

A Glimpse of Non-Ptolemaic Astronomy in Early *Hay'a* Work – Planetary models in ps. Mashā'allāh's *Liber de orbe**

Taro Mimura

Abstract: Recently I identified two Arabic manuscripts containing the Arabic original of a Latin work entitled *Liber de orbe* attributed to Māshā'allāh, and identified the title and author of the Arabic *Liber de orbe* as *Book on the Configuration of the Orb* written by Dūnash ibn Tamīm. This identification confirms that it is one of the earliest works on *'ilm al-hay'a* in Western Islam. In this article, I attempt to illustrate the details of its astronomical contents, and to determine its significance as an early *hay'a* work in Western Islam. The analysis reveals that although it explicitly refers to the name of Ptolemy, this work transmits non-Ptolemaic planetary system based on an eccentric-epicycle model. And by using a piece of the non-Ptolemaic materials that this work accidentally preserves in the name of Ptolemy as a criterion of determining what was the original achievement by Ptolemy, I show that one of his innovations was building a lunar model by using an epicycle model of the Sun and introducing an eccentric to it.

Keywords: Ps. Mashā'allāh's *Liber de orbe*, non-Ptolemaic astronomy, Theon of Smyrna, Calcidius, Pliny, Ptolemaic lunar model

1. Introduction

* This article is a revised version of my paper "Planetary models in pseudo-Mashā'allāh's *Liber de orbe* in the early *'ilm al-hay'a* tradition" presented at 24th International Congress of History of Science, Technology and Medicine, at Manchester, 24th July 2013. I am grateful to Julio Samsó and the two anonymous referees for giving many precious comments on an early draft.

Ptolemy's *Almagest* was translated into Arabic in the early ninth century C.E., and with recourse to this attainment, Islamic scholars worked on the quantitative determination of the planetary motion. They also articulated qualitative and physical cosmology by using composition of celestial spheres inspired by the Ptolemaic planetary models. This genre of astronomical research was called *'ilm al-hay'a*,¹ on which Qusṭā ibn Lūqā (c. 820–c. 912/3) is one of the earliest authors. This astronomical genre was popularized by Ibn al-Haytham (965–c. 1040)'s *On the Configuration (hay'a) of the World*, and later it was standardized by al-Khiraqī (d. 1138/9) in *Al-Tabṣira fī 'Ilm al-hay'a* and then by Naṣīr al-Dīn al-Ṭūsī (1201–1274) in *Tadhkira fī 'ilm al-hay'a*. After the appearance of the *Tadhkira*, it became a main stream of astronomy in the Islamic world, and quite a few scholars wrote books on it, where they focused on the qualitative features of the cosmos, deliberately skipping astronomical quantitative determination usually written in *zīj*es, namely, astronomical handbooks with tables.²

Whereas we have wealthy information on *'ilm al-hay'a* after the time of Ṭūsī, research on its history in the pre-Qusṭā ibn Lūqā days is not an easy task, because of the scarcity of available documents, most of which are fragmentary; however, there is a book containing rich contents of physical cosmology attributed to a scholar flourishing before the days of Qusṭā: that is, *Liber de orbe* ascribed to Māshā'allāh (d. c. 815), a court astrologer in the Abbasid dynasty.

The main topic of the *Liber de orbe* is not astrology but cosmology, including elements in the sublunary world, meteorology, geology, and astronomy. The author structures his arguments by describing the mechanism of sublunary phenomena with recourse to the theory of the elements, while explaining superlunary phenomena with geometrical reasoning, using plenty of diagrams. The reason for his thoroughly logical explanation of all phenomena lies in his ambition to show how rational the construction of the World is, which in turn proves that its creation was impossible except by the wisest God.

Until recently, the *Liber de orbe* had been known in two Latin versions: the long version consisting of 40 chapters and the short version consisting of 27 chapters.³ This Latin translation was made in the 1130s, and became one of the

¹ On this genre, see F. Jamil Ragep, *Naṣīr al-Dīn al-Ṭūsī's Memoir on Astronomy*, 2 vols., New York: Springer-Verlag, 1993, vol. 1, pp. 29–46.

² E. S. Kennedy, "A Survey of Islamic Astronomical Tables", *Transactions of the American Philosophical Society*, vol. 46 part 2 (1956), pp. 123–177 gives a survey of the *zīj*es. An updated version is found in D. A. King, J. Samsó and B. R. Goldstein, "Astronomical Handbooks and Tables from the Islamic World (750–1900): an Interim Report", *Suhayl* (2001) 2, pp. 9–105.

³ For an overview of the *Liber de orbe*, see Barbara Obrist "William of Conches, Māshā'allāh, and Twelfth-Century Cosmology", *Archives d'histoire littéraire et doctrinale du Moyen Age* (2009) 76,

earliest Latin sources of Aristotelian physics. In fact, William of Conches (1090–after 1154)⁴ and the anonymous author of *De secretis philosophie*,⁵ a twelfth-century cosmography, utilized and paraphrased a part of it to compose their physical and cosmological sections. However, no one had been able to locate its Arabic original. Since this work had long been considered as remaining only in the Latin translation, its importance as an early *‘ilm al-hay’a* work had not been well-recognized, probably except for David Pingree and F. Jamil Ragep.⁶

In the course of examining volumes of Arabic codices on exact sciences, however, I identified two manuscripts containing its Arabic original: Berlin, Staatsbibliothek zu Berlin, Ms. or. oct. 273 (henceforth MS B), and Philadelphia, Pennsylvania University Library, MS LJS 439 (henceforth MS P).⁷ By

pp. 29–87; Barbara Obrist, “Twelfth-Century Cosmography, the *De secretis philosophie*, and Māshā’allāh (attr. to) *Liber de orbe*”, *Traditio* (2012) 67, pp. 235–276.

⁴ See Barbara Obrist, “Guillaume de Conches: cosmologie, physique du ciel et astronomie. Textes et images”, in B. Obrist and I. Caiazzo (eds.), *Guillaume de Conches: philosophie et science au XIIIe siècle*, Micrologus Library 42, Florence: SISMEL, 2011, pp. 123–196.

⁵ See Barbara Obrist, “Twelfth-Century Cosmography, the *De secretis philosophie*, and Māshā’allāh (attr. to) *Liber de orbe*”, *Traditio* (2012) 67, pp. 235–276.

⁶ See David Pingree, “Māshā’allāh: Some Sasanian and Syriac Sources”, in F. Hourani (ed.) *Essays on Islamic Philosophy and Science*, Albany: State University of New York Press, 1975, pp. 5–14; Ragep, *Naṣīr al-Dīn al-Ṭūsī*, vol. 1, pp. 29–30.

⁷ Taro Mimura, “The Arabic Original of (ps.) Māshā’allāh’s *Liber de orbe*: its date and authorship”, *The British Journal for the History of Science*, 48 (2015), pp. 321–352 describes the details of these two manuscripts and the identification of the author of the Arabic *Liber de orbe*. The following paragraphs are a summary of it. In this article, when I quote a text from *ArLO*, I critically edit it based on MSS B and P, and note variant readings at the bottom of it, where I use the following sigla:

[Separates reading in the text from the variant
:	Separates variant and manuscript sigla
+	Added in
–	Missing from
[...]	Indicates a damaged and unreadable part
()	My comments
ب	MS B
ف	MS P
طا	مطموس (unreadable)
ها	هامش (margin)

As I described in Mimura, “The Arabic Original”, pp. 333–335, I generally follow MS B, which contains the complete set of the chapters with minor modification, and emend its scribal errors by comparing it with MS P. Henceforth, I indicate, for example, MS B, folio 1a, lines 1–2 by “B: 1a, 1–2”, and MS P, page 1, lines 1–2 by “P: 1, 1–2” (for MS P has page numbers instead of folio numbers). Note that MS P does not have the entire text of *ArLO*. When I emend a reading of a text only preserved in MS B, I note the original reading at the bottom of the text.

comparing preliminarily the two Arabic manuscripts with the Latin long version and short version, I showed that these two Arabic manuscripts contain the Arabic original of this Latin work, and that the short version is a truncated version of it, while the long version is paraphrased from it. And by analyzing the contents of the Arabic *Liber de orbe* (henceforth *ArLO*), I denied its attribution to Māshā'allāh, and identified its author as Dūnash (or Dūnas) ibn Tamīm, who was a disciple of Isaac Israeli (c.855–c.955), a Jewish physician and philosopher in the Fatimid court.⁸

The biographical information of Dūnash is very scarce, but two of his works are descended to us, that is, *Treatise on Armillary Sphere*, and *Commentary on the Sefer Yezira*. The *Treatise on Armillary Sphere* is extant only in Istanbul, Ayasofya MS 4861.⁹ In this work, he describes in great details how to construct and operate an armillary sphere. We are also informed from the *Commentary on the Sefer Yezira* that he wrote other astronomical works.

