 | I4 | I;42 | 3 I |
| 4;30 | 25;30 | I 5 | I;42 [sic] | 36 |
| ... | ... | $\ldots$ | $\ldots$ | $\ldots$ |
| I; 0 | 29; 0 | 22 | I;5I | 56 |
| 0;30 | 29;30 | 23 | I;52 [sic] | 57 |
| O; 0 | O; о | 24 | I;5I | 57 |

Table i i. "Table for lunar eclipses with the hours of their phases"

Comments:
(I) The heading for the first two columns is tannin yareah (lit. the node of the Moon).
(2) A similar table appears in Bonjorn's set of tables with the same presentation. Columns I and 2 are the same, column 3 in Table II can be derived from Bonjorn's col. 3 by rounding, and the entries in columns 4 and 5 are slightly different. ${ }^{40}$

(Continued)
40. Chabás 1992, p. 25 I.

| I | I2;38 | $0 ; 57$ | $\mathrm{I} 5 ; 57$ | 2 | $25 ; 45^{\mathrm{b}}$ | $\mathrm{I} ; 56$ | $2 ; 3 \mathrm{I}$ | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| O | $\mathrm{I} 3 ; \mathrm{II}$ | $0 ; 59$ | $\mathrm{I} 6 ; 38$ | 2 | $26 ; 2 \mathrm{I}$ | $\mathrm{I} ; 58$ | $3 ; \mathrm{I} 3$ | 8 |

a. Read: 15;22.
b. Read: 25;48.

Table i2. "Table for the mean position of the Moon, the mean solar anomaly, the true lunar anomaly, together with the minutes of proportion, for each day from hour to hour up to 15 days".

| hours <br> of the <br> mean <br> conj. <br> or opp. <br> after <br> noon | Number of days from mean conjunction to mean opposition or vice versa |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | I4[d] |  |  |  | I5[d] |  |  |  |
|  | mean <br> Moon | solar anomaly | true lunar anomaly |  | mean <br> Moon | solar anomaly | true lunar anomaly |  |
|  | 5[s] | o [s] | $5[\mathrm{~s}]$ |  | 6[s] | o [s] | 6[s] |  |
|  |  |  |  | minutes of proportion | minutes of proportion |  |  |  |
| 23h | 21;52 ${ }^{\circ}$ | 12;51 ${ }^{\text {o }}$ | 14;16 ${ }^{\circ}$ | 6 | $5 ; \mathrm{I}^{\mathrm{o}}$ | 13;50 ${ }^{\circ}$ | $0 ; 51^{\circ}$ | I |
| 22 | 22;23 | 12;54 | 14;57 | 6 | 5;34 | 13;53 | 1;32 | I |
| 2 I | 22;56 | 12;56 | 15;39 | 5 | 6; 7 | 13;55 | 2;15 | I |
| 20 | 23;29 | 12;59 | 16;20 | 5 | 6;40 | 13;58 | 2;56 | I |
| 19 | 24; 2 | I3; I | 17; I | 5 | 7;13 | 14; 0 | 3;38 | I |
| 18 | 24;35 | I3; 4 | 17;43 | 5 | 7;49 ${ }^{\text {b }}$ | 14; 3 | 4;20 | I |
| ... | - | - | - | - | - | - | - | - |
| 9 | 29;32 | 13;26 | 23;57 | 3 | 12;42 | 14;25 | 10;34 | o |
| 8 | 6[s] 0; 5 | 13;28 | 24;38 | 3 | 13;15 | 14;27 | II; 15 | o |
| 7 | 0;38 | I3;3I | 25;19 | 2 | 13;48 | 14;30 | II; 57 | o |
| 6 | I; II | 13;33 | 26; I | 2 | 14;2I | 14;32 | 12;39 | o |
| 5 | 1;43 | 13;36 | 26;52 ${ }^{\text {a }}$ | 2 | 14;54 | 14;35 | I3;2I | o |

(Continued)

| 4 | $2 ;$ I6 | I3;38 | $27 ; 24$ | 2 | - | - | - | - |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $2 ; 49$ | I3;4I | $28 ; 6$ | 2 | - | - | - | - |
| 2 | $3 ; 22$ | I3;43 | $28 ; 47$ | 2 | - | - | - | - |
| I | $3 ; 55$ | I3;45 | $29 ; 28$ | I | - | - | - | - |
| O | $4 ; 28$ | I3;48 | $6[$ s] 0;10 | I | - | - | - | - |

a. Read: 26;42.
b. Read: 7;46.

