# Two Early Persian Texts on Shadow Schemes and the Regulation of the Prayer Times ${ }^{1}$ 

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#### Abstract

In this article, we shall elucidate the use of shadow lengths for regulating prayer times among Zoroastrians, focusing on a part of a Zoroastrian religious book entitled Shāyest Nāshāyest, written in Middle Persian (Pahlavī) language probably around the $9^{\text {th }}$ c. A.D. Then, we will compare this text with a part of another Persian work entitled Yawāqīt al- 'Ulūm wa Darārī al-Nujūum which is written in the second half of the $6^{\text {th }}$ century after Hijra (the $12^{\text {th }} \mathrm{c}$. A.D.) by Abū Muḥammad al-Najjār in Islamic civilization. For this purpose, we will recalculate the given values for the shadow lengths of the gnomon - taken as the height of a man - in both texts and find the best fitted latitudes for them. Finally, we will mention the similarities of the two texts to investigate the possibility of a historical relationship between them. Moreover, the transmission of this timekeeping method among some civilizations will be discussed.


Keywords: shadow schemes, prayer times, timekeeping, Shāyest Nāshhāyest, Yawāqīt al- `Ulūm wa Darārī al-Nujūm, Zoroastrians, gnomonics

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## 1. Introduction

Several kinds of sundials were commonly used in various civilizations as astronomical instruments for telling the time of day. We have quite considerable materials to elucidate the designs of sundials and the methods for their construction in such civilizations as Egypt, Greece, Rome, China, India and Islamic civilization. However, our information is too limited to analyze the use of sundials by Zoroastrians in Iranian pre-Islamic astronomy due to the lack of original sources.

We can find some evidence of the use of a gnomon shadow length for determining the time which has elapsed since sunrise or noon, or remains until noon or sunset in some ancient texts. In addition to the Babylonian origin for arithmetical foundations of the shadow schemes, it seems that there was a purely Greek origin, by which we can trace the measurement of time based on the length of one's shadow in terms of his feet back to the fourth or fifth century B.C. in Greek astronomy. ${ }^{2}$

In another example two Indian sources named Arthaśāstra (shortly before 300 B.C.) and Śārdūlakarņāvadāna (anti-caste tract) include the values of noonshadow length of a gnomon for various dates probably inspired by Mesopotamian sources. ${ }^{3}$

In about 450 A.D. Palladius Ritilius Taurus Emilianus indicated shadow lengths based on one's height for each hour of the day during each month of the year in Sicily. ${ }^{4}$ This timekeeping method was used in the Mediterranean and spread in some other regions. Tiberius ${ }^{5}$ timekeeping table (around 850 A.D.) for the probable latitude $53^{\circ}$ and Bede's table (between 700-730 A.D., England) are other examples of this timekeeping method. ${ }^{6}$

New evidence which has recently been found shows that Zoroastrians used this method for regulating their prayer times as did Muslims. ${ }^{7}$ In the pre-Islamic Sassanid dynasty (226-652 A.D.), Zoroastrians were a majority in Iran and used

[^1]the Sassanid calendar. ${ }^{8}$ We have two sources that show us the existence of Zoroastrians' knowledge about gnomonics at that time. The first one is Ifrād alMaqāl fī Amr al-Zilāl of al-Bīrūnī, according to which there was a part in the Shāh $Z_{\bar{l} j}$ (Royal Astronomical Handbook) compiled in the Sassanid dynasty. Unfortunately, no copy of it has survived. The fragment quoted by al-Bīrūnī is about determining the solar hour angle based on the shadow length of a gnomon by using trigonometric methods. ${ }^{9}$