The *Sefer Yezira* “Book on Creation” is a Hebrew esoteric book on cosmogony, and several scholars wrote commentaries on it.¹⁰ Among them, the extant earliest is the Arabic commentary by Saadia Gaon (884–942), a contemporary with Isaac Israeli.¹¹ Dūnash wrote the next earliest commentary in Arabic. However, we do not have a complete Arabic manuscript of it. Fortunately, about 75 percent of the text is salvaged by George Vajda and Paul Fenton from Judeo-Arabic documents, and the whole text is transmitted in several Hebrew versions.¹²

In this work, Dūnash comments on the passages of the *Sefer Yezira* one by one. We must remark that he gives some information about himself, especially in the commentary on 1.5,¹³ which is only extant in the Hebrew translations.¹⁴

In the middle of this part, Dūnash comments on the passage “the limit of good and the limit of bad”, and remarks that good and bad are not substances but attributes, and he declares the oneness of God. Then he refutes the dualists who believe the existence of the good and the bad as the light and the dark, by using

⁸ On his biography, see Georges Vajda (ed. by Paul B. Fenton), *Le Commentaire sur le Livre de la Création de Dūnaš ben Tāmīm de Kairouan (Xe siècle)*, Leuven: Peeters, 2002, pp. 3–6.

⁹ S.M. Stern, “A Treatise on the Armillary Sphere by Dunas ibn Tamīm”, In *Homenaje a Millás-Vallcrosa*, 2 vols., Barcelona: Consejo Superior de Investigaciones Científicas, vol. 2, pp. 373–382, gives the Arabic text of its preface.

¹⁰ A. Peter Hayman, *Sefer Yezira: edition, translation and text-critical commentary*, Tübingen: Mohr Siebeck, 2004.

¹¹ Vajda, *Le Commentaire sur le Livre de la Création*, pp. 11–12.

¹² Vajda, *Le Commentaire sur le Livre de la Création*, pp. 10–20.

¹³ On this passage of the *Sefer Yezira*, see Hayman, *Sefer Yezira*, pp. 74–76.

¹⁴ Vajda, *Le Commentaire sur le Livre de la Création*, pp. 56–69.

the example of lunar phases which show that light is produced by the Sun and dark occurs when something blocks the light. This example is concluded by the following note:¹⁵

We have already explained this [i.e., lunar phases] and have put figures about it in our book which we composed and sent to Abū Yūsuf Ḥasdāy (c.915–c.975) to reply to the questions which reached us from the city Constantinople. It consists of three parts: the first part is on the science of the configuration of the orbs (*'ilm al-hay'a*); the second part is on the knowledge of the orbs according to calculation; and the third is on the judgement of stars [i.e. astrology].

In this note, he explicitly mentions one of his astronomical works containing a section on *'ilm al-hay'a*.

Afterwards, Dūnash comments on the passage “the limit of east and the limit of west”. Here he focuses on the unreality of the east and the west, because there are unlimited terrestrial points, each of which has each east and west. Then he adds the following remark:¹⁶

Therefore, we have said in our treatise entitled “on the weakness of the principles of the judgement of stars [i.e. astrology]”: As for constellations determined among stars, the truth cannot be found in them. Even if a scholar on geometry and the science of the orbs draws a horoscope according to a place and it is adjusted in detail, the east cannot be set according to the region of this horoscope because of various reasons, and surely if one wants to know the region of this place and its east. This subject is that which is in this treatise; namely, if it were repeated here, it would become long. This treatise is the second section of our *Book on the Configuration of the Orbs*, which we have written for al-Manṣūr Ismā'īl ibn al-Qā'im.

In this note, he tells us that he explained the unreality of the determination of constellations (i.e. zodiacal signs) in the treatise on the weakness of the principles of astrology, which is the second chapter of *Book on the Configuration of the Orbs* dedicated to the third Fatimid Caliph, al-Manṣūr (r. 945–952). As is evident from its title, this book is a work on *'ilm al-hay'a*.

These quotations show that Dūnash wrote at least two books containing *'ilm al-hay'a*. Moreover, what is remarkable is that the contents of his comments on 1.5 indicate Dūnash's sharing some distinctive opinions of the author of *ArLO*:

¹⁵ The text is found in Georges Vajda, “Le commentaire kairouanais sur le «Livre de la Création» [1]”, *Revue des études juives* 107 (1946–7), pp. 99–156, p. 147, lines 19–23.

¹⁶ Vajda, “Le commentaire kairouanais”, p. 146, lines 7–14.

1) Dūnash defends “the creation of the world by the one God” and rejects opinions contrary to his thesis, such as the dualism; this is also the main thesis of *ArLO*.

2) He has explained lunar phases with a diagram in one of his books on *‘ilm al-hay’a*; *ArLO* also has a section (Chapter 19) on lunar phases with a diagram.

3) He is sceptical about astrology, and his remark on “the unreality of constellations” is comparable to Chapter 29 of *ArLO*, where the author describes that the places of the zodiacal signs in the orb of the zodiacal signs are not actual.

These similarities make Dūnash a promising candidate of the author of *ArLO*. As elucidated in great detail,¹⁷ comparison between Dūnash’s two works and *ArLO* confirms Dūnash’s authorship of *ArLO*.

Criticism to astrology in *ArLO* is most evident in the following part of Chapter 14:¹⁸

“However, if someone says and claims that the orb has the four natures, many predecessors on astronomy and our contemporary scholars who demand astronomy say about the twelve zodiacal signs: Aries, Leo, and Sagittarius are in the same nature of the fiery zodiacal signs, [namely] hot and dry; Taurus, Virgo, and Capricorn are in the same nature of the earthy zodiacal signs, [namely] cold and dry; Gemini, Libra, and Aquarius are in the same nature of the airy zodiacal signs, [namely] hot and moist; and Cancer, Scorpio, and Pisces are in the same nature of the watery zodiacal signs, [namely] cold and moist”. They also say, “Some of them are tropical; some of them are fixed; some of them are bicorporeal; some of them are masculine; and some of them are feminine”, and [say], “some of them are luminous, and some of them are dark”, and [say], “some of the five planets are benefic; some of them are malefic; some of them are benefic when they are accompanied with benefic [planets], and they are malefic when they are accompanied with malefic [planets]”. Indeed, we see the Sun being blackened, burnt, and dried; if the Sun were not hot in itself in actuality, it could not do that, because just as we see fire being burnt and blackened by its heat in actuality, the Sun is like that. We see the Moon being moistened and putrefying; if that [i.e., the Moon] were not like that in its nature, it could not do that.

Then we say: as for the matter about the zodiacal signs and stars that you describe, if this is said about them, they are neither hot in themselves nor cold nor wet nor dry nor light nor heavy nor benefic nor malefic, even if they [i.e., these scholars] say these concepts when they manifest relationships of them [i.e. the zodiacal signs and stars], what they indicate by a different relationship, and what [ordinary] people receive from them.

¹⁷ Mimura, “The Arabic Original”, pp. 347–352.

¹⁸ The Arabic text is found in Mimura, “The Arabic Original”, p. 341.

We will give an explanation about each of them [i.e. the astrological notions] and its characteristics, and we will tell the reason why they mention them, in great details in a more specific place, if God—may He be exalted—will”.

In this quotation, he explains that many scholars use astrological concepts as metaphors to convince ordinary people, and he ends this section with a promise to explain these astrological concepts in a later section; however, we cannot find a relevant section in *ArLO*. We must remark that this is unusual for *ArLO*, because all the topics promised to describe are explained except for this astrological matter. This anomaly leads us to think that *ArLO* is a part of a large work, lacking a section on astrology. Given that the only sacred phrase in *ArLO* quoted to verify the main thesis “the existence of the one God” is from the Qur’ān,¹⁹ this work was probably written for Muslims. Consequently, the above analysis gives us a promising candidate of the author and the title of *ArLO*: that is, Dūnash ibn Tamīm, *Book on the Configuration of the Orbs* (dedicated to al-Manṣūr), although it lacks the astrological section “on the weakness of the principles of astrology”.

