Bīrūnī's Telescopic-Shape Instrument for Observing the Lunar Crescent

S. Mohammad Mozaffari and Georg Zotti

Abstract: This paper deals with an optical aid named *barbakh* that Abū al-Rayḥān al-Bīrūnī (973–1048 AD) proposes for facilitating the observation of the lunar crescent in his *al-Qānūn al-Mas'ūdī* VIII.14. The device consists of a long tube mounted on a shaft erected at the centre of the Indian circle, and can rotate around itself and also move in the vertical plane. The main function of this sighting tube is to provide an observer with a darkened environment in order to strengthen his eyesight and give him more focus for finding the narrow crescent near the western horizon about the beginning of a lunar month. We first briefly review the history of altitude-azimuthal observational instruments, and then present a translation of Bīrūnī's account, visualize the instrument in question by a 3D virtual reconstruction, and comment upon its structure and applicability.

Keywords: Astronomical Instrumentation, Medieval Islamic Astronomy, Bīrūnī, *Al-Qānūn al-Mas'ūdī*, *Barbakh*, Indian Circle

Introduction: Altitude-Azimuthal Instruments in Islamic Medieval Astronomy.

Altitude-azimuthal instruments either are used to measure the horizontal coordinates of a celestial object or to make use of these coordinates to sight a heavenly body. They belong to the "empirical" type of astronomical instruments.[1] None of the classical instruments mentioned in Ptolemy's *Almagest* have the simultaneous measurement of both altitude and azimuth of a heavenly object as their main function.[2] One of the earliest examples of altitude-azimuthal instruments is described by Abū al-Rayḥān al-Bīrūnī for the observation of the lunar crescent near the western horizon (the horizontal coordinates are deployed in it to sight the lunar crescent). In this paper, we attempt to make a virtual reconstruction of this instrument and discuss the aspects of its efficiency and applicability from a technical point of view. Another early example is that proposed by Bīrūnī's contemporary, Ibn Sīnā (Avicenna), who presumably built a model of it in his observatory at Iṣfahān (central Iran) between 1024 and 1037 AD (this was used in order to measure the altitude and azimuth of an object in any direction in the sky).[3]

Nearly three centuries later, four altitude-azimuthal instruments appeared in the first period of activity at the Maragha observatory (1259–1283 AD). These were constructed by Mu'ayyad al-Dīn al-'Urḍī (d. 1266 AD), the main instrument maker of the observatory at the time. We provide the names and brief descriptions of these instruments for later reference:

(1) *Ālat dhāt al-rub'ayn*, "Instrument having two quadrants" or, as al-Kāshī (1380–1429 AD) calls it, *dhāt al-samt wa-l-irtifā'*, "(Instrument) having azimuth and altitude",[4]

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[1] Here, we follow the classification set forth in Charette 2006, p. 123. Research on medieval Islamic observational instruments is insufficient from many aspects (King 2004/5, Vol. 2, pp. 11–27 gives a general perspective of it), while other types of operational and demonstrational instruments such as various models of astrolabes, sundials, portable quadrants, and so on, have been investigated to the finest details (e.g., see King [1987] 1995; Charette 2003).

[2] Of the seven antique observational devices, only Two Circles, Mural Quadrant (*Almagest* I.12), and Parallactic Instrument (*Almagest* V.12) are installed in the plane of the meridian and are used for measuring the altitude/zenith distance of a celestial object when it passes the meridian (see Toomer 1998, pp. 61–63, 133, 217–219, 244–247, 252 and 404–407).

[3] Ibn Sīnā describes this instrument in his *Maqāla fī 'l-ālāt al-raṣadiyya* ("Essay on observational instruments"); concerning it, see Wiedemann and Juynboll 1926; for a figure of the instrument, see Sezgin and Neubauer 2010, Vol. 2, pp. 26–27. In spite of what Sayılı ([1960] 1988, pp. 156–158) and the other sources blindly following him (e.g., Sezgin and Neubauer 2010, *loc cit.*) claim, Ibn Sīnā's observatory was located at Iṣfahān, not in Hamadhān, as his pupil, Abū 'Ubayd al-Jūzjānī, says definitely that Ibn Sīnā "wrote the *Maqāla fī 'l-ālāt* in Iṣfahān during his observations for 'Alā' al-Dawla [d. 1041 AD, Daylamī military leader and founder of the short-lived but significant Kakuyid dynasty]" (Gohlman 1974, pp. 104–105).

[4] Kāshī, *Sharḥ*, S: ff. 13r–14r, P: pp. 23–26, M: pp. 36–38, T: f. 116r; Kennedy 1961, pp. 101–103, 106.

a double azimuthal quadrant built from copper inside a circular wall, capable of measuring, at the same time, azimuth and altitude of two celestial objects;[5]

(2) *Dhāt al-jayb wa-l-samt*, "(Instrument) having sine and azimuth", an instrument to determine the sine of the zenith distance of a heavenly body and its azimuth using a wooden bar rotating on an iron axis inside a circular wall, on one end of which the alidade can slide, the other end sliding up a vertical central pillar;[6]

(3) *Dhāt al-jayb wa-'l-sahm*, "(Instrument) having sine and versed sine", a similar instrument to determine the sine and versed sine of the altitude and/or zenith distance of a celestial object;[7] and

(4) *Āla al-kāmila*, "Perfect instrument", consisting of a rotating parallactic rule inside a circular wall.[8] Al-ʻUrḍī mentions that he had built a model of his "Perfect instrument" for Malik Manṣūr, the ruler of Ḥimṣ (now Homs, Syria), in 650 H/1252–3 AD, in presence of the latter's vizier, Najm al-Dīn al-Lubūdhī.[9] (1) and (3) are described by al-Kāshī.