## Comments:

(I) Days I to 3 are on 99b; days 4 to 6 on Iooa; days 7 to 9 on Ioob; days io to I2 on IoIa; and days 12 to I5 on ioib. We only display full entries for days I and 2, and days I4 and I5.
(2) For a given date, the mean position of the Moon and the mean solar anomaly are found by adding the entries in the first and second columns under the day number to the appropriate entries in Tables 2, 3, and 4 (radices, mean motions in cycles of 8 years, and mean motions for syzygies within a year). The sum of the entries in Tables 2, 3, and 4, yield the mean positions of the Moon and the mean solar anomaly at the preceding syzygy. The entries in Table 12 give the increments since the preceding sygyzy.
(3) The argument is the number of hours preceding the number of days at the head of subsequent columns. So, for example, the entry for day I, 23h, means 23 h before the end of day I, counted from the preceding mean sygyzy, or Ih following the preceding mean syzygy. This is certainly an unusual way to designate time.
(4) The entry for day I 4 , oh, in the column for the mean Moon, is $6 \mathrm{~s} 4 ; 28^{\circ}$ (= $\left.184 ; 28^{\circ}\right)$. With Levi's parameter, in I4 days the mean motion of the Moon is I $84 ; 28^{\circ}$, in agreement with the text. ${ }^{41}$
(5) Similarly, in I4 days the solar anomaly has progressed I3;48 . In Table 3 the mean motion in solar anomaly was $0 ; 59,8, \mathrm{II}, 6^{\circ} / \mathrm{d}$; hence, in I4 days the solar anomaly would advance $13 ; 47,55^{\circ} \approx \mathrm{I} 3 ; 48^{\circ}$, as in the text.
(6) We now turn to the true anomaly and the minutes of proportion in the third column under each day. Unlike the previous two columns, this column does not display a mean motion. Rather, it displays the true lunar anomaly cor-

[^6]rected by the equation of center which, in the Parisian Alfonsine Tables, is tabulated in a column just after the argument; the minutes of proportion are displayed in the next column there..$^{42}$ The entries in these columns are the same as in al-Battānī's tables and many other medieval zijes. However, Farissol Botarel did not use signs of $60^{\circ}$ as in the first printed edition of the Parisian Astronomical Tables for, as we have noted, he used signs of $30^{\circ}$, as in Bonfils's planetary tables for I340. So it seems more likely that for this column in Table 12 Farissol Botarel depended on Bonfils (Munich, MS Heb. 386, 32b-33b). On the other hand, as we shall see, in Tables I3 and I4 Farissol Botarel definitely depended on the Parisian Alfonsine Tables. The argument for these two columns is the double elongation between the Moon and the $\operatorname{Sun}\left(\approx 24 ; 22,53^{\circ} / \mathrm{d}\right)$, here represented by the time since syzygy. For example, the entry for the true anomaly in Table 12 for 2 days oh is Is $3 ; 13^{\circ}$ (= $33 ; 13^{\circ}$ ) which corresponds to a double elongation of $48 ; 46^{\circ}$ and a mean anomaly of $26 ; 8^{\circ}$. The difference between the mean and true anomaly is $7 ; 5^{\circ}$, as in the Parisian Alfonsine Tables and in Bonfils's planetary tables for I340. ${ }^{43}$ We have checked I6 entries scattered throughout Table I2 and the differences between text and computation were mostly zero with a few cases of I or 2 minutes. The entry for the minutes of proportion for 6 d oh can serve as an example. The entry in Table 12 is $54^{\prime}$ and it corresponds to a double elongation of $146 ; 17^{\circ}$. In the Parisian Astronomical Tables the entry for the minutes of proportion corresponding to $146 ; 17^{\circ}$ is $54^{\prime}$ and this is also the case in Bonfils's planetary tables for I340.44 The double elongation at mean syzygy is always $0^{\circ}$ (the elongation at mean conjunction is $0^{\circ}$ and at mean opposition it is $180^{\circ}$ ); hence, the true anomaly at any given time is the sum of the mean anomaly at mean syzygy plus the progress in true anomaly since mean syzygy. The mean anomaly at mean syzygy is found by adding the appropriate entries in Tables 2, 3, and 4. To find the true position of the Moon, one adds the mean position of the Moon and the appropriate entry in Table I3, where the entries in Table I2 serve as the arguments: see the comments to Table I3, below. Levi's lunar model is quite different from the Ptolemaic tradition, and his tables for the lunar equations are unrelated to those in Faris-