Suggested by the title (*Book on the Configuration of the Orbs*), *ArOL* is clearly a book on *‘ilm al-hay’a*. In fact, the author shows the physical aspect of the cosmos in great details with a number of geometrical diagrams, but he does not mention how to calculate astronomical quantities by using them. And at the end of Chapter 20 (entitled “Discourse on lunar eclipses”),²⁰ he remarks the absence of the quantitative description in this book as follows:²¹

ولو أنا لم نقصد في كتابنا هذا غير تفسير العلة من أجلها ينكسف القمر، لا وضحنا وجوب كسوفه على ما هو في الأزياج.

(“If we only aim in our book to explain the cause by which the Moon is eclipsed, we do not describe the necessity of its eclipse according to what is in *zījes*”).

This quotation confirms his conscious concentration on the qualitative explanation. What is remarkable is that *ArLO* is distinctively characteristic as a *‘ilm al-hay’a* work, because whereas books of this genre normally exclude

¹⁹ See Mimura, “The Arabic Original”, p. 339.

²⁰ Since in both of MSS B and P each chapter has the heading without numbering, I number the chapters for the sake of convenience. Mimura, “The Arabic Original”, pp. 325–331 lists the whole of the chapter headings.

²¹ B: 40b, 2–4.

subjects on physics and meteorology, it covers more comprehensive topics about the cosmos to show its creation by the one God.

The identification of *ArLO* reveals its importance in the history of astronomy. This work is definitely one of the earliest works on *'ilm al-hay'a* in Western Islam. In fact, as far as we know, there is no *hay'a* work in Western Islam before *ArLO* except the *Kitāb al-Hay'a* written by Qāsim ibn Muṭarrif al-Qaṭṭān, a Cordoban scholar contemporary with Dūnash.²² Thus, this work provides us with valuable materials for investigating the formation of the *'ilm al-hay'a* tradition in Western Islam. In this article, I attempt to illustrate the details of its astronomical contents by analyzing the Arabic original, and to determine its significance as an early *hay'a* work in Western Islam.

2. Astronomy in *ArLO*

Among the 39 chapters of *ArLO*, the following 22 are on astronomy:

Chapter 13: Discourse on the roundness of the orb and its motion and nature

الكلام في تدوير الفلك وحركته وطبعه

Chapter 14: Account and discourse on the revolution of the orb and the Sun

القول والكلام في دوران الفلك والشمس

Chapter 15: Discourse on the fact that any change does not reach the orb in its essence and in its motion

الكلام¹ في أنه لا يلحق الفلك تغيير² في ذاته ولا في حركته
¹ الكلام [ب = طاف² أنه لا يلحق الفلك تغيير] ب = انَّ الْفَلَكُ لَا يَلْحَقُهُ تَغْيِيرٌ: ف.

Chapter 16: Discourse on circles, chords, and points

الكلام في الدوائر والأوتار والنقط

Chapter 17: Discourse on the difference of the Sun's rising and setting in [various] countries

الكلام في اختلاف طلوع¹ الشمس ومغيبها على البلدان
¹ اختلاف طلوع [ب = ا | ... | و:ع: ف.

Chapter 18: Account on the knowledge of the Sun's magnitude

القول في معرفة عظم الشمس

Chapter 19: Discourse on the Moon's borrowing of the Sun's light

الكلام في استعارة القمر الضياء من الشمس

Chapter 20: Discourse on lunar eclipses

²² On this work, see Josep Casulleras, "The Contents of Qāsim ibn Muṭarrif al-Qaṭṭān's *Kitāb al-hay'a*", in M.I. Fierro and J. Samsó (eds.), *The Formation of al-Andalus, Part 2: Language, Religion, Culture and the Sciences*, Aldershot; Brookfield, Vt.: Ashgate, 1998, pp. 339-358. I am grateful to Julio Samsó for reminding me of the importance of this work and giving this reference.

الكلام في كسوف القمر

Chapter 21: Discourse on planets' borrowing of the Sun's light

الكلام في استعارة النجوم الضياء¹ من الشمس
¹ [الضياء] الضياء: ب.

Chapter 22: Discourse on the cause of solar eclipses

الكلام في علّة كسوف الشمس¹
¹ علّة كسوف الشمس] ب = كُسُوفِ الشَّمْسِ وَ عَلَّةٌ ذَلِكَ: ف.

Chapter 23: Discourse on why the Moon becomes visible to some people and does not become visible to others and why it appears small or large

الكلام في القمر لِمَ صار يراه قوم ولا يراه آخرون ولم يستهلّ صغيراً أو كبيراً

Chapter 24: Account on the difference of the Moon's light and stars' light in [various] cities

القول¹ في اختلاف ضياء القمر والنجوم على المدائن²
¹ القول] ب = الكلام: ف ² المدائن] ب = اهل المدائن: ف.

Chapter 25: Discourse on the number of the Moon's orbs

الكلام في عدد¹ أفلاك القمر
¹ عدد] ب = عدّة: ف.

Chapter 26: Discourse on the two orbs of the Sun

الكلام في فلكي الشمس

Chapter 27: Discourse on the number of the orbs and their motions

الكلام في عدّة الأفلاك وحركاتها

Chapter 28: Discourse on the motion of the greatest orb

الكلام في حركة¹ الفلك الأعظم
¹ حركة] ب = جزيّة: ف.

Chapter 29: Discourse on the orb of the zodiacal signs

الكلام في فلك البروج

Chapter 30: Discourse on the alteration of the natures of the seasons

الكلام في تبديل طبائع الأزمان¹
¹ الأزمان] ف = الزمان: ب.

Chapter 31: Discourse on the orbs of Saturn

الكلام في أفلاك زحل

Chapter 32: Discourse on Saturn's retrogradation and its returning into the zodiacal sign from which it has left

الكلام في قهقرة زحل وانصرافه إلى البرج¹ الذي خرج منه²
¹ البرج] ف = ب - منه² + من البروج: ب.

Chapter 33: Discourse on the orb of the fixed stars

الكلام في فلك النجوم الثابتة

Chapter 34: Discourse on the knowledge of how large the whole Earth is in miles

الكلام في علم كم من ميل في الأرض كلها

In these chapters, *ArLO* refers to only one authority, Ptolemy. Throughout this work, Dūnash quotes three statements of Ptolemy.²³ One of them is found at the beginning of Chapter 27:²⁴

قال بطلميوس إننا نرى في السماء حركتين مختلفتين إحداهما من المشرق إلى المغرب والأخرى من المغرب إلى المشرق. والتي من المغرب إلى المشرق يتفق فيها جميع حركات¹ الكواكب والنيرين² على قدر سعة³ أفلاكها، والحركة التي⁴ من المشرق⁵ تسوق جميع هذه الكواكب⁶ على خلاف حركاتها التي هي⁷ من المغرب إلى المشرق وهي حركة الفلك الأعظم.
¹ حركات [ف = حركات: ب² والنيرين] والنسراحين: ب = والشمس والقمر: ف³ سعة [ف = سرعه:
 ب⁴ والحركة التي] ب = والتي: ف⁵ من المشرق] + الي المغرب: ف⁶ هذه الكواكب] + والشمس
 والقمر: ف⁷ هي] ب = ف.

(“Ptolemy said: we see in the sky two motions. One of them is from east to west, and the other is from west to east. In the [motion] from west to east, all motions of the stars and the two luminaries are in accordance with the widths of their orbs, whilst the motion from east carries all these stars contrary to their motions from west to east, that is, the motion of the greatest orb”).

The idea described in the first part of this quotation can be found in the *Almagest* i. 8 (entitled “That there are two different primary motions in the heavens”),²⁵ where Ptolemy explains the motion according to the equator and the motion according to the ecliptic. But in the second part (“all motions of the stars and the two luminaries are in accordance with the widths of their orbs”), Ptolemy seems to declare the proportionality of the rotation period of a planet and the width of its orb.²⁶ Whilst this thesis on the planets’ constant velocity was employed by several early Greek astronomical works, such as Cleomedes (fl. ca. 200)’ *On the Heavens* (ii. 1) and Geminus (the first century B.C.)’ *Introduction*

²³ I.e., two in Chapter 27, and one in Chapter 29.

²⁴ B: 48a, 2–7; P: 17, 2–5.

²⁵ Greek text: J.L. Heiberg (ed.), *Claudii Ptolemaei Opera quae exstant omnia*, Lipsiae: B.G. Teubner, 1898-1903, 2 vols, vol. 1, pp. 26-27; English translation: G.J. Toomer (tr.), *Ptolemy’s Almagest*, London: Duckworth, 1984, pp. 45-46.