In the second period of activity at the Maragha observatory (1283–*ca.* 1320 AD), 12 instruments were invented by Ghāzān Khān, the seventh ruler of the Īlkhān dynasty of Iran (r. 1295–1304 AD) on the basis of a totally novel approach. 11 of these instruments are altitude-azimuthal (the remaining is a pinhole image device used to observe solar eclipses).[10]

The altitude-azimuthal instruments appear to have reached their most evolved stage at the Maragha observatory, since no significant innovation in the construction and design of these instruments can be recognized at the Samarqand and Istanbul observatories. In Book III of his *Sidrat al-muntahā*, Taqī al-Dīn Muḥammad b. Maʻrūf, the director of the short-lived observatory at Istanbul (1526–1585 AD), introduces some unprecedented observational instruments, of which his improved models of the Ptolemaic parallactic instrument (one is called *dhāt al-jayb*, "(Instrument) having the sine" and the other, *dhāt al-shuʻbatayn*, "(Instrument) having two branches/legs", - which is the standard name for the antique parallactic instrument in Islamic sources, too - can be classified as

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[5] Seemann 1929, pp. 72–81.
[6] Seemann 1929, pp. 87–92.
[7] Seemann 1929, pp. 92–96.
[8] Seemann 1929, pp. 81–87, 96–104. For an illustration of the instruments of the first period of the Maragha observatory, see Sezgin and Neubauer 2010, Vol. 2, pp. 38–52.
[9] Seemann 1929, p. 97.
[10] About them, see our extensive study in Mozaffari and Zotti 2012; 2013.

altitude-azimuthal instruments. The eight instruments built at the Istanbul observatory are described in detail in an anonymous treatise in Turkish named the *Ālāt-i raṣadiyya li Zīj-i Shāhanshāhiyya*.[11]

Concerning the history of the altitude-azimuthal type of observational instruments prior to the beginning of the past millennium, namely, before Bīrūnī's time, we should first consider the fact that, although not a single example of such instruments can be found in Greek and Roman sources,[12] various examples can be traced in Indian astronomical sources from the time before the rise of astronomy in medieval Islam. For instance, in the *Súrya Siddhánta* and in Brahmagupta's *Brāhmasphuṭasiddhānta* (628 AD), some prototypes of altitude-azimuthal instruments can be recognized that enabled observers to measure altitude, azimuth, time, the solar declination and amplitude, and the longitudes of the sun, moon, and planets. For example, *cakra* was a wooden wheel suspended by a string and graduated to 360 degrees that could be used for measuring the altitude and zenith distance of a heavenly object at any azimuth, the angular distance between the sun and the moon, and the time elapsed from sunrise or remaining till sunset. Also, the longitude of a planet can be measured by this instrument from the longitude of a reference star already known (two other variants with 180º and 90º graduations are also described, which are called, respectively, *dhanuṣ*, "semi-circle", and *turyagola*, "quadrant"). Horizontal variants of the same instruments were also used: *pīṭha* was a horizontally placed *cakra* on whose circumference the cardinal points are marked off, and in whose centre a vertical axis, equal in length to the radius of the circle, is erected. *Kapāla* and *kartarī* were two variants of *dhanuṣ*; the first consists of a semicircle installed to the north of the east-west line with a vertical gnomon in its centre, and the latter consists of two semicircles located in the planes of the equator and the meridian. These instruments were used in timekeeping. A similar instrument was *yaṣṭi*, a staff which can be utilized with two circles engraved on the ground, of which one

[11] Taqī al-Dīn invented and built some other new observational instruments, which were set up at the Istanbul observatory in addition to the classical instruments (e.g., a mural quadrant with radius of 13 cubits ~ 6.5 m that he installed at the observatory on 22 April 1574, and an armillary sphere with a radius of its meridian ring equal to $9^1/_6$ cubits ~ 4.6 m) and those already invented by his Muslim predecessors, like al-'Urḍī's "Two Quadrants". Taqī al-Dīn gives a detailed account of his instruments in his *Sidrat* (K: f. 14v–15r). An illustrated version of the treatise *Ālāt-i raṣadiyya li Zīj-i Shāhanshāhiyya* is also appended to MS. K of Taqī al-Dīn's *Sidrat* (ff. 48v–50r); it is clear that it is not Taqī al-Dīn's since he is referred to as *mawlānā*, "our master", therein. About his instruments, see Tekeli 1963; Sezgin and Neubauer 2010, Vol. 2, pp. 53–61. About his observations carried out in Istanbul in the 1570s, see Mozaffari and Steele 2015.

[12] See, e.g., Evans 1999; 2015.

represents the azimuthal circle and the other stands for the sun's diurnal circle at a given day.[13] In later Hindu sources, based on Brahmagupta, instructions are given how to convert the ortive amplitude of the sun, as observed at sunrise/sunset or as derived from three observations of the solar altitude at a given day by means of this instrument, to its declination, and then to its longitude.[14] It is worth mentioning that Brahmagupta's *Brāhmasphuṭasiddhānta* and *Khaṇḍakhādyaka* (665 AD) were translated into Arabic or adapted in the early ʿAbbāsid period.

In the second place, it seems necessary to discuss some anachronistic misunderstandings attached to the history of altitude-azimuthal instruments in the early Islamic period, especially in the two systematic astronomical observational programs associated with the rule of al-Ma'mūn (ʿAbbāsid caliph from 812 to 833 AD) in Baghdad and Damascus. In the following, we will evaluate whether such instruments were available at the time and/or were employed in the observations.

The above-mentioned misunderstanding is related to the Ma'mūnic observational program made in Damascus, and is rooted in some later Islamic sources belonging to the period from the turn of the fifteenth century onwards, the writings of the already mentioned al-Kāshī and Taqī al-Dīn Muḥammad b. Maʿrūf. From these writings, the misunderstanding was taken over into the modern scholarly literature. When describing two of al-ʿUrḍī's altitude-azimuthal instruments, al-Kāshī says that these did not exist and were not employed in ancient (i.e., pre-Islamic) observations,[15] and adds that *dhāt al-samt wa-l-irtifāʿ* (i.e., al-ʿUrḍī's azimuthal double quadrants) had been built first in Damascus (without specifying at what time) and then at Maragha. Also, in his *Sidrat* III, Taqī al-Dīn says that this instrument is an invention of Islamic observational astronomy.[16] In the *Ālāt-i raṣadiyya li Zīj-i Shāhanshāhiyya*, it is added that it was designed by Islamic astronomers in Damascus, next brought over to Maragha, and then used by Ibn al-Shāṭir in Damascus.[17] Sayılı argues that the first statement refers to the Ma'mūnic observations in Damascus.[18] But, as Kennedy remarks,[19] al-Kāshī's reference