[^7]sol Botarel's astronomical tables. ${ }^{45}$ It is noteworthy that Farissol Botarel accepted Levi's mean motions and combined them with equations based on Ptolemy's model.

| [true] lunar anomaly | Minutes of proportion at $5^{\prime}$ intervals |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | o | 5 | IO | ... | 55 | 60 |
| $\begin{array}{ll} \mathrm{O}[\mathrm{~s}] \\ \text { Ari } & \mathrm{I}^{\circ} \end{array}$ | 7;35 ${ }^{\circ}$ | 7;35 ${ }^{\circ}$ | 7;34 ${ }^{\circ}$ |  | 7;32 ${ }^{\circ}$ | 7;32 ${ }^{\text {o }}$ |
| 5 | 7;16 | 7;15 | 7;12 |  | 7; 6 | 7; 4 |
| 10 | 6;53 | 6;51 | 6;49 |  | 6;41 | 6;29 |
| 15 | 6;29 | 6;26 | 6;23 |  | 5;57 | 5;54 |
| 20 | 6; 6 | 6; 2 | 5;58 |  | 5;23 | 5;19 |
| 25 | 5;45 | 5;40 | 5;34 |  | 4;51 | 4;46 |
| 30 | 5;23 | 5;17 | 5;1 I |  | 4;19 | 4;13 |
| ... |  |  |  |  |  |  |
| $\begin{array}{ll} \hline 5[\mathrm{~s}] & \\ \text { Vir } & 5 \end{array}$ | 5;25 | 5;19 | 5;12 | ... | 4;1 I | 4; 4 |
| 10 | 5;50 | 5;44 | 5;39 |  | 4;48 | 4;42 |
| 15 | 6;17 | 6;13 | 6; 8 |  | 5;29 | 5;25 |
| 20 | 6;44 | 6;4I | 6;38 |  | 6;1I | 6; 8 |
| 25 | 7;12 | 7;10 | 7;7 |  | 6;56 | 6;54 |
| 30 | 7;40 | 7;40 | 7;40 |  | 7;40 | 7;40 |
| ... |  |  |  |  |  |  |
| I [ [s] |  |  |  | ... |  |  |
| Psc 5 | 9;36 | 9;4I | 9;46 |  | 10;30 | 10;35 |
| 10 | 9;14 | 9;18 | 9;24 |  | 9;57 | IO; I |
| 15 | 8;51 | 8;53 | 8;57 |  | 9;23 | 9;26 |
| 20 | 8;27 | 8;29 | 8;31 |  | 8;49 | 8;51 |
| 25 | 8; 4 | 8; 5 | 8; 6 |  | 8;15 | 8;16 |
| 30 | 7;40 | 7;40 | 7;40 |  | 7;40 | 7;40 |

Table i3. "Table for the correction of the Moon that has to be added to the mean positions to find the true [positions]".
45. For Levi's lunar model, see Goldstein I974, pp. 53-74, 212-2 I7.