²⁶ I.e., all the planets move the same distance in an equal period.

to the *Phenomena* (i. 19),²⁷ he never adopted such hypothesis, so the attribution of this statement to him cannot be true.²⁸

After this quotation, however, Dūnash illustrates this thesis, and then he quotes Ptolemy's explanation utilizing layers of different pulleys (*bakra*), as follows:²⁹

وقد ضرب في هاتين الحركتين بطلمبيوس مثلاً¹ فقال²: لو أنّ بكرة تجري من المشرق في كلّ يوم وليلة دورة³، ثمّ تكون⁴ في البكرة دائرة صغيرة قريبة⁵ من وسطها، وفوقها⁶ دائرة مثل ضعفيها⁷، ثمّ أخرى⁸ مثل ثلاثة أمثال الأولى⁹، ثمّ دائرة¹⁰ مثل أربعة أمثال الأولى¹¹، ثمّ دائرة¹² مثل خمسة أمثال الأولى¹³، ثمّ دائرة¹⁴ مثل ستة أمثال الأولى¹⁵، ثمّ دائرة¹⁶ مثل سبعة أمثال الأولى¹⁷، ثمّ دائرة¹⁸ مثل ثمانية أمثال الأولى¹⁹، ويكون في²⁰ كلّ دائرة من هذه الدوائر دائرة²¹ تدور من المغرب إلى المشرق في دائرتها، والبكرة تدور²² من المشرق إلى المغرب في يوم وليلة دورة²³ كاملة وهي حركة الفلك الأعظم، فإذا دارت²⁵ الأولى دائرتها مرّة، دارت²⁶ التي تليها نصف دورتها²⁷، ثمّ²⁸ الثالثة ثلث دورتها²⁹، ثمّ الرابعة ربع دورتها³⁰، ثمّ الخامسة خمس دورتها³¹، ثمّ السادسة سدس دورتها³²، ثمّ السابعة سبع دورتها³³، ثمّ الثامنة ثمن دورتها³⁴. فإذا دارت الثامنة دائرتها كلّها، دارت³⁵ الأولى دائرتها ثمانية³⁶ مرّات. وفي دوران هذه الثمانية³⁷ حركات تدار مراراً³⁸ إلى المغرب، وهي في ذاتها³⁹ تجري إلى المشرق وتقطع علامة بعد علامة⁴⁰ من البكرة. فإذا كملت كلّ دور فلكها، ابتدأت بدور ثاني⁴¹.

¹ وقد ضرب في هاتين الحركتين بطلمبيوس مثلاً] ب = واضع في هاتين الحركتين الثنتين من المشرف إلي المغرب ومن المغرب الي المشرف مثلاً ضربه بطلمبيوس ليهنّدي به الي فهم ما حكيناها ان شاء الله: ف² فقال] ب = بيفول: ف³ دورة] ب = دؤراً: ف⁴ تكون] ف = يكون: ب⁵ قريبة] قوبيه: ب = قريباً: ف⁶ وفوقها] ف = وفودها: ب⁷ ضعفيها] ف = ضعيفها: ب⁸ أخرى] ب = فوفها دايرة: ف⁹ الأولى] ب = الدايرة الاولي الصغيرة: ف¹⁰ دائرة] ب = فوفها دايرة تكون: ف¹¹ الأولى] ب = الدايرة الاولي: ف¹² دائرة] ب = فوفها دايرة تكون: ف¹³ الأولى] ب = الدايرة الاولي: ف¹⁴ دائرة] ب = فوفها دايرة تكون: ف¹⁵ الأولى] ب = الدايرة الاولي: ف¹⁶ دائرة] ب = فوفها دايرة تكون: ف¹⁷ الأولى] ب = الدايرة الاولي: ف¹⁸ دائرة] ب = فوفها دايرة تكون: ف¹⁹ الأولى] ب = الدايرة الاولي: ف²⁰ ويكون في] ب = ثم من فؤف: ف²¹ دائرة] ف = دوره: ب²² تدور] ب = ف دورة] ف = دورا: ب²⁴ فإذا] ب = وإذا: ف²⁵ دارت] ف = ادارت: ب²⁶ دارت] ف = داره: ب²⁷ دورتها] ب = دايرتها: ف²⁸ ثم] ب = ف دورتها] ب = دايرتها: ف³⁰ دورتها] ب = دايرتها: ف³¹ دورتها] ب = دايرتها: ف³² دورتها] ب = دايرتها: ف³³ دورتها] ب = دايرتها: ف³⁴ دورتها] ب = دايرتها: ف³⁵ دارت] ب = ففد دارت: ف³⁶ ثمان] ف = ثمان: ب³⁷ الثماني] ف = الثمان: ب³⁸ تدار مراراً] تُدارُ «بحل» زاند في الهامش مع رمز «صح» مراراً: ف = يدارها: ب³⁹ ذاتها] ب = في حركاتها: ف⁴⁰ علامة] + وشيا بعد شيء: ف⁴¹ فإذا كملت كلّ دور فلكها، ابتدأت بدور ثاني] ب = حتي تُدور كلّ دايرة منها دؤرها ثم تبندئ

²⁷ See Alan C. Bowen and Robert B. Todd, *Cleomedes' Lectures on Astronomy: A Translation of The Heavens with an Introduction and Commentary*, Berkeley: University of California Press, 2004, p. 116, footnote 70.

²⁸ Pingree, "Māshā'allāh", p. 11 already pointed out the peculiarity of the attribution of this thesis to Ptolemy.

²⁹ B: 49b, 4–50a, 2; P: 7, 8–8, 2.

دَوْرَانِ ثَانٍ بِهِدَا بَيَانٌ لِمَا بَسْرَنَاهُ وَشَرَحْنَاهُ مُخْتَصَرًا وَهَكَذَا جَرِيَةُ الْكَوَاكِبِ وَالشَّمْسِ وَالْقَمَرِ مِنَ الْمَغْرِبِ
الِي الْمَشْرِفِ وَجَرِيَةُ الْعَالَمِ الْأَعْظَمِ وَفَلَكِ الْبُرُوجِ مِنَ الْمَشْرِفِ وَالِي الْمَغْرِبِ وَلَا قُوَّةَ إِلَّا بِاللَّهِ: ف.

(“Ptolemy gave an example about these two motions³⁰ [i.e. the ecliptic and equatorial motions], and said: if a pulley rotates from east one rotation in every day and night, and a small circle is on the pulley in the vicinity of its middle, and on the [small circle] is a circle equal to the double of it, and [on it] is another [circle] equal to the three times of the first [circle], and [on it] is a circle equal to the four times of the first, and [on it] is a circle equal to the five times of the first, and [on it] is a circle equal to the six times of the first, and [on it] is a circle equal to the seven times of the first, and [on it] is a circle equal to the eight times of the first, and on each of these circles is a circle rotating from west to east on its circuit, while the pulley rotates from east to west during day and night one complete rotation, that is, the motion of the greatest orb, and when the first [circle] rotates once on its circuit, the [circle] adjacent to it rotates a half of its rotation, and the third [circle] rotates a third part of its rotation, and the fourth rotates a fourth part of its rotation, and the fifth rotates a fifth part of its rotation, and the sixth rotates a sixth part of its rotation, and the seventh rotates a seventh part of its rotation, and the eighth rotates an eighth part of its rotation. When the eighth [circle] rotates completely on its circuit, the first rotates on its circuit eight times. In the rotation of these eight motions, they [i.e. the circles] are rotated to west many times, while they rotate to east by themselves and cut a sign after a sign of the pulley. When they complete each rotation of their orbs, they begin to rotate the second time”).³¹

Again this quotation cannot be derived from Ptolemy, because he had no need of explaining this thesis.

The above two quotations confuse us about Dūnash’s source: although he declares Ptolemy’s name as his authority, his source definitely contains a non-Ptolemaic doctrine. To determine more clearly the relationship between *ArLO* and Ptolemy, let us look into the planetary models described by him and try to find the source.

³⁰ “Ptolemy gave an example about these two motions”: MS P has the following: “I will give an example offered by Ptolemy about the two motions from east to west and from west to east in order that one is directed to understand what we have said, if God will.”

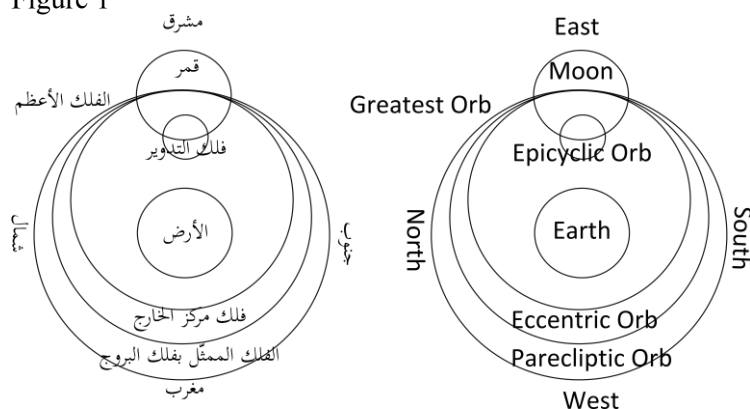
³¹ “When they complete each rotation of their orbs, they begin to rotate the second time”: MS P has the following: “until each circle rotates its rotation, and then it begins to rotate the second time. This is clear from what we have elucidated and explained briefly. The motion of stars, the Sun and the Moon from west to east and the motion of the greatest orb and the orb of the zodiacal signs from east to west are like that. There is no power except by God.”