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[13] *Súrya Siddhánta* XIII.20: [1860] 1997, pp. 306–307; *Brāhmasphuṭasiddhānta* XXII.8–45: see Sarma 1986/7, pp. 68–69.
[14] E.g., *Siddhánta Śiromaṇi* XI.10–15, 28–39 ([1861] 1974, pp. 212–213, 218–221).
[15] Kāshī, *Sharḥ*, S: ff. 13v–14r, P: p. pp. 25–26, M: pp. 37–38, T: f. 116r; Kennedy 1961, pp. 101, 103.
[16] Taqī al-Dīn, *Sidrat*, K: f. 15v.
[17] Taqī al-Dīn, *Sidrat*, K: f. 49r.
[18] Sayılı [1960] 1988, pp. 73–74.
[19] Kennedy 1961, p. 106.

to Damascus may merely be a case of confusion with al-ʻUrḍī's "Perfect Instrument" which, as mentioned above, he says to have built at Homs before joining the Maragha team. It is quite probable that al-Kāshī was a source for Taqī al-Dīn, as his other works were read and annotated by the latter,[20] and that al-Kāshī's mention of Damascus, just after his mention that the instrument was not available in antiquity, might have led Taqī al-Dīn to ascribe the instrument to the early Islamic observers in Damascus. After all, it does not seem reasonable to assume that these ambiguous and very late sources can be taken as reliable testimony for the existence of an azimuthal quadrant in Damascus in the ninth century.

Nevertheless, a lost episode of altitude-azimuthal instruments in medieval Middle Eastern astronomy appears to pertain to the Ma'mūnic observational program carried out by Yaḥyā b. Abī Manṣūr in Baghdad. It merits noting that both observational programs carried out in Baghdad and Damascus in the first half of the ninth century concentrated on the determination of the solar parameters,[21] and that the most important result achieved was the discovery of the motion of the solar apogee.[22] Bīrūnī reports four values measured for the obliquity of the ecliptic[23] and also the times estimated for five

[20] E.g., a note on f. 90v of MS. K of the *Sidrat* concerning a computational error committed by al-Kāshī.

[21] On the astronomical activities in the Ma'mūnic period, see Sayılı [1960] 1988, chapter 2 (note that this source contains numerous statements that should be considered with caution; e.g., see note 26, below), a brief summary of which is given in Charette 2006, p. 125; about the solar observations carried out in them, see Mozaffari 2013, esp. Part 1, pp. 327–329.

[22] See Mozaffari 2013, Part 1, pp. 322, 326, the discussion of the discovery of the solar apogee's motion in *ibid*, Part 2, pp. 403–408.

[23] Yaḥyā, working in Baghdad, measured ε = 23;33º. Concerning Khālid's observations in Damascus, Bīrūnī, in his *Taḥdīd*, first mentions that the maximum solar noon-altitude of 80;3,55º was obtained in 217 H/832 AD and the two values 32;56º and 32;55º for the minimum solar noon-altitude were observed, respectively, in 216 H/ 831 AD and 218 H/833 AD, from which he derives ε = 23;33,57,30º and 23;34,27,30º. He also mentions that Sanad b. ʻAlī, who supervised the observations, reported 23;33,52º which is closer to the first value. Bīrūnī follows that in a table in which the solar noon altitudes observed by Khālid in Damascus were written down, he found the following values for the extremal meridian altitudes of the sun: on Sunday–Tuesday, 13–15 Jumādā I 217 H/21–23 Urdībihisht 201 Yazdigird (16–18 June 832, JDN 2025113–5): 80;4,10Ķ, 80;4,30º, and 80;4,28º, and on Monday–Wednesday, 19–21 Dhū al-Qaʻda 217 H/24–26 Ābān 201 Y (16–18 December 832, JDN 2025296–8): 32;55,0º, 32;54,58º, and 32;55,28º. Bīrūnī notices that the max and min noon-altitudes in this table are, respectively, 80;4,30º and 32;54,48º which result in ε = 23;34,51º, but doing some extrapolations between the abovementioned values, he computes 23;34,57,30º (Bīrūnī 1967, pp. 60–64; Bīrūnī 1954–6, Vol. 1, p. 363–4; also, see Kennedy 1973, pp. 32–39. The true modern value at the time ~ 23;35,33º). This seems to have been the

autumnal equinoxes (829–832, and 843 AD) from these observations.[24] In general, three sets of values for the solar orbital elements were measured in the observational programs in Baghdad and Damascus during 829–832 AD and, some 11 years later, again in Baghdad in 843–844 AD.[25] Bīrūnī associates an armillary sphere and a "**circle**" (about whose nature he says nothing further) with the Baghdad observations and a 10-cubit (~ 5 m) gnomon made of iron and a quadrant made of marble with a radius of the same length with the Damascus observations. Bīrūnī mentions the existence of a defect in an instrument Yaḥya made use of, but does not specify which instrument this was. The only deficiency Bīrūnī mentions about the instruments employed in the Ma'mūnic observations is related to the gnomon erected in the monastery Murrān near Damascus;[26] as Bīrūnī says, its length changed by one *sha'īra* (1/144 cubit ~ 3.5 mm) between early morning and evening, because of the decrease in temperature during the night, and consequently, it did not allow an accurate measurement of the true length of the year.[27]

We will consider Yaḥyā's instrument simply named "circle" in more detail. Since the observers in Baghdad measured the obliquity of the ecliptic (see note 23), this instrument may have been a solstitial armilla, maybe similar to the "Two Circles" in

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authoritative source for the later astronomers to associate the value 23;35° with the Damascus observations. It is noteworthy that Khālid's extremal altitudes are in error by less than –1′ (see Said and Stephenson 1995, esp. pp. 122, 125).

[24] Bīrūnī 1954–6, Vol. 2, p. 640, nos. 9–13. As he mentions (*ibid*, Vol. 2, p. 653), Khālid b. 'Abd al-Malik al-Marwarūdhī, Sanad b. 'Alī, and 'Alī b. 'Īsā al-Ḥarranī were engaged in the observations carried out in Baghdad in 843 for determining the solar orbital elements. See also the next note. Note that the table of vernal equinoxes in the Hyderabad edition of Bīrūnī's work is in disorder. The correct order can be easily obtained by means of the times of observations as reckoned from the beginning of the Nabonassar era: in nos. 11–13, the years should be 1579, 1580, and 1591, as in the manuscript "B" listed in the preface of the Hyderabad edition (Vol. 1, before p. 1), noted in the apparatus of the table.