## Comments:

(I) This is a double argument table where one variable is the true lunar anomaly and the other is minutes of proportion, which represent the double elongation. The values for the true lunar anomaly are found from Tables 2,3 , and 4 plus the value in Table I2. The values for the minutes of proportion are found in Table I2. In the Ptolemaic tradition, a lunar position is computed by means of the following formulas:

```
\(\alpha=\bar{\alpha}+c_{3}(\bar{\alpha})\)
and
\(c(\alpha, 2 \eta)=c_{6}(\alpha)+c_{5}(\alpha) \cdot c_{4}(2 \eta)\),
```

where $\bar{\alpha}$ is the mean anomaly, $\alpha$ is the true anomaly, and $2 \eta$ is the double elongation. In the first formula $c_{3}$ refers to entries in column 3 of the Parisian Alfonsine Tables, headed equation of center. In the second formula $c_{6}$ refers to entries in column 6, headed equation of anomaly; $\mathrm{c}_{5}$ refers to entries in column 5 representing the increment; and $c_{4}$ refers to entries in the column 4 headed minutes of proportion. ${ }^{46}$ Table I2 deals with the first formula, and Table I3 with the second formula. The arrangement of Table I3 is unusual: the entries in the column headed o minutes of proportion yield directly $\mathrm{c}_{6}(\alpha)$ because in the second formula $c_{4}(2 \eta)=0$. Similarly, the entries in the in the column headed 60 minutes of proportion the entries yield $c_{6}(\alpha)+c_{5}(\alpha)$ because $c_{4}(2 \eta)=60$ (where $\left.60^{\prime}=I\right)$. Hence it is no longer necessary to tabulate $c_{4}(2 \eta), c_{5}(\alpha)$, or $c_{6}(\alpha)$, in contrast to almost all other medieval tables for the lunar corrections.