3. Planetary Models in *ArLO*

In Chapter 26, *ArLO* explains the motion of the Sun by an eccentric model, in which the eccentric orb moves with the Sun “from west to east”.³² As for the rest of the planets including the Moon, we find in Chapters 25, 31 and 32 that this book uses an epicycle-eccentric model: the epicycle moves with a planet “from west to east”, while the eccentric moves with the epicycle “from west to east”.

For example, in Chapter 25, Dūnash describes the four orbs of the Moon, namely, the greatest orb (for daily rotation), the parecliptic orb (which is concentric to the ecliptic orb), the eccentric (deferent) orb and the epicyclic orb, and concludes the chapter by presenting a diagram (see Figure 1³³).

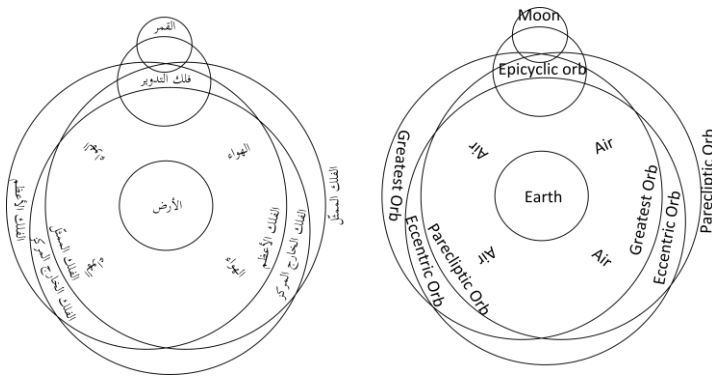
Figure 1



Cf. The figure in MS P

³² Dūnash denotes the direction of a rotation by using “east”, “west”, “north” and “south”. Note that he never presents a solar epicycle model.

³³ This is based on MS B (folio 47a). MS P (page 14) has a slightly different figure which is transcribed as the figure attached with Figure 1.



Although his diagram (Figure 1) is planar without giving a three-dimensional idea, he clearly thinks about not only the longitudinal motion, but also the latitudinal motion, of the Moon. In fact, when he introduces the second orb (i.e., the pareiptic), he mentions that the Moon deviates latitudinally from the pareiptic, as follows:³⁴

وله¹ فلك ثاني² يقال له الفلك الممثل بفلك البروج وهو الذي يظهر به³ داخلاً⁴ في البروج وخارجاً منها⁵. فإذا كانت البروج شمالية أمال⁶ إليها، وإذا كانت جنوبية هبط إليها⁷، فهو في حاله لا يفارق مجرى فلك البروج⁸.

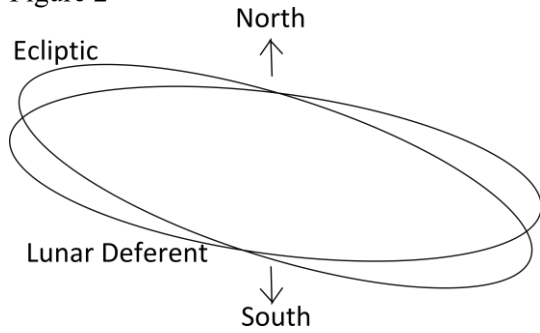
¹ وله + أيضًا: ف فلك ثاني] ب = الفلك الثاني: ف³ به] ف = منه فيه: ب⁴ داخلاً] ف = داخله: ب⁵ منها] ف = عنه: ب⁶ أمال] ب = أمالته: ف⁷ هبط إليها] ب = هبطته إليها أيضًا: ف⁸ البروج] + ولا يُزِيله: ف.

(“It [the Moon] has the second orb called the pareiptic orb, which shows that it [the Moon] enters the zodiacal signs and departs from them. When the zodiacal signs are northern, it deviates to them; when they are southern, it descends to them; however, it does not leave the course of the orb of the zodiacal signs in its condition”).

Given that the Moon is on the plane of the deferent (i.e. the third orb), its deviations described here should be due to the inclination of its deferent to the ecliptic: as Figure 2 illustrates, in the northern side of the ecliptic, the Moon on the deferent “deviates” towards the ecliptic, while in the southern side, it “descends” to it.

³⁴ B: 46a, 12–15; P: 13, 10–13.

Figure 2



Thus, the above explanation of its latitudinal motion confirms that his introduction of the deferent inclined to the ecliptic as the third orb is for its ecliptic latitudinal anomalies.

Here it is noteworthy that the third orb is called “the eccentric orb”, because its center does not agree with the middle of the Earth. In fact, he completes its explanation with the following remark:³⁵

فوجب أن يهبط القمر¹ إلى الأرض من جهة ويتباعد² من أخرى.³
¹ أن يهبط القمر [ب = للقمر هبوطاً: ف² ويتباعد] ب = وتباعد من الأرض: ف³ أخرى [ب = جهة
 أخرى كما يجعل: ف.

(“It is necessary that the Moon descends toward the Earth in one side and becomes distant in the other side”).

Since the deferent is set for its latitudinal deviations, this note suggests that its eccentricity is due to its asymmetrical latitudinal anomalies with respect to the center of the cosmos, i.e. the Earth.

At last, the author describes the fourth orb, i.e., the epicycle orb, which is on the eccentric deferent. Using this orb, he explains the motion of the Moon, as follows:³⁶

فإذا كان القمر¹ في أعلاه كان أسرع مسيراً، وإذا كان في أسفله كان أبطأ مسيراً² وكان مقهقراً³ إلى
 المغرب⁴ لأنه يعرض⁵ له ما يعرض للكواكب من السرعة والبطء⁶ والقهقرة، غير أنه لا تظهر فيه⁷ لسرعة
 مجرى فلك⁸ مركز الخارج.

³⁵ B: 46a, 3–4; P: 13, 15–16.

³⁶ B: 46a, 6–10; P: 13, 18–20.

¹ فإذا كان القمر [ب = وَالْقَمَرُ إِذَا كَانَ: ف 2 مسيراً] ب = ف 3 مقهراً + جَارِيًا: ف 4 المغرب [ب = جَهَّةَ الْمَغْرِبِ: ف 5 لأنه يعرض] ب = وَعَرَضَ: ف 6 والبطء [ب = وَالإِبْطَاءُ: ف 7 غير أنه لا تظهر فيه] ب («يطهر» بدلاً من «تظهر») = وَلَا يَكُن لَيْسَ يَظْهَرُ ذَلِكَ: ف 8 فلك [ب = ف.

(“When the Moon is in the uppermost of it [the epicycle], it is in the swiftest motion. When it is in the lowermost, it is in the slowest motion and is in retrogradation to west. That is because the swiftness, the slowness and the retrogradation which occur to the planets occur to it, although it [i.e., the retrogradation] does not appear to it due to the swiftness of the motion of the eccentric orb”).

In this part he explains by the epicycle the anomalistic change of planetary velocity and the retrogradation. It is remarkable that he adds the reason why the Moon does not retrograde in this model: the motion of its eccentric deferent is fast.

Regarding the remaining planets, Dūnash only explains the case of Saturn in Chapter 31, where he introduces the same model as presented in the case of the Moon by using the same four orbs, and ends with the following note:³⁷

فهذه أربع حركات لزحل ولجميع الدراري¹ خلا² الشمس فإن لها فلكين.³
¹ ولجميع الدراري [ب = وَكَذَلِكَ لَجَمِيعِ الْكَوَاكِبِ وَالْقَمَرِ: ف 2 خلا [ف = خِلاف («ف» مشطوب): ب 3 فلكين] + فَذُ حَكَيْتَاهُمَا فِي مَا تَقَدَّمَ مِنْ كِتَابِنَا هَذَا: ف.

(“These [four motions caused by the four orbs] are the four motions belonging to Saturn as well as to all planets except for the Sun, since it has two orbs”).³⁸

This remark shows that in *ArLO* the planets except the Sun have the same model, an epicycle-eccentric.