[25] Ibn Yūnus (L: p. 104) remarks unspecifically that the time of the autumnal equinox of 844 was observed by "a group of scientists" (see Said and Stephenson 1995, p. 128).

[26] None of the later Islamic astronomers, including Bīrūnī (1954–6, Vol. 2, p. 637), say that this monastery was on Mt. Qāsiyūn, as stated commonly in the secondary literature (e.g., Sayılı [1960] 1988, p. 71; Charette 2006, p. 125). According to the available historical sources, the monastery Murrān that was near Damascus (there was another monastery with the same name on a mount overlooking Kafarṭāb near al-Mu'arra) had nothing to do with Mt. Qāsiyūn; e.g., in his *Mu'jam al-buldān* (Vol. 2, pp. 533–534, Vol. 4, pp. 295–296), Yāqūt states that the monastery is on a *hill* overlooking saffron farms, which is named after the monastery itself by some authors such as Bīrūnī and Ḥabash in his *Kitāb al-ajrām wa 'l-ab'ād* (cf. Langermann 1985, p. 120), while Mt. Qāsiyūn overlooks the city itself.

[27] Bīrūnī 1954–6, Vol. 1, p. 363, Vol. 2, pp. 637, 778.

Almagest I.12 or to later alterations that substituted an alidade for its inner circle.[28] In the primary sources earlier than Bīrūnī, e.g., in the prologue of the two extant versions of the *Mumtaḥan zīj* and in Ḥabash's *Kitāb al-ajrām wa 'l-ab'ād* ("Book on the volumes and distances [of celestial bodies]"), the "circle" Yaḥyā made use of is unspecifically named *dā'irat al-Shammāsiyya*, the "*Shammāsiyya* circle".[29] These sources also mention that in his observation of the vernal equinox of 829 AD by means of this instrument, Yaḥyā measured, in Baghdad, a longitude of 179;43° for the sun at sunrise on Sunday, 19 September 829 (JDN 2024112), from which the time of the autumnal equinox was deduced as 4/5 hours after noon on this date.[30] Bīrūnī only mentions the time but not the solar longitude. In the *Talkhīṣ al-Majisṭī* (*Compendium of the* Almagest), Muḥyī al-Dīn al-Maghribī, the most prominent astronomer of the Maragha observatory (d. 1283 AD),[31] correctly derives from Yaḥyā's value for the solar longitude the time of the autumnal equinox of 829 AD as 6;54 hours after the moment of the observation or 0;2,15 days after noon.[32] He also provides us with a clue for understanding the nature of Yaḥyā's "circle" by calling it *al-dā'irat al-samtiyya*, the "azimuthal circle".

Muḥyī al-Dīn is presumably the earliest source referring to Yaḥyā's instrument as an "azimuthal circle". Of course, this can neither be verified, nor is it sufficient in itself to decide on the type and form of the instrument. Moreover, it is nearly impossible to decide which reading of the name of the instrument is correct, because the names may be scribal confusions of each other (شمّاسيّة ↔ سمتيّة).[33] However, the principal question is

[28] E.g., the *ḥalqat al-'aḍudiyya* al-Ṣūfī made use of about two centuries later; see Mozaffari and Zotti 2012, p. 402.

[29] See Vernet 1956, p. 508; Langermann 1985, p. 121.

[30] This value is in error by ~ +7 hours (cf., also, Said and Stephenson 1995, p. 128).

[31] About Muḥyī al-Dīn, see Saliba 1983, 1985, 1986 and Mozaffari 2014. A monograph about his unique contribution to observational and practical astronomy at the Maragha observatory on the basis of a thorough analysis of his documented observations in the *Talkhīṣ al-Majisṭī* is in preparation by one of us (SMM).

[32] Al-Maghribī, *Talkhīṣ*, f. 58r. On the equinoctial days the true daily motion of the sun is approximately equal to its daily mean motion (about 0;59,8°); the time for the sun to travel 0;17° to reach the point of the autumnal equinox is calculated as 0;17/0;59,8 ≈ 6.9 hours. Note that half the length of an equinoctial day is equal to 6 hours, and so the time of the equinox counted from sunrise minus 6 hours yields the time elapsed from noon to the equinox: 6;54 – 6 = 0;54 hours or 0;2,15 days.

[33] The reasons why the two primary sources, i.e., the available MSS of the *Mumtaḥan zīj* and Ḥabash's *Kitāb al-ajrām wa 'l-ab'ād*, might be, in our estimate, not much more reliable than al-Maghribī's 13th-century *Talkhīṣ* lies in the following facts: the two known MSS of the *Mumtaḥan zīj* (see Yaḥya b. Abī Manṣūr in the bibliography at the end of the paper; also, e.g., van Dalen 2004; Kunitzsch 2003; Viladrich

how Yaḥyā could have been able to measure both the obliquity of the ecliptic and the longitude of the sun at sunrise on the equinoctial day by a single "circle" or "ring". There are only two possibilities: either the instrument was a circle suspended by a string or rotating about a vertical axis and could thus be used for observing altitude, zenith distance, the obliquity of the ecliptic, etc., or it consisted of a circle installed parallel to or on the ground and thus allowed measuring, for example, the azimuth of a celestial object when it was rising or setting.

We present a hypothesis on the nature of Yaḥyā's circular instrument based upon the assumption of Indian influence. As mentioned earlier, Brahmagupta's *Brāhmasphuṭasiddhānta* had already been translated into Arabic or adapted in Islamic astronomy in the early 'Abbāsid period. It contains detailed descriptions of some circular altitude-azimuthal instruments. The procedure for measuring the ortive amplitude of the sun by means of a horizontally installed circle, and then converting the result to the solar longitude is described in it as well as in other Indian sources in connection with circular altitude-azimuthal instruments. If this hypothesis were true, it can be said that Yaḥyā's "circle" is probably the first example of a horizontal instrument in Islamic astronomy.[34] However, further concrete data is needed to establish the structure and application of Yaḥyā's circular instrument with certainty.