Further evidence of probable astronomical knowledge of the Zoroastrians applied in gnomonics appears in the $21^{\text {st }}$ chapter of the Shāyest Nāshāyest, a religious book of the Zoroastrians, where the text explains the way by which one can use the shadow of a gnomon for regulating his prayer times. ${ }^{10}$ It seems that the main parts of this book were originally compiled in the late Sassanid era ${ }^{11}$ (the $6^{\text {th }}$ and $7^{\text {th }}$ c. A.D.) by an anonymous author and probably other parts were added in Islamic period around the $9^{\text {th }}$ c. A.D. However the earliest remaining manuscripts of this work were copied later, sometime between 1351 and 1397 A.D. ${ }^{12}$ and it is not certain that the $21^{\text {st }}$ chapter of this book is a part of the original text. Furthermore, it is noteworthy that a large number of remaining Zoroastrian religious texts including materials on praying, liturgy and worship ceremonies were written in Middle Persian (Pahlavī) language even after the $9^{\text {th }}$ c. A.D. ${ }^{13}$ Before the time of Zoroaster (whose religion flourished almost continually from the $6^{\text {th }}$ c. B.C. to the $7^{\text {th }}$ c. A.D. ${ }^{14}$ ), pagan Iranians prayed three times a day, at sunrise, noon and sunset. These times divided the daylight hours into two periods. Hāwan̄̄ was the morning one, uzerīn (or uzayra) was the afternoon one and the aiwisrothrem was the evening one. Now Zoroaster

[^2]introduced two other times for praying. One of these new times, named rapithwan, began at noon and extended into the end of the first part of the afternoon. The second time, whose name was ushah (or ushahīn), began at the midnight and extended until daybreak of the next day. ${ }^{15}$ So, the five prayer times of the Zoroastrians were: hāwan̄̄, rapīthwan, uzerīn, aiwīsrothrem and ushahīn.
In the first part of our text, the values of the length of one's shadow in terms of one's feet are given for noon when the sun is at the beginning and the middle of each zodiacal sign during a year. ${ }^{16}$ In the second part, the values of shadow length are given for determining the uzerīn prayer time at solstices and the beginning of Leo. In addition, the author presented a simple arithmetic rule for calculating the uzerīn prayer time for each month. Probably he cited the uzerīn prayer time at solstices as a clue to use this rule, and since this rule would not be valid for the beginning of Leo, ${ }^{17}$ he cited the uzerīn prayer time for this longitude separately. ${ }^{18}$

## Shadow Schemes in the Islamic Period

In Islam, there were two distinct traditions in astronomy and astronomical timekeeping. The first was based on mathematics, especially trigonometry, and involved extensive tables for timekeeping by the sun and stars. This does not concern us here, the second was folk astronomy, based on numerical, schemes. It is to this tradition that our material directs us. Widespread use of a gnomon shadow schemes in Islamic folk astronomy has been shown by giving some examples of different Islamic sources which include one or more parts about the use of gnomon shadow length for different purposes such as geographical or religious aims. ${ }^{19}$ As we know, Muslim geographers divided the northern hemisphere of the earth into seven climates (iqlīm), following (perhaps) Ptolemy. ${ }^{20}$ They distinguished the climates based on one's own shadow length in

[^3]terms of one's feet at midday on the equinoxes. ${ }^{21}$ We can trace this way for determining latitude back to Hipparchus. ${ }^{2}$
We can also find similar texts in the Islamic sources in which shadow lengths of a gnomon for performing two prayers of the Muslims were determined. ${ }^{23}$ One of them, is a kind of encyclopedia of sciences entitled Yawāqīt al- 'Ulūm wa Darārī al-Nujūm which was written in Persian by Abū Muḥammad al-Najjār probably in the second half of the $6^{\text {th }}$ century after the Hijra (the $12^{\text {th }}$ c.A.D.), in Qazwīn. ${ }^{24}$ In the $27^{\text {th }}$ chapter of this book ("On the Science of Astronomy") the author has indicated three values of midday shadow lengths in feet for each zodiacal sign when the sun is in the first, second or third ten days of each zodiacal sign. ${ }^{25}$ In the following, we will survey the values of the shadow length which are indicated in this book. We can also find some other examples among Islamic sources in which one's shadow lengths is indicated for determining the time of day. ${ }^{26}$ We now turn to the translations of sections of the two treatises that are the principal topic of this paper.