This table by Farissol Botarel is very similar to a table for the same purpose in the Almanac of Jacob ben Makhir Ibn Tibbon of Montpellier for I300, composed in Hebrew. The only substantial difference is that Jacob's table depends on the lunar equation of anomaly found in the Toledan Tables and in the zij of al-Battānī with a maximum value of $5 ; \mathrm{I}^{\mathrm{o}}$, whereas Farissol Botarel's table is based, as will be seen below, on the Alfonsine Tables, where the maximum value of this equation is taken as $4 ; 56^{\circ}$. In this sense, Farissol Botarel adapted to Alfonsine astronomy a table in the Almanac of Jacob ben Makhir, which was widely diffused in a Latin version beginning in the early years of
46. Ratdolt I483, e4r-e6v; see also Poulle 1984, pp. I48-153, and Chabás and Goldstein 2012, pp. 67-73.
the fourteenth century. ${ }^{47}$ The only other example we have found of a table displaying the lunar corrections in a similar arrangement is in the astronomical tables of Joseph Ibn Waqār (Castile, fourteenth century), extant only in Munich, MS Heb. 230. ${ }^{48}$ Ibn Waqār's double argument table has two variables: (I) the true lunar anomaly, and (2) the minutes of proportion at IO' intervals from o' to 60' displayed in 7 columns.
(2) In canon 5 Farissol Botarel gives the instruction to subtract 7;40 ${ }^{\circ}$ after adding the mean position and the correction in Table I3. ${ }^{49}$ This displacement is exactly the same as that used by Jacob ben Makhir in his table, and its purpose is to avoid subtractions in the procedure..$^{50}$
(3) We now turn to the underlying parameters for this table. The minimum entry for $0^{\prime}$ is $2 ; 44^{\circ}$ at $3 \mathrm{~s} 5^{\circ}\left(=95^{\circ}\right)$, and the maximum entry for $0^{\prime}$ is $12 ; 36^{\circ}$ at $8 \mathrm{~s} 25^{\circ}$ $\left(=265^{\circ}\right)$, which are symmetric about $7 ; 40^{\circ}$, for $7 ; 40^{\circ}-2 ; 44^{\circ}=4 ; 56^{\circ}$, and I2;36 $6^{\circ}-7 ; 40^{\circ}=4 ; 56^{\circ}$. Similarly, the mininimum entry for $60^{\prime}$ is $0 ; 6^{\circ}$ at $95^{\circ}$ and the maximum for $60^{\prime}$ is $15 ; 14^{\circ}$ at $8 \mathrm{~s} 20^{\circ}$ and $25^{\circ}\left(=260^{\circ}\right.$ and $\left.265^{\circ}\right)$. The sum of $0 ; 6^{\circ}$ and $\mathrm{I} 5 ; 14^{\circ}$ is $15 ; 20^{\circ}$ and half of it is $7 ; 40^{\circ}$. That is, $7 ; 40^{\circ}-0 ; 6^{\circ}=$ $7 ; 34^{\circ}$ and $15 ; 14^{\circ}-7 ; 40^{\circ}=7 ; 34^{\circ}$. In the corresponding tables in the Parisian Alfonsine Tables for $95^{\circ}\left(=\mathrm{I}, 35^{\circ}\right)$ the entries for $\mathrm{c}_{6}$ and $\mathrm{c}_{5}$ are $-4 ; 56^{\circ}$ and $-2 ; 38^{\circ}$, whose sum is $-7 ; 34^{\circ}$; and for $265^{\circ}\left(=4,20^{\circ}\right)$ the entries for $\mathrm{c}_{6}$ and $\mathrm{c}_{5}$ are $4 ; 55^{\circ}$ and $2 ; 39^{\circ}$, whose sum is $7 ; 34^{\circ}$. For an intermediate case, consider the entries in Table I3 for a lunar anomaly of $5 \mathrm{~s} 5^{\circ}\left(=\mathrm{I} 55^{\circ}=2,35^{\circ}\right)$ : for $0^{\prime}$ and $60^{\prime}$ they are $5 ; 25^{\circ}$ and $4 ; 4^{\circ}$, respectively. The corresponding entries for $\mathrm{c}_{6}$ and $\mathrm{c}_{5}$ in the Parisian Alfonsine Tables are $-2 ; 15^{\circ}$ and $-\mathrm{I} ; 2 \mathrm{I}^{\circ}$ whose sum is $-3 ; 36^{\circ}$. When we add $7 ; 40^{\circ}$ to $-2 ; 15^{\circ}$ the result is $5 ; 25^{\circ}$, as in Table 13 for $0^{\prime}$, and when we add $7 ; 40^{\circ}$ to $-3 ; 36^{\circ}$ the result is $4 ; 4^{\circ}$, as in Table I3 for $60^{\prime}$. The underlying parameters for Table I3 are characteristic of the Parisian Alfonsine Tables and thus indicate dependence on them.
(4) Farissol Botarel changed the presentation of this table while maintaining the model and the underlying parameters of the lunar corrections in the Parisian Alfonsine Tables. This is consistent with a general trend in the late Middle Ages to make astronomical tables more "user-friendly", that is, reducing the

[^8]number of steps in the computation of the position of the celestial body, simplifying the table so that the only interpolation required can be done at sight, and eliminating subtractions which were complicated to put in the instructions before the invention of negative numbers. ${ }^{51}$

|  | Signs of the mean solar anomaly |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| degrees | o[s] | I [s] | 2[s] | 3[s] | ... | 8[s] | 9[s] | Io[s] | I I [s] |
| I | Cnc $0 ; 40^{\circ}$ | $\begin{aligned} & \text { Cnc } \\ & 29 ; 37^{\circ} \end{aligned}$ | Leo $28 ; 50^{\circ}$ | Vir $28 ; 32^{\circ}$ | ... | Psc $2 ; 38^{\circ}$ | Ari $2 ; 52^{\circ}$ | Tau 2;31 ${ }^{\circ}$ | Gem $\mathrm{I} ; 42^{\circ}$ |
| 2 | I;38 | $\begin{aligned} & \text { Leo } \\ & \text { o;35 } \end{aligned}$ | 29;49 | 29;32 | ... | 3;39 | 3;5I | 3;30 | 2;4I |
| 3 | 2;37 | I;33 | Vir 0;48 | Lib $0 ; 32$ | ... | 4;40 | 4;5I | 4;29 | 3;39 |
| ... | ... | ... | ... | .. | ... | ... | ... | ... | ... |
| 6 | 5;29 | 4;28 | 3;45 | 3;32 | ... | 7;43 | 7;50 | 7;25 | 6;32 |
| $\cdots$ | ... | $\cdots$ | ... | ... | ... | ... | ... | ... | ... |
| 12 | 11;16 | 10;18 | 9;40 | 9;34 | ... | 13;47 | 13;47 | 13;16 | 12;20 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 18 | 17; 4 | I6; 8 | 15;36 | 15;37 | ... | 19;50 | 19;44 | 19; 6 | 18; 8 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 24 | 22;51 | 21;59 | 21;33 | 2I;4I | ... | 25;52 | 25;39 | 24;56 | 23;55 |
|  |  |  |  |  | ... | Ari | Tau | Gem |  |
| 30 | 28;39 | 27;5I | 27;32 | 27;47 | ... | I;52 | I;33 | 0;45 | 29;42 |