Dūnash’s explanation of his planetary system impresses on us that he did not follow the Ptolemaic models in strict sense; e.g., he never mentioned the notion of “equant”. His attitude towards Ptolemy is characteristics as compared with Islamic astronomers in the Abbasid courts including Qusṭā ibn Lūqā and Farghānī, who rigorously followed the Ptolemaic system, so that some of them began to realize its defects.³⁹

³⁷ B: 52a, 5–6; P: 5, 5–6.

³⁸ MS P adds the following: “We have already explained the two [orbs] in the previous part of this book of ours.”

³⁹ See George Saliba, “Early Arabic Critique of Ptolemaic Cosmology: A Ninth-Century Text on the Motion of the Celestial Spheres”, *Journal for the History of Astronomy* 25 (1994), pp. 115–141.

Since Dūnash clearly attached Ptolemy's name to his quotations, we do not need to doubt his attribution of them to Ptolemy. His peculiar understanding of the Ptolemaic astronomy might be explained by the circumstance of the Ptolemaic works in the Maghrib and al-Andalus in his day, where some scholars obtained information on Ptolemy not through Ptolemy's works themselves but through intermediary works containing Ptolemaic doctrines. In fact, Qāsim ibn Muṭarrif al-Qaṭṭān, the above mentioned Cordoban scholar contemporary with Dūnash, referred to Ptolemy's *Planetary Hypothesis* in his *hay'a* work, but Casulleras shows that his reference depended on *Kitāb al-A'lāq al-Nafīsa* by ibn Rustah.⁴⁰ The case of Qāsim ibn Muṭarrif al-Qaṭṭān suggests the possibility that Dūnash utilized an indirect book on the Ptolemaic astronomy. And if so, his source might be "contaminated" by arguments coming from other books than the Ptolemaic works, so that his quotations attributed to Ptolemy could contain non-Ptolemaic arguments. Therefore, when we determine the source of non-Ptolemaic elements found in *ArLO*, it is useful to examine astronomical works in Greek and Latin written by other scholars than Ptolemy which might be integrated into the source on Ptolemy used by Dūnash. Then, when we begin to explore these Greek and Latin astronomical works, we realize the fact that there are surprisingly a few works on epicycle and eccentric planetary models. However, Theon of Smyrna (the second century A.D.) and Calcidius (the fourth century A.D.) exceptionally give wealthy accounts on them.

3. Planetary Models Explained by Theon of Smyrna and Calcidius

Theon of Smyrna, a Platonist, composed a book entitled *On the Mathematics Useful for Reading Plato* in Greek.⁴¹ This work is a summary of mathematical sciences necessary for understanding Plato: that is, arithmetic, music and astronomy. What is remarkable about the astronomical part is that he explicitly mentioned his source several times, that is, a Peripatetic philosopher Adrastus of Aphrodisias (the second century A.D.). And we must note that Plato's works inspired curiosity of astronomy not only among Greek philosophers, but also among Latin Platonists.

⁴⁰ See Casulleras, "The Contents of Qāsim ibn Muṭarrif al-Qaṭṭān's *Kitāb al-hay'a*", p. 341.

⁴¹ Greek text: Eduardus Hiller, *Theonis Smyrnaei philosophi platonici Expositio rerum mathematicarum ad legendum Platonem utilium*, Lipsiae: in aedibus B. G. Teubneri, 1878. Alexander Jones, "Theon of Smyrna and Ptolemy on Celestial Modelling", in Vincenzo De Risi (ed.), *Mathematizing Space: The Objects of Geometry from Antiquity to the Early Modern Age*, Cham: Springer International Publishing, 2015, pp. 75–103 gives updated information about Theon of Smyrna and a useful overview of his *On the Mathematics Useful for Reading Plato*.

In the Latin Platonic tradition, Calcidius' *Commentary on Plato's Timaeus* is the earliest extant commentary on Plato's works.⁴² For explaining Plato's statements in the *Timaeus* 17a–53c, this commentary includes extensive sections on physics, arithmetic and astronomy, consisting of two parts. Among them, Part 1, Section 5 “On the fixed stars and the planets” (Chapters 56–97) and Section 6 “On the heavens” (Chapters 98–118) are on astronomy. By comparing the astronomical contents in Theon's book with those in Calcidius' *Commentary*, we realize that they resemble each other very much, sometimes in almost the same wording. As I have mentioned, Theon's source is apparent, that is, Adrastus, but Calcidius does not give any information. Since his arguments sometimes differ from Theon's, it is difficult to determine whether he refers to Theon's *On the Mathematics* or to Theon's source written by Adrastus or to another book containing Adrastus' astronomical statements.

The core of the planetary system shared by Theon and Calcidius can be summarized as follows:

1) All the planets have constant velocity.⁴³ Theon notes that he owes this thesis to Adrastus.

2) They, however, appear to have anomalistic velocity, and some of them appear to have retrogradation and station. First, Theon and Calcidius (thus Adrastus, too) explain the motion of the Sun by an eccentric as well as by an epicycle.⁴⁴ Then, they claim that the remaining planets can also be described in the same way.⁴⁵

When they explain the planetary motion, Theon and Calcidius (thus also Adrastus) clearly think that all planetary anomalies can be saved by an eccentric or by an epicycle model. Each planet has a different anomaly with respect to the motion of its deferent moving from west to east as well as to the motion of its epicycle. As a result, the five planets appear to have retrogradation and station, whereas Sun is always in direct motion due to the equality of the velocity of the

⁴² Latin text and French translation: Béatrice Bakhouche (ed. and tr.), *Calcidius: Commentaire au Timée de Platon*, 2 vols., Paris: J. Vrin 2011. Anna Somfai, “Calcidius's *Commentary* to Plato's *Timaeus* and its place in the commentary tradition: the concept of *analogia* in text and diagrams”, in P. Adamson, H. Baltussen, and M. W. F. Stone (eds.), *Philosophy, Science and Exegesis in Greek, Arabic and Latin Commentaries*, in 2 vols, (*Supplement to the Bulletin of the Institute Of Classical Studies* 83), 1-2, London 2004, vol. 1, pp. 203–220 elucidates the importance of this work in the Latin Platonic tradition.

⁴³ I.e., they move the same distance during an equal period. Theon: ed. Hiller, pp. 151–152; Calcidius: ed. Bakhouche, vol. 1, p. 296.

⁴⁴ Theon: ed. Hiller, pp. 152–172; Calcidius: ed. Bakhouche, vol. 1, pp. 296–308.

⁴⁵ Theon: ed. Hiller, pp. 172–173; Calcidius: ed. Bakhouche, vol. 1, pp. 308–310.

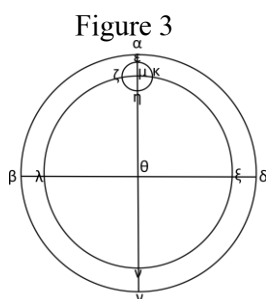
epicycle and the deferent, and the Moon is also always in direct motion because the swiftness of its deferent's motion prevents the occurrence of retrogradation.⁴⁶

As for the direction of epicycle rotation, Calcidius sets the epicycle of the Sun moving from east to west, whilst those of other planets are set from west to east. Theon gives the same direction to the epicycle of the Sun and the epicycles of the five planets respectively as Calcidius does. However, he seems to be confused in the case of the Moon.

When he explains the epicycle models of all planets, Theon describes them as follows:⁴⁷

τὸν <δὲ> εἴηκ ἐπίκυκλον ἔχοντα τὸν πλανώμενον κατὰ τὸ ε φέρεσθαι
 πάλιν περὶ τὸ μ κέντρον, ἐπὶ μὲν ἡλίου καὶ σελήνης ἐπὶ τὰ αὐτὰ τῷ παντί,
 ἐπὶ δὲ τῶν ἄλλων καὶ τοῦτον ὑπεναντίως τῷ παντί·

(“The epicycle εἴηκ (Figure 3) carrying the planet on ε turns around the center μ, in the case of the Sun and the Moon in the same direction as the universe [i.e. from east to west]; in the case of other planets in the opposite direction”).



But, when he offers the eccentric model of a planet with a central small circle corresponding to its epicycle, he differentiates the direction of the rotation of the small circle for the Sun (from east to west) from that for other planets (from west to east).⁴⁸ Apparently this classification of the planets (the Sun vs. the rest of the planets) concerning their epicycles' directions contradicts the explanation found in the previous quotation (the Sun and the Moon vs. the five planets). Given that, however, he generally divides the case for the Sun from that for the remaining planets such as he does in the description of a mechanical device demonstrating

⁴⁶ Theon: ed. Hiller, p. 174, lines 12–15; Calcidius: ed. Bakhouché, vol. 1, p. 310, lines 4–6.