[^4]
# 2a. Translation of the Fragment of the Shāyest Nāshāyest on Finding Prayer Times ${ }^{27}$ 

Chapter XXI, translation ${ }^{28}$

1. I write of the indication of the midday shadow; may it be auspicious.
2. (when) the sun (is) in Cancer, (the shadow is) the sole of a man's foot; at the fifteenth (degree) of Cancer, (it is) one foot; (When) the sun (is) in Leo, (it is) one foot and a half; at the fifteenth of Leo, (it is) two feet; (When) the sun (is) in Virgo, it (is) two feet and a half; at the fifteenth of Virgo, (it is) three feet and a half; at Libra, four feet and a half; at the fifteenth of Libra, five feet and a half; at Scorpio, six feet and a half; at the fifteenth of Scorpio, seven feet and a half; at Sagittarius, eight feet and a half; at the fifteenth of Sagittarius, nine feet and a half; at Capricorn, ten feet; at the fifteenth of Capricorn, nine feet and a half; at Aquarius, eight feet and a half; at the fifteenth of Aquarius, seven feet and a half; at Pisces, six feet and a half; at the fifteenth of Pisces, five feet and a half; at Aries, four feet and a half; at the fifteenth of Aries, three feet and a half; at Taurus, two feet and a half; at the fifteenth Taurus, two feet; at Gemini, one foot and a half; at the fifteenth of Gemini, one foot.
3. May the end of the (indication of) the midday shadow be good.
4. I write of the indication of the afternoon; by the help of the Yazda, may it be good and auspicious.
5. When the day is on the increase and the sun enters the beginning of Cancer and the shadow becomes six feet and two parts, one keeps the uzerīn $g a \bar{h}$.
6. Every thirty days, (the shadow) increases always by one foot and one-third, now for every period of ten days, the reckoning is always half a foot; (When) the sun (is) at the beginning of Leo, the shadow (is) seven feet and a half.

[^5]7. In conformity with this, in every zodiacal sign, similarly, and in all months, similarly, till the sun enters the beginning of Capricorn, (when) the shadow becomes fourteen feet and two parts.
8. In Capricorn, it decreases again by one-third of a foot; from there (where) it turns back (its course) just like the decrease of night and the increase of day, (so) each one of the months decreases always by one foot and one-third; so every ten days the reckoning is always half a foot till it comes again to six feet and two parts. Every zodiacal sign similarly, and the months similarly.

8a. I have written and finished it, I, the servant of the Faith, Mihr-Ābān son of Kay-Husraw, priestly-born.

8b. The writing is mine; I (am) the teacher, Ērbad Pēšōtan, son of Rām

## 2b. Translation of $27^{\text {th }}$ Chapter of the Yawāqīt al- 'Ulū̀m wa Darārī al-Nujūm ${ }^{29}$ on Finding Midday Shadow Length

The question twelve: How to know the solar transit (zawāl) based on the [length of one's shadow in terms of his own] feet?

Answer: [They] know the time of the solar transit based on the increase of the shadow length of gnomons. [And their method] is such that they pierce a straight stick to the earth [perpendicularly], so that its shadow stretches towards west at the sunrise. In this case, when the sun rises the shadow length shortens and reverts from west, and stops when the sun reaches its culmination. Then, the shadow length increases again. Whenever the beginning of increasing of the shadow length can be known, that time is the time of the solar transit. And in the divine wisdom, the sun has passed the meridian before this time, but in the legal elaborations, whenever [the decreasing of the solar altitude] can be detected is the time of midday. And the midday shadow length differs in various times and locations. The maximum value of the shadow length is eleven feet in Qazwīn ${ }^{30}$ and Rayy ${ }^{31}$. But its example is that when the sun enters Aries, during the first ten days [the length of one's shadow] is four feet and a quarter ${ }^{32}$, and during the second ten days it is four feet and one sixth [of a foot] ${ }^{33}$, and during the third ten days it is three feet and a half ${ }^{34}$.