Table i4."Table for the true position of the Sun by means of [be-emsa'ut] its mean anomaly".
Comments:
(I) Table 14 displays the true solar longitude. The entries result from adding three terms: the mean solar anomaly, $\bar{x}$, taken here as the argument; $c(\bar{x})$, the solar equation, based on the corresponding table in the Parisian Alfonsine Tables where the maximum solar equation is $2 ; 10^{\circ} ; 52$ and the quantity $89 ; 42^{\circ}$, taken here as the longitude of the solar apogee, $\lambda_{A}$. In general

$$
\lambda=\lambda_{\mathrm{A}}+\bar{\chi}+\mathrm{c}(\bar{\chi}) .
$$

5I. Cf. Chabás and Goldstein 2013.
52. Ratdolt I483, e2v-e3v; see also Poulle 1984 , pp. 145-I47.

So for $\bar{x}=92^{\circ}, \mathrm{c}(\bar{x})=-2 ; 10^{\circ}$. Hence $\lambda=89 ; 42^{\circ}+92^{\circ}-2 ; 10^{\circ}=179 ; 32^{\circ}$ (= Vir $29 ; 32^{\circ}$ ) as in Table I4. For another example, consider $\overline{\mathcal{\gamma}}=30^{\circ}$ for which argument in the Parisian Alfonsine Tables $c(\bar{x})=-1 ; 3^{\circ}$. Then $89 ; 42^{\circ}$ $+30^{\circ}-\mathrm{I} ; 3^{\circ}=\mathrm{II} 8 ; 39^{\circ}\left(=\right.$ Cnc $\left.28 ; 39^{\circ}\right)$ as in Table 14.
(2) The solar apogee, Gem $29 ; 42^{\circ}\left(=89 ; 42^{\circ}\right)$, is the entry for mean anomaly I Is $30^{\circ}\left(=360^{\circ}=0^{\circ}\right)$, and this is consistent with what we noted in Comments to Table 2 § 5, above.
(3) As was the case for Table I3, Table I4 is arranged to avoid subtractions.

| hours | o, ', ' | hours | ', ', ', |
| :---: | :---: | :---: | :---: |
| minutes of an hour | ',','" | minutes of an hour | ',','"' |
| seconds of an hour | ", '"', '"' | seconds of an hour | ','"', '"' |
| I | 0, 2,27 | 3 I | I, I6, 2 I |
| 2 | 4,55 | 32 | 18,49 |
| 3 | 6,23 | 33 | 21,16 |
| 4 | 9,5 I | 34 | 23,45 |
| 5 | 12,19 | 35 | 26,13 |
| ... | ... | $\ldots$ | ... |
| 10 | 24,38 | 40 | 38,33 |
| ... | ... | ... | ... |
| 15 | 36,57 | 45 | 50,52 |
| ... | ... | $\ldots$ | ... |
| 20 | 49,16 | 50 | 2, 3, I I |
| ... | ... | ... | ... |
| 24 | 0,59, 8 | ... | ... |
| 25 | I, I,35 | 55 | 15,30 |
| .. | ... | ... | ... |
| 30 | 1,13,55 | 60 | 2,27,50 |

For arg. 3, the entry should be $7 ; 23$.
Table i5. "Table for the motion of the Sun in hours, etc.; with it you may determine the moment when the Sun enters each zodiacal sign".