Bīrūnī's Instrument

Bīrūnī's *al-Qānūn al-Mas'ūdī* VIII.14 in two chapters is devoted to the observation of the lunar crescent.[35] In the second chapter, the author explains how the azimuth of the new-moon should be calculated and then describes a telescopic-shape instrument by

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1988; Vernet 1956). were compiled after Ibn al-A'lam (d. 985 AD) and ultimately go back to the same recension of the *Mumtaḥan zīj*, presumably compiled in the tenth century (see van Dalen 2004, esp. p. 11); consequently, they belong to a time some two centuries after the original. Also, Ḥabash's treatise is extant in a Judeo-Arabic codex written not earlier than 1144 H/1731 AD (see Langermann 1985, pp. 108, 113). Thus, the possibility of scribal errors and other mistakes in these manuscripts is equally likely as that in the sources available to al-Maghribī.

[34] Under this assumption, Yaḥyā would have read off the ortive amplitude as ~0;10° towards the north from the azimuthal circle. The explanation is that the ortive amplitude η is computed as follows: $\tan \eta = \tan \delta / \cos \varphi$, where $\sin \delta = \sin \lambda \times \sin \varepsilon$, δ is the solar declination, λ the solar longitude, ε the obliquity of the ecliptic (23;33° or 23;35° as measured by Yaḥyā and his colleagues; see above, note 23), and φ the latitude of Baghdad (the historical value used at the time: 33;20° in agreement with the modern measurements). For $\lambda = 179;43°$, this yields $\eta = 0;8°$.

[35] About Bīrūnī's criteria for the visibility of lunar crescent, see Rizvi 1980; 1991.

means of which the finding and observation of a narrow lunar crescent is expected to be facilitated. The device is based upon the optical theory of visual radiation, according to which the visual flux is emitted from the eye.

In what follows, we present a translation of the passage in question:[36]

[I] "After the azimuth of the lunar crescent from the equinoctial west is known, a shaft (*rumḥ*) is set up on the circumference of the Indian circle pointing to its direction. The observer stands in the centre of the Indian circle and looks for the crescent (by observing) from the upper end of the shaft. By doing so, the eyesight is concentrated on it and the visual rays do not become dispersed (*mutafarriq*^an^). It will be easier to install another shaft at the centre of the Indian circle and look for the crescent in the direction of the upper ends of both shafts, that is, from a position at which the one covers another.

[II] "This work can also be done by a sighting tube (*barbakh*, lit. "pipe") that is installed on a perpendicular shaft and has two motions: the one made by the rotation of the perpendicular shaft around itself, so that the sighting tube rotates through the entire of the (azimuthal) directions; The other, the motion provided by a hinge/joint (*narmādhaja*)[37] by which it would be possible for the sighting tube to move in the plane of the circle of altitude in such a way that it would not go away from it. The length of the sighting tube should not be less than five cubits (i.e., ≥ 2.5 m) and its width (*sa'a*, i.e., the diameter of its circular base) should not be less than one cubit (i.e., ≥ 50 cm), so that the eyesight is concentrated (on the lunar crescent) in it and strengthened by its darkness and shadow, and it also can be further strengthened by blackening its inside surface. When the perpendicular shaft is set up at the centre of the Indian circle, it will be rotated around itself until the plumb line of the sighting tube reaches the line marking the azimuth of the lunar crescent. Then, the sighting tube is moved vertically in order to

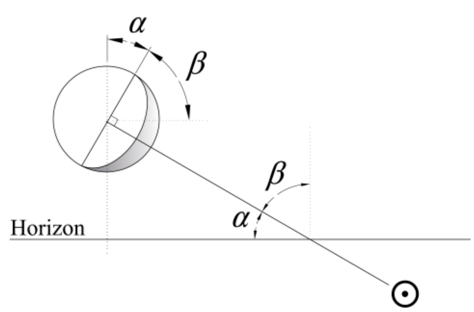
[36] Since the Hyderabad edition of Bīrūnī's *al-Qānūn al-Mas'ūdī* is not always trustworthy, we also made use of two MSS of this work available to us: (*i*) MS. Paris, Bibliothèque nationale de France, no. Ar. 6840, copied in Iṣfahān about the end of Ramaḍān 501 H/the first half of May 1108 AD (f. 205v), referred to henceforth with siglum **F**, which was used in the Hyderabad edition, and (*ii*) a late MS. preserved in the Staatsbibliothek zu Berlin, no. or. oct. 275 (= Ahlwardt 5667), dated to *ca.* 1250 H/1834 AD, marked hereafter with siglum **B**, which was *not* used in the Hyderabad edition. The most important variants of the Hyderabad edition and these two MSS. that are decisive in a better understanding of the text are noted in the Appendix. Bīrūnī 1954–6, Vol. 2, pp. 964–965; F: f. 137v; B: ff. 180v–181r.

[37] Etymologically, this term was Arabicized from the Persian verbal noun/gerund *narmādagī*/*narmādigī* that consists of *nar* ("male") + *mādah*/*mādih* ("female"), and that means "hinge", "joint", and any device used to lock doors and fasten clothes such as "staple-and-hasp", "latch", "snap", etc.

make an angle equal to the altitude of the crescent with the surface of the earth. This becomes easier by adding the quadrant of a circle graduated to 90º to the perpendicular shaft, so that it can rotate with the perpendicular shaft in parallel with the sighting tube. When the instrument is pointed to the crescent, as we described, the observer looks at it from the lower end of the sighting tube to the direction in the sky in which the crescent, the observation of which is possible, does not become faint […].