⁴⁷ Ed. Hiller, p. 175, lines 12–14.

⁴⁸ Ed. Hiller, pp. 175–177.

planetary motion by an epicycle model,⁴⁹ and that today the astronomical part of this work is preserved only in one manuscript,⁵⁰ the passage “and the Moon” (καὶ σελήνης) in the quotation (“in the case of the Sun and the Moon”) is likely to be added later, which implies that Theon and Calcidius have the same planetary system.

The above analysis shows that the epicycle model for the five planets and the Moon in the works of Theon and Calcidius and that in *ArLO* are almost identical. Notably, both of them adopt the “planets’ constant velocity” hypothesis, and employ the same reasoning for why the Moon does not retrograde. But the model of *ArLO* is “reformed” with the introduction of an eccentric deferent inclined to the ecliptic: whereas the concentric deferent in Theon and Calcidius also has inclination to the ecliptic, the eccentric deferent in *ArLO* represents asymmetrical planetary anomalies in ecliptic latitude. In non-Ptolemaic astronomical works, however, we can find some descriptions of an eccentric deferent.

4. Eccentric Deferent in Non-Ptolemaic Astronomy

For example, Cleomedes (*On the Heavens*, ii. 5) describes an eccentric model as follows:⁵¹

Ὑψουμένων δὲ καὶ ταπεινουμένων πάντων τῶν πλανήτων ἐπ’
ἴσης ἕκκεντροι πάντων αὐτῶν εἰσὶν οἱ κύκλοι, ἐπεὶ γε διὰ τὰ
ὔψη καὶ τὰ ταπεινώματα μὴ πάντοθεν τὸ ἴσον τῆς γῆς ἀφεςτᾶσι.
 (“Since all the planets are heightened and lowered, all of their circuits
are comparably eccentric, since because of the variation in their
heights they are not equidistant from the Earth in every distance”).

This quotation suggests that he uses an eccentric for representing asymmetrical latitudinal anomalies as does *ArLO*, although he does not introduce an epicycle in this book.

The second example is found in Pliny (23–79)’s *Natural History* Book 2.⁵² First, he explains the constant velocity of the planets,⁵³ and introduces an

⁴⁹ Ed. Hiller, pp. 177–189. Jones, “Theon of Smyrna”, pp. 95–101 gives an English translation of this part.

⁵⁰ See Hiller’s introduction (pp. v–viii).

⁵¹ Greek text: Robert Todd (ed.), *Cleomedis Caelestia*, Leipzig: BSB B.G. Teubner, 1990, p. 133, lines 139–141 ; English translation: Bowen and Todd, *Cleomedes’ Lectures*, p. 152.

⁵² Latin text and English translation: H. Rackham (ed. and tr.), *Pliny: Natural History*, 10 vols, Cambridge, Mass.: Harvard University Press, 1956–63, vol. 1. Alexander Jones, “Pliny on the

eccentric deferent in a very vague manner.⁵⁴ Then he describes asymmetrical latitudinal anomalies, e.g. of Mercury, as follows:⁵⁵

ab his Mercurii stella laxissime, ut tamen e duodenis partibus — tot enim sunt latitudinis — non amplius octonas pererret, neque has aequaliter, sed duas in medio eius et supra quattuor, infra duas.

(“Among them, Mercury is the most elongated planet, but without wandering over more than 8 of the 12 degrees of ecliptic latitude, and these not equally but two in the middle, four above it and two below it”).

Following this, again in an ambiguous manner, he describes an epicycle by using the term “altitude (*altitudo*)”, as follows:⁵⁶

Convenit stellas in occasu vespertino proximas esse terrae et altitudine et latitudine, ... perinde confessum est motum augeri, quamdiu in vicino sint terrae; cum abscedant in altitudinem, minui. quae ratio lunae maxime sublimitatibus adprobatur.

(“It is accepted that the planets are nearest the Earth in altitude and latitude at evening setting. ... Moreover, it is granted that the motion increases as long as they are in the neighbourhood of the Earth; and when they depart in altitude, [the motion] decreases. This account is especially confirmed by the Moon’s apogee”).

From his very vague statement, we can say at least that he tries to explain planetary motion by an eccentric-epicycle model, although it is not clear that he intends to apply this model not only to the five planets, but also to the Sun and the Moon. And the direction of epicycle rotation is not indicated in this model.

There is another example of an eccentric-epicycle model found in an anonymous commentary on Ptolemy’s *Handy Tables*.⁵⁷ This commentary contains a fragment written by Apollinarius (the second century A.D.) on the

Planetary Cycles”, *Phoenix* 45 (1991), pp. 148–161 gives a detailed analysis of Pliny’s planetary system.

⁵³ Ed. Rackham, *Pliny: Natural History*, vol. 1, pp. 188–190.

⁵⁴ Ed. Rackham, *Pliny: Natural History*, vol. 1, p. 210.

⁵⁵ Ed. Rackham, *Pliny: Natural History*, vol. 1, p. 214, lines 1–4.

⁵⁶ Ed. Rackham, *Pliny: Natural History*, vol. 1, p. 214, lines 12–17.

⁵⁷ Greek text and English translation: Alexander Jones, *Ptolemy’s First Commentator* (Transactions of the American Philosophical Society, 80.7), Philadelphia: American Philosophical Society, 1990.

lunar periods, where he gives an eccentric-epicycle model to the five planets and the Moon, although the direction of epicycle rotation is not clear.⁵⁸

These planetary systems show that the direction of epicycle rotation is inconsistent in them. Among these, Apollinarius' fragment includes no indication of the direction; Pliny does not mention direction of his epicycles. As for Theon's *On the Mathematics*, he uses an epicycle with rotation from east to west when he describes retrogradation in general,⁵⁹ but as explained above, he chooses an epicycle with the opposite rotation to construct the model of the five planets and the Moon without reasoning. Calcidius also illustrates retrogradation by an epicycle with rotation from east to west,⁶⁰ and then he explains how mathematicians use an epicycle with the opposite rotation in their model, and declares that he too adopts their epicycle in his model.⁶¹ This Calcidian decision indicates that the authors of these non-Ptolemaic astronomical works had no concrete evidence to determine the directions of their epicycles. Then, we become interested in examining how Ptolemy did select the direction.

5. Ptolemy's Choice of the Direction of His Epicycle

For the five planets (the *Almagest* ix. 5), Ptolemy first constructs an eccentric-epicycle model.⁶² His introduction of the eccentric is for elucidating their anomaly with respect to the ecliptic, that is, "the time from least speed to mean is always greater than the time from mean speed to greater" (τὸν ἀπὸ τῆς ἐλαχίστης κινήσεως ἐπὶ τὴν μέσην χρόνον μείζονα γινόμενον αἰεὶ τοῦ ἀπὸ τῆς μέσης ἐπὶ τὴν μεγίστην).⁶³ This illustrates that he uses the eccentric to explain the anomaly of planetary velocity, not latitude, according to the ecliptic. Then, he uses the epicycle for their anomaly relating to the Sun, and it is set to rotate from west to east, because "the time from greatest speed to mean is always greater than the time from mean speed to least" (τὸν ἀπὸ τῆς μεγίστης κινήσεως ἐπὶ τὴν μέσην χρόνον μείζονα πάντοτε γινόμενον τοῦ ἀπὸ τῆς μέσης ἐπὶ τὴν ἐλαχίστην).⁶⁴ In this way, he merely gives rough observations of their velocities

⁵⁸ Ed. Jones, *Ptolemy's First Commentator*, pp. 38–44; see also Jones, *Ptolemy's First Commentator*, p. 55–56.

⁵⁹ Ed. Hiller, pp. 158–162.

⁶⁰ Ed. Bakhouché, vol. 1, pp. 310–312.

⁶¹ Ed. Bakhouché, vol. 1, pp. 312–314.

⁶² Ed. Heiberg, *Claudii Ptolemaei*, vol. 2, pp. 250–253; tr. Toomer, *Ptolemy's Almagest*, pp. 426–443.

⁶³ Ed. Heiberg, *Claudii Ptolemaei*, vol. 2, p. 251, lines 14–16; tr. Toomer, *Ptolemy's Almagest*, p. 442.

⁶⁴ Ed. Heiberg, *Claudii Ptolemaei*, vol. 2, p. 250, lines 18–20; tr. Toomer, *Ptolemy's Almagest*, p. 442.

to determine the direction, and he completes the first model which is identical to the eccentric epicycle model in *ArLO*.