## Comments:

(I) One can enter this table with the argument in hours, or minutes of an hour, or seconds of an hour. The entries are then in degrees, or minutes, or seconds, respectively, as indicated in the heading for the second column. For example, Ih corresponds to $0 ; 2,27^{\circ} ; 0 ; \mathrm{Ih}$ corresponds to $0 ; 0,2,27^{\circ}$, etc.
(2) A mean solar velocity of $0 ; 2,27,50^{\circ} / \mathrm{h}$ corresponds to a mean solar velocity of $0 ; 59,8^{\circ} / \mathrm{d}$, a crude value for this parameter that makes it difficult to specify Farissol Botarel's source. The entries in this table are simply truncated submultiples of the last entry. A few entries were not computed correctly, e.g., for 3I the entry should be I,I6,22 (truncated from I,I6,22,50), and for 33 the entry should be $\mathrm{I}, 2 \mathrm{I}, \mathrm{I} 8$ (truncated from I, $2 \mathrm{I}, \mathrm{I} 8,30$ ).
(3) Given the time when the true position of the Sun is close to the beginning of a zodiacal sign, one begins by finding its distance in longitude from the beginning of the sign. Then one can either subtract successively the motion of the Sun in the time intervals displayed in this table, or one can simply divide the distance by the mean solar motion to obtain the time interval that is sought.
(4) The purpose of Table I5, as stated in its title, is to aid in the determination of the moment when the Sun enters a zodiacal sign. This topic was addressed by various authors, who computed tables for specific dates and places; in particular, it is discussed in John of Saxony's canons to the Parisian Alfonsine Tables, ch. $23 .{ }^{53}$ From an astrological point of view, the entry of the Sun into Aries (i.e., the vernal equinox) is most important because from the horoscope for that moment world affairs for the coming year can be forecast. ${ }^{54}$ For examples of the astrological significance of the Sun's entry into different zodiacal signs see, e.g., Kūshyār Ibn Labbān's Introduction to Astrology, II. 6 [5-6]. ${ }^{55}$
53. For the Hebrew version of this chapter, see Munich, MS Heb. 126, I8b-igb. For the Latin version, see Poulle 1984, pp. 86-91. For tables displaying the entry of the true Sun into the zodiacal signs, see Chabás and Goldstein 2012, pp. 84-85; Chabás and Goldstein 2000, 47-49; and Langermann 1988, pp. 25-26.
54. See, e.g., Māshā’allāh's Book on Eclipses, ch. 4, in Sela 2010, p. 247; Sela 2013, p. 214 , in notes to Abraham Ibn Ezra's Book of Nativities. See also Ptolemy, Tetrabiblos, ii.Io, in Robbins (ed. and tr.) 1964, pp. 197-199.
55. Yano 1997, p. 59; see also pp. 75-77.

Appendix: Moses Farissol Botarel

on the solar eclipse of July 29, I478

The following passage (Munich, MS Heb. 343, 92a) has many problems and obscurities which make translating it difficult. So what follows is a free translation or a paraphrase ${ }^{56}$

We heap [lit. inspect] scorn and disdain on the haughty [cf. Ps. 123:4], the [would-be] experts in the laws of the heavens: insult, shame, and disgrace of the Nations be poured upon them like water, and especially upon those who rely on tables that are far removed from the truth. Due to the hurriedness and the hastiness of their work - concerning a solar eclipse that took place on July 29, I478 according to the Christian reckoning-they truly displayed their ignorance by positing this solar eclipse as total, ${ }^{57}$ because they did not follow the astronomy of Alfonso and of Master Leo [Levi ben Gerson], and those like them, who assign to this eclipse 8 digits or $8^{1 / 2}$ digits...