[**III**] "The two horns (*al-qarnayn*) of the crescent are also the evidence for it(s visibility). The line connecting the centres of the luminaries passes through the middle of the horns. The vertical tilt (*intiṣāb*) of the crescent is equal to the horizontal tilt of that line (*iḍṭijāʿ*), and the horizontal tilt (*istilqāʾ*) of the crescent is equal to the vertical tilt of that line."[38]

[38] This paragraph is not related to the instrument, but we have translated it here in order to clarify the mistakes and incorrect interpretations found in the entry "Barbakh" in the *Encyclopaedia Islamica* (in Persian), in which the author gives an account of this instrument on the basis of Bīrūnī's *al-Qānūn* (see Nāṭiq 1996). The paragraph [**III**] in Bīrūnī's account has been badly misunderstood and deformed in this entry as follows: "The two diametrically-opposed protrusions (*al-qarnayn*) were installed on the upper rim of the *barbakh* that could rotate around its axis. When looking for the crescent, the two protrusions were arranged in such a way that the imaginary line joining the two protrusions would be aligned with the line connecting the centres of the sun and moon. So, the place of the crescent was known, and an observer was looking for the crescent in the correct spot [in the sky]". This entirely incorrect explanation cannot be found in the original, where Bīrūnī solely refers to the horizontal and vertical alignments of the horns of the lunar crescent with respect to the line joining the two luminaries; what he intends to say can be simply illustrated with the figure below:



Another problematic interpretation in the entry in question occurs in connection with paragraph [**II**]. We are told: "[H]e (i.e., Bīrūnī) adds that another sighting tube is sometimes coaxially attached to the main sighting tube in order to facilitate the arrangement of the direction of vision." This unclear statement cannot be found in the original either.

Reconstruction and Technical Assessment

According to Bīrūnī, the horizontal coordinates, i.e., the altitude and azimuth, of the crescent are already computed, and then an observer attempts to find it in the sky.

In both methods proposed by our author in doing so, it is first necessary to draw the Indian circle, a circle on a plane ground with marks indicating the four cardinal points. From Bīrūnī's account, it makes sense to imagine that the azimuths are marked on the Indian circle by radii going from the centre to the graduated divisions of the circle.

The first way Bīrūnī puts forward, in paragraph [**I**], is the simple observation of the crescent by the aid of a shaft erected at a point on the circumference of the Indian circle that indicates the azimuth of the crescent, namely, this peripheral shaft points to the direction of the lunar crescent near the western horizon. In doing so, Bīrūnī posits the two options: in the first, the observer standing in the centre of the Indian circle looks for the crescent by fixing his sight on the upper end of the peripheral shaft (see Figure 1). An alternative is that another shaft is set up at the centre of the Indian circle, and then an observer looks through the upper ends of both shafts, i.e., the central and the peripheral shaft, in the fixed direction pointing to the crescent (see Figure 2).

The second method is making use of a specified device named *barbakh* (see Figures 3 and 4). It consists of a cylindrical tube larger than 2.5 m in length and 0.5 m in diameter, the inside surface of which is blackened in order to provide a dark environment to help sharpening the eyesight. With the specified dimensions, the field of view for the eye centered in the rear end is limited to $w = 2\tan^{-1}(0.25/2.5) = 11.42^{\circ}$.

The tube is attached by a hinge to the upper end of the central shaft erected in the centre of the Indian circle (see Figure 5), and has a motion with two degrees of freedom; one is provided by the rotation of the central shaft around itself (the base shown in the reconstruction is however not described in the text) and the other by the hinge connecting the sighting tube to the outer extremity of the central shaft. A plumb is drawn from the sighting tube which indicates its azimuthal position on the marked circle on the ground.

In order to ease putting the sighting tube towards the location of the crescent, we are told that it is equipped with a quadrant, which is attached to the central shaft, naturally in parallel with the sighting tube. The position of the quadrant as seen in Figure 3 and 4 is not specified in the text but was chosen for practical reasons. It is obvious that it should be mounted on the tube to indicate the altitude it is pointing at. The lunar crescent is expected in low altitudes, so that this position, where it would collide with the central shaft when pointed to high altitudes, causes no problem, else, it could also be mounted on the side of the tube. Bīrūnī gives no further data about the size of the quadrant and how it is connected with the central shaft. But, since it is said in the text that the quadrant should be graduated from 0º to 90º, a quadrant of radius of 50 cm (~ 1 cubit), as used in our reconstruction, would be sufficient for being engraved to each degree of arc because the space between every two adjacent degrees would amount to about 9 mm, which is practically enough for a secure reading.

For the operation, first, the sighting tube lying horizontally on the central shaft is rotated, so that the plumb on the end indicates the computed azimuth of the lunar crescent on the ground, and consequently, the sighting tube would be aligned with the azimuth of the crescent. Then, the tube is pointed upwards until the computed altitude is indicated on the quadrant. After the instrument is set up and positioned manually, an observer puts his head into the blackened sighting tube from its lower extremity and looks through its outer end in order to find the crescent with less distraction by stray light.

Conclusion

Bīrūnī's optical device for lunar crescent observation, which he described about the turn of the past millennium, is one of the two earliest examples of altitude-azimuthal instruments in Middle-Eastern medieval astronomy on whose structure and application we have clear information. The other similar (and contemporary) instrument was Ibn Sīnā's, which was used for the measurement of horizontal coordinates, and which has some basic features in common with the altitude-azimuthal instruments fabricated at the Maragha observatory in the mid-thirteenth century. Ibn Sīnā's instrument may thus be considered as the ancestor of the Marāgha ones, although there is no explicit evidence that al-'Urḍī actually employed Ibn Sīnā's instrument as a prototype for his own altitude-azimuthal instruments nearly three centuries later. We have also discussed that a horizontally installed circle may have served as an altitude-azimuthal instrument in the first systematic observations carried out by Yaḥyā b. Abī Manṣūr in Baghdad in the first half of the ninth century.

Bīrūnī's optical device has a peculiar property that makes it substantially distinct from the other medieval altitude-azimuthal instruments: we do not know of any similar medieval observational aid that was especially designed for resolving the difficulties in the observation of one specific celestial object. In this respect, since neither in Greek nor in Indian astronomy there was an urgent need to observe the lunar crescent, it does not appear to be far from the truth to assume that the *barbakh* was an original Islamic instrument and possibly Bīrūnī's own invention.

It is not known whether such an instrument was actually manufactured and employed in observation or remained solely a design, the execution of which was never tested in practice. Later astronomers appear to have been satisfied by sighting the lunar crescent with the alidade of a simple portable astrolabe or quadrant mounted on a perpendicular stick erected in the centre of the Indian circle (for example, Wābkanawī in his *Muḥaqqaq zīj* IV.9.5).[39]

Acknowledgements: The authors owe a debt of gratitude to Prof. Benno van Dalen, Prof. David King, and Prof. Julio Samsó for their kind help.