As for the case of the first lunar model (the *Almagest* iv. 5),⁶⁵ Ptolemy uses a concentric epicycle with rotation from east to west, without giving the reason; instead, he begins the description only by saying: “We shall use the method of establishing the theorem which Hipparchus, as we see, used before us” (ἐπι δὲ τῆς προηγουμένης ἀποδείξεως ἀκολουθήσομεν ταῖς τοῦ θεωρήματος ἐφόδοις, αἷς καὶ τὸν Ἰππαρχὸν ὁρῶμεν συγκεκρημένον).⁶⁶ This statement suggests that his choice depends on Hipparchus.

Comparison of this first lunar model with the lunar models that we have examined beforehand leads us to realize a particular characteristic of Ptolemy’s model – that he makes a lunar model by borrowing the solar epicycle model, whilst the others construct one model for the Sun and another for the remaining planets. And the above quotation indicates that he owes his use of the solar model for the Moon to Hipparchus. But after choosing the concentric-epicycle for his first model, he points out that demonstrating the same motion by using an eccentric is equally possible. This remark, along with the contents of the *Almagest* iv. 11,⁶⁷ implies that Hipparchus, when he constructed his lunar model by introducing an epicycle, might not have been aware that this motion could be explained with either an eccentric or an epicyclic. Or, he might have been pessimistic about such a possibility altogether.

Ptolemy’s construction of his models shows that even he did not have a concrete criterion when he set the direction of epicycle rotation. But this was only the first step of a long process of completing his system. Only after modifying the first model with several geometrical devices, and then determining parameters based on a number of dated observations, were his models able to predict the positions of the planets. In this final stage, these models at last gained legitimacy, and consequently, the directions of the rotations of their epicycles also became legitimized.⁶⁸

6. Again Dūnash’s Source

⁶⁵ Ed. Heiberg, *Claudii Ptolemaei*, vol. 1, pp. 294–300; tr. Toomer, *Ptolemy’s Almagest*, pp. 180–190.

⁶⁶ Ed. Heiberg, *Claudii Ptolemaei*, vol. 1, p. 294, lines 21–23; tr. Toomer, *Ptolemy’s Almagest*, p. 181.

⁶⁷ Ed. Heiberg, *Claudii Ptolemaei*, vol. 1, pp. 338–348; tr. Toomer, *Ptolemy’s Almagest*, pp. 211–216.

⁶⁸ Bernard R. Goldstein, “What’s New in Ptolemy’s *Almagest*”, *Nuncius* 22 (2007), pp. 261 – 285 gives a new insight into Ptolemaic innovations.

The above examination of the Greek and Latin astronomical works written by other scholars than Ptolemy revealed that some of them share with *ArLO* the planetary model of the five planets and the Moon including the reason why the Moon's retrogradation does not occur. The comparison between *ArLO* and these non-Ptolemaic books suggested a popular planetary model for the five planets and the Moon outside the Ptolemaic works and highlighted Ptolemy's new approach in his lunar model: building a lunar model by using an epicycle model of the Sun and introducing an eccentric to it. In fact, before the appearance of this Ptolemaic program, no one was successful in the determination of the direction of epicycle rotation.

Given that Dūnash shared with these non-Ptolemaic astronomical works this standard planetary model for the five planets and the Moon⁶⁹ as well as the hypothesis "planets' constant velocity" in the name of Ptolemy, it is reasonable to think that he used an indirect book on the Ptolemaic astronomy containing popular non-Ptolemaic arguments. The affinity of *ArLO* with these non-Ptolemaic works is also suggested by the last example of the three quotations attributed to Ptolemy, which is found in Chapter 18.

In this chapter, Dūnash discusses the magnitude of the Sun, and at the beginning, he refers to Ptolemy's *Composition of the Orbs* as follows:⁷⁰

وهذا الباب قد ذكر بطليموس في كتاب تركيب الأفلاك فأوردنا¹ من ذكر الحقّ لنألا يخلو² الكتاب منه. فقال إنّ الشمس لا تخلو³ من أن تكون مثل الأرض أو أصغر منها أو أكبر⁴. وأقول إنّ ضياء القمر والنجوم كلّها من الشمس وإنّها لا ضوء لها إلا من استعارتها من الشمس وإنّها مظلمة في ذواتها⁵. وسنبيّن ذلك في باب الأخصّ به إن شاء الله. فلنرجع إلى ما أشرطناه فنقول: إن كانت الشمس مثل الأرض، فواجب أن يكون ظلّ الأرض الذي هو الليل⁶ خارجاً من الأرض على قدر قطر الأرض وقطر الشمس مازاً إلى السماء ذاهباً في العلو⁷ إلى ما لا⁸ نهاية له، ويلحق⁹ الكواكب الثابتة من الفلك الثامن أن تنكسف من عدمها ضياء الشمس إذا كانت الأرض بينها وبين ضياء الشمس، ويجب من ذلك كسوف القمر في كلّ شهر ويكون كسوفه عامّة الليل بسعة¹⁰ قطر الأرض إذا¹¹ ضياؤه من الشمس. وليس يرى ذلك كذلك.

¹ فأوردنا] فأوردن: ب² يخلو] يخلوا: ب³ تخلو] تخلوا: ب⁴ أكبر] كبر: ب⁵ ذواتها] ذراتها: ب⁶ الليل] الميل: ب⁷ العلو] العلو: ب⁸ لا] لا: ب⁹ ويلحق] ويلحق: ب¹⁰ بسعة] تسعه: ب¹¹ إذ] اذا: ب.

("Ptolemy already discussed this subject [i.e. the magnitude of the Sun] in the book *Composition of the Orbs*, so we will offer a part of the account of the essence in order that the book might not lack it.

⁶⁹ Note that Dūnash never mentioned the possibility of an epicycle model for solar motion. This point is owed to one of the anonymous referees.

⁷⁰ B: 35b, 9–36a, 13. I have briefly mentioned this part in Mimura, "The Arabic Original", p. 343.

He said: the Sun cannot be free from being either equal to the Earth or smaller or larger than it.

I say: the light of the Moon and all planets is from the Sun, and they have light only by their borrowing [it] from the Sun, and they are dark in themselves. We will explain that in a specific chapter, if God will.⁷¹

Let us return to what we have prepared, and we say: if the Sun were equal to the Earth, it would be necessary that the shadow of the Earth at night would come from the Earth according to the size of the Earth's diameter as well as the Sun's diameter, crossing towards the sky and going up towards what has no end, and it would cause the fixed stars of the eighth orb to be eclipsed because of their lacking the Sun's light in the case of the Earth being between them and the Sun's light; and that would make a lunar eclipse necessary every month and its eclipse would exist during the greater part of the night by means of the width of the Earth's diameter, since its light is from the Sun. But that is not seen like this.

Then he describes the case when the Sun is smaller than the Earth, and refutes it by presenting several absurdities caused by it. As a result, he concludes that the Sun is larger than the Earth").

As for the above quotation, it is difficult to determine which part is a quotation from Ptolemy's work and which part is an explanation by Dūnash; however, we can say at least that for determining the Sun's size, Ptolemy seems to classify the three cases: when the Sun is equal to the Earth, or when it is smaller than it, or when it is larger than it. Although we cannot find in Ptolemy's works any argument corresponding to it, we must note that this classification was well-known in non-Ptolemaic astronomical works, such as Cleomedes' *On the Heavens* (ii. 2), Theon's *On the Mathematics*,⁷² Calcidius' *Commentary* (Chapters 89-90), and Pliny's *Natural History* (ii. 51).⁷³ Thus, this quotation also suggests that Dūnash utilizes a popular non-Ptolemaic argument in the name of Ptolemy.

Since there exist a few non-Ptolemaic Greek and Latin astronomical works and there is almost no information about whether these works were transmitted into the Islamic world or not, we cannot precisely determine how Dūnash obtained these non-Ptolemaic doctrines as arguments by Ptolemy. However, given that in the Maghrib and al-Andalus in his day, scholars sometimes got information on Ptolemy through intermediate books, the striking similarity between these non-Ptolemaic works and *ArLO* about the planetary models, the planets' constant velocity thesis, and the classification of the three cases on two

⁷¹ I.e., Chapter 21.

⁷² Ed. Hiller, pp. 195–197.

⁷³ See Bowen and Todd, *Cleomedes' Lectures*, p. 129, footnote 9.

illuminated bodies, leads us to think that it might be rational to accept the possibility that he acquired Ptolemaic astronomy through an intermediary book in which some popular non-Ptolemaic elements were integrated, even if we cannot identify his direct source without discovery of a new material.