ומפני כי השגחנו הלעג והבוז לגאיונס יודעי חקות שמיס כמיס תשפוך עליהס כלימת הגויס בושת וקלון וביחוד לסומכיס בלוחות רחוקות מנקדת האמת לקלות מרוצתס ומהירות מלאכתס על לקות חמה שהיה יוס כ״ט גולי שוּ שנת תע״ח לחשבון הנצריס ובאמנה לבשו כתונת קדרות השמש בתתס אותו כולל ולא הביטו אחרי תכונת אלפונצו ומא’ ליאון ודומיהס הנותניס אותו הקדרות ח’ אצבעות או ח’ וחצי....

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56. The Hebrew text was published in Steinschneider I899, p. 45.
57. Lit. the solar eclipse covered [with Steinschneider, read לבשה instead of לבשו] them (like a) cloak when they posited it as total.

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[^0]:    I. We are grateful to Gad Freudenthal for his help with some issues in Hebrew terminology, and to Shlomo Sela for his assistance in astrological matters.
    2. Steinschneider 1964, p. 200.
    3. See n. I5, below.
    4. Oxford,Bodleian Library,MS Reggio I4, 44a-56a; London, British Library,MS Or. 3658, ia-Iob.

[^1]:    I3. For the Parisian Alfonsine Tables, see Ratdolt 1483 and Poulle 1984; for Levi's Astronomy, see Goldstein I974 and I985b; for Bonfils's Six Wings (restricted to the motions of the Sun and the Moon), see Solon I968 and I970; for Bonfils's planetary tables for I340, see Goldstein and Chabás 2017, and Munich, MS Heb. 386; for Bonjorn, see Chabás i99I and I992. Bonjorn was also known as ha-po'el, that is, the (table-)maker. The Hebrew text of Bonfils's Six Wings was published in Zhitomir in 1872.

[^2]:    I4. Oxford, Bodleian Library, MS Reggio I4, 44a:Io-II. A copy of the Hebrew version of the Alfonsine Tables by Moses of Nîmes ( $f l$. I460) is preserved in Munich, MS Heb. I26, and it includes a translation of the canons by John of Saxony (ff. 4a-22a). Steinschneider 1895, p. 78; cf. Steinschneider 1964, p. I96.

    I5. On the Oxford Tables, see North 1977 and Chabás and Goldstein 2016; on Solomon ben Davin of Rodez (in the south of France), see Steinschneider I964, pp. I66-167. Mordekhai Finzi translated the Oxford Tables of I348 into Hebrew in I44I: see Langermann i988, pp. 26-28.

    I6. See, e.g., Chabás and Goldstein 2012, and F. S. Pedersen 2002.
    17. See Munich, MS 343, 92 b.

[^3]:    Kennedy 1987. See also Comes 1994 for discussion of the five base meridians (including one in the Far East) in Islamic sources.
    20. Goldstein 1974, p. 218.

    2 I. Goldstein 1974, p. 226.
    22. Goldstein 1974, p. Io6.
    23. Goldstein 1974, p. 170.
    24. Goldstein 1974, p. 171.

[^4]:    34. For an analysis of Bonfils's Wing 2, see Solon 1970, pp. 3-4. For Bonfils's Six Wings we have consulted several Hebrew manuscripts, including Munich, MS Heb. 343, Ia-26a. Cf. Solon 1968.
[^5]:    35. Cf. Munich, MS Heb. 343, 93 a.
[^6]:    4I. Chabás and Goldstein 2012, p. 58.

[^7]:    42. Ratdolt I483, e4r-e6v; see also Poulle 1984, pp. I48-I53.
    43. Ratdolt I483, e4v; see also Poulle I984, p. I49, and Munich, MS Heb. 386, 32a.
    44. Ratdolt I483, e6r; see also Poulle I984, p. I52, and Munich, MS Heb. 386, 33 a.
[^8]:    47. See Boffito and Melzi d'Eril I908, especially pp. 98-Io9.
    48. The lunar correction tables in this manuscript are on folios $37 \mathrm{a}-38 \mathrm{~b}$. Cf. Chabás and Goldstein 2015, pp. 583, 607.
    49. Munich, MS Heb. 343, 94b.
    50. On displaced tables, see Chabás and Goldstein 2013.