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[39] Wābkanawī, *Zīj*, T: f. 83r, P: f. 125r, Y: f. 145r. Wābkanawī gives here a thorough comparative analysis of the criteria put forward for lunar crescent visibility by his Muslim predecessors, and finally concludes that as he himself repeatedly observed the lunar crescent by the same method, i.e., using an astrolabe or portable quadrant installed on a stick, he reached the result that al-Khāzinī's criterion (*qawl*, lit. "saying") is closer to the truth, [because it gives better results] at some times when [the situations of] the crescent visibility computed on the basis of the other criteria (*a'māl*, lit. "operations"), especially that in the *Īlkhānī zīj*, are not in agreement with what was observed. Wābkanawī's interesting materials on crescent visibility will be discussed by one of us (SMM) in a separate paper.

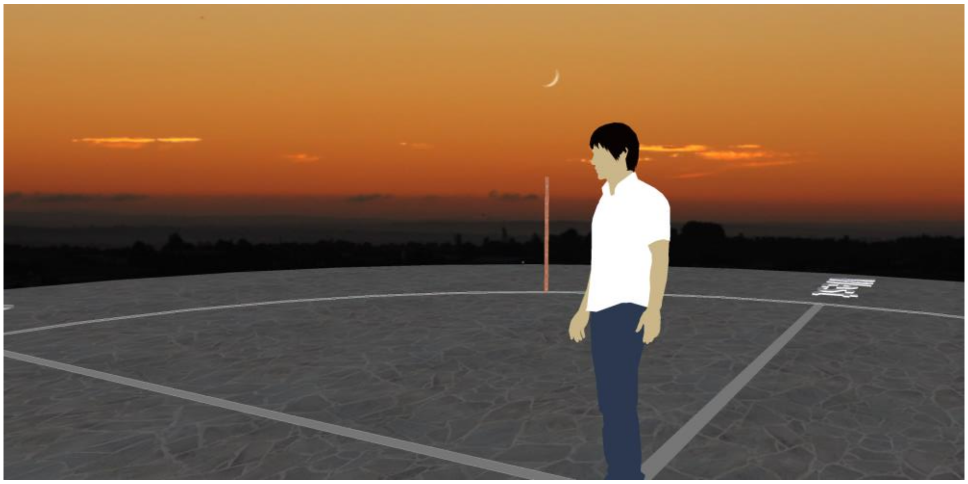

Figure 1: The observer has prepared a pole on the expected azimuth of the lunar crescent along the outer rim of the Indian Circle and observes from its center.

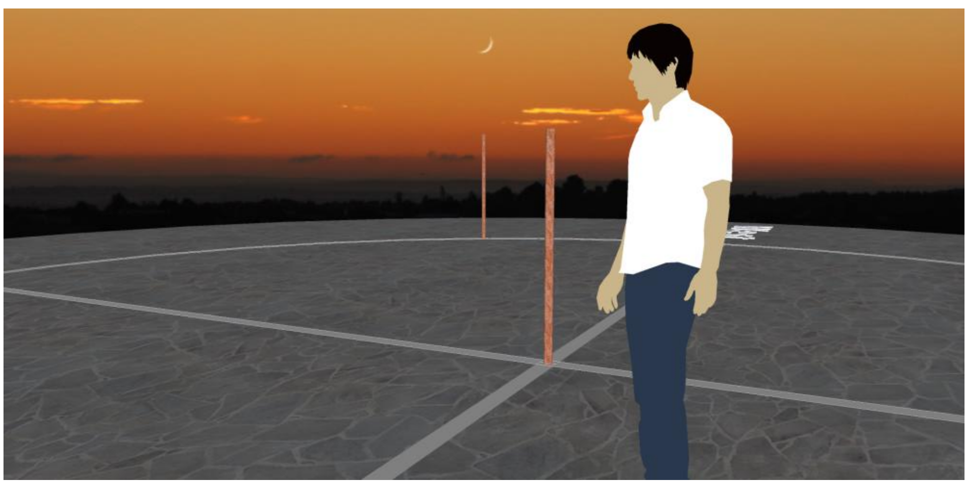

Figure 2: A more accurate line of sight can be achieved with another pole in the center of the Indian Circle and observing over both poles.

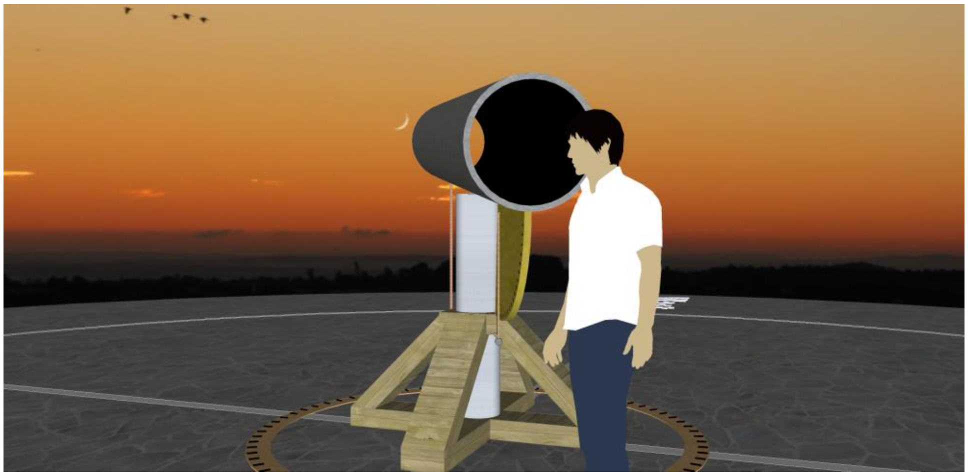

Figure 3: A possible reconstruction of Bīrūnī’s sighting tube.

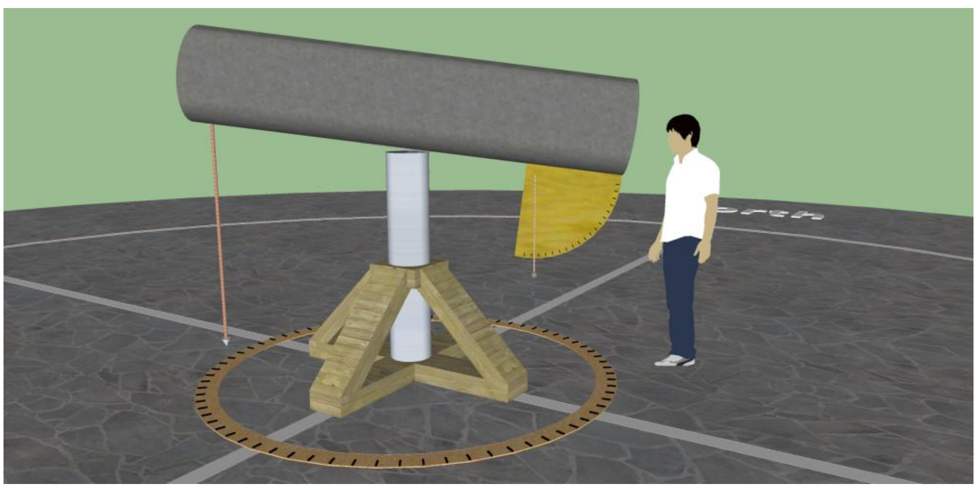

Figure 4: A possible reconstruction of Bīrūnī’s sighting tube.

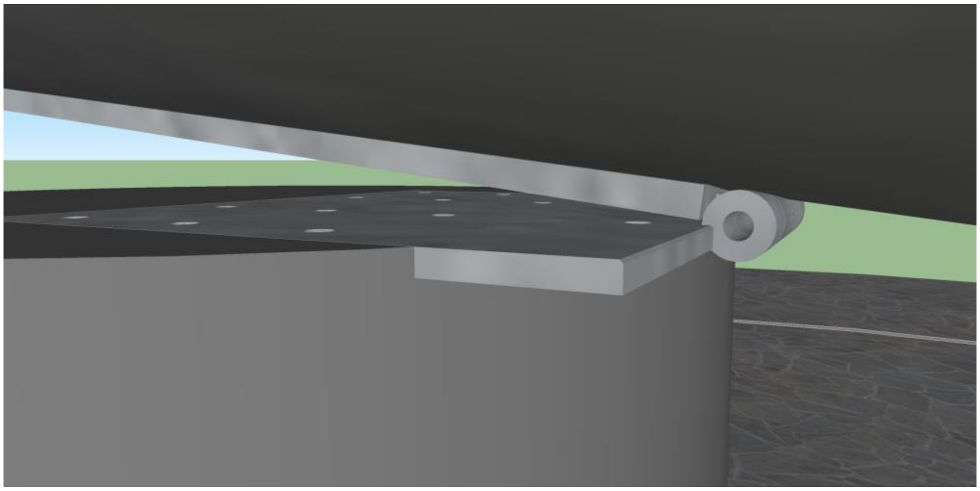

Figure 5: The hinge connecting the sighting tube to the central shaft.

Appendix: Bīrūnī's Text

بسم الله الرحمن الرحيم

القانون المسعودي لأبي الريحان البيروني

المقالة الثامنة

الباب الرابع العشر: فى رؤية الهلال

الفصل الثانى: فى سمت الهلال ورؤيته ونصب(1) البرخ عليه

H. Ed.: Vol. 2, p. 964. **MS. F:** f.137v. **MS. B:** f. 180v.

[I] [...] فسمت الهلال عن مغرب الاعتدال معلوم. فإذا(2) نصب عليه رُمح وكان الناظر فى مركز الدائرة وطلب الهلال على انتصابه، اجتمع البصر عليه ولم يذهب شعاعاً(3) متفرّقاً، بل(4) إن نصب رمح آخر على مركز الدائرة وطلب الهلال على مسامتة كلّيهما – أعنى، من الموضع الذّى يستر فيه أحدهما الآخر– كان أسهل.

[II] وعلى هذا عمل البرنخ[5] الّذى ينصب على عمود. له حركتان: إحداهما[6] على نفسه حتّى يدير البرنخ فى جميع الجهات؛ والأُخرى[7] بنرماذجة يمكن بها أن يحرّک[8] البرنخ[9] فى سطح دائرة الارتفاع الّذى هو فيها[10] لايزول عنه. وأمّا البرنخ فلايقصر عن خمسة أذرع وسِعَتُه عن ذراع ليجتمع[11] فيه[12] البصر ويقوى بظلّه وظلمته. ويزاد[13] فى ذلک بتسويد[14] جوفه من داخله. فمتى کان العمود منصوباً على مركز الدائرة الهندية، فأدير[15] على نفسه حتّى يحصّل شاقول البرنخ على خطّ سمت الهلال. ثمّ حرّک[16] بالحركة الأخرى حتّى أحاط البرنخ مع وجه الأرض بزاوية تساوى زاوية ارتفاع الهلال. وذلک سهل بربع دائرة مقسوم[17] بتسعين يضاف إلى العمود حتّى يدور معه فى موازاة البرنخ. وإذا نصب على الهلال كما وصفنا، ثمّ نظر الناظر| إليه من طرفه الأسفل إلى ما يسامته من السماء لم يخفّف فيه الهلال الممكن الرؤية [...].

MS. B: f. 181r.

[III] وأمّا قرناه فإنّها[18] أيضاً من الأدلّة عليه. والخطّ الواصل بين مركزي النيرين تمرّ بين القرنين.| فيكون انتصاب الهلال بقدر اضطجاع ذلک الخطّ واستلقاء الهلال بقدر انتصاب الخطّ. وذلک ماقصدناه.

H. Ed.: Vol. 2, p. 965.

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(1) H. Ed.: وقربه ونصف / MS. B: وقوسه ونصب | (2) H. Ed.: واذا | (3) MS. B: شعاعه | (4) H. Ed.: متفرقا قابلا | (5) H. Ed.: وعلى هذا البرنخ | (6) H. Ed.: احدهما / MSS. B & F: احديهما | (7) H. Ed.: والآخر | (8) MS. B: يمكن ان يحرّک بها / H. Ed.: تحرک | (9) H. Ed.: الزيج | (10) H. Ed.: فيما | (11) H. Ed.: يجتمع | (12) MS. B: فى | (13) MS. B: فيزاد | (14) H. Ed.: بالتسويد | (15) H. Ed. & MS. F: وادير | (16) MS. B.: حول | (17) H. Ed.: مقسومة | (18) MSS. B & F.: فانها.

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