And when [the sun] enters Taurus, during the first ten days [the length of one's shadow] is on sixth of a foot less than three feet, and during the second ten days it is two feet and two parts of a foot, and during the third ten days it is two feet and one sixth [of a foot]. ${ }^{35}$

And when [the sun] enters Gemini, during the first ten days [the length of one's shadow] is one foot and four sixths of a foot, and

[^6]during the second ten days it is one foot and two parts of a foot, and during the third ten days it is one foot and a half.

And when [the sun] enters Cancer, during the first ten days [the length of one's shadow] is one foot and one third of a foot, and during the second ten days it is one foot and a half, and during the third ten days it is two feet and one sixth of a foot.

And when [the sun] enters Leo, during the first ten days [the length of one's shadow] is two feet and four quarters ${ }^{36}$ of a foot, and during the second ten days it is three feet and one $\operatorname{six}^{37}{ }^{37}$ of a foot, and during the third ten days it is three feet and a half ${ }^{38}$.

And when [the sun] enters Virgo, during the first ten days [the length of one's shadow] is four feet and one sixth of a foot, and during the second ten days it is four feet and four sixths of a foot, and during the third ten days it is five feet and one sixth of a foot.

And when [the sun] enters Libra, during the first ten days [the length of one's shadow] is six feet, and during the second ten days it is six feet and three fourths of a foot, and during the third ten days it is seven feet and a quarter of a foot.

And when [the sun] enters Scorpio, during the first ten days [the length of one's shadow] is eight feet and one sixth of a foot, and during the second ten days it is nine feet and one sixth ${ }^{39}$ of a foot, and during the third ten days it is nine feet and three fourths of a foot.

And when [the sun] enters Sagittarius, during the first ten days [the length of one's shadow] is ten feet, and during the second ten days it is ten feet and four sixths of a foot, and during the third is eleven feet.

And when [the sun] enters Capricorn, during the first ten days [the length of one's shadow] is ten feet and four sixths of a foot, and during the second ten days it is ten feet, and during the third ten days it is nine feet and a half.

And when [the sun] enters Aquarius, during the first ten days [the length of one's shadow] is nine feet, and during the second ten days it

[^7]is ten feet and three fourths ${ }^{40}$, and during the third ten days it is nine feet and one sixth of a foot ${ }^{41}$.

And when [the sun] enters Pisces, during the first ten days [the length of one's shadow] is seven feet and a half ${ }^{42}$, and during the second ten days it is six feet, and during the third ten days it is five feet and a quarter.

And this is closer to the truth. And it would be better if the muezzin trusts a gnomon, observes its midday shadow and demarcates it daily, and determines the 'assr prayer time [based on] seven feet beyond [to the midday shadow length]. And this is the figure of the gnomon:

[Fig. 1. A kind of gnomon which was used for determining the 'asr prayer time by Muslims ( Yawāqīt al- 'Ulūm wa Darār̄̄ al-Nujūm, ms. preserved in Loghatnameh Dehkhoda Institute, no. 95, f. 64 r)]

And this is famous among Sufis and pious and devout that they concern the shadow, for performing [their] prayers. The prophet Muhammad has told: "The most devoted to God are those who love God and His devotees, and those who

[^8]observe the sun and stars and shadows to remember God". [The muezzin] has to be careful to know the 'astr prayer time. [For this purpose, he has to] demarcate the shadow length which corresponds to the midday, then add the height of the gnomon to the midday shadow length. When the shadow length reaches to this value, it is the 'assr prayer time. And God is the wisest as to what is correct. (This notion suffices, although the subject can be more comprehensive)

## 3. Commentary <br> A) Numerical Values of One's Shadow Length at Noon of the Beginning and Middle of Each Zodiacal Sign in the Shāyest Näshāayest

As we mentioned, the values of a gnomon shadow length for determining the Zoroastrians' prayer times which are cited in the Shāyest Nāshāyest have similarities with some Islamic astronomical sources on this issue which were influenced by Greek and Indian background. ${ }^{43}$ Some of the astronomical information which is cited in such sources is reckoned as folk astronomy. ${ }^{44}$ Here we should note that since the Shāyest Nāshāyest is written for laity and it is not an astronomical book, the shadow lengths which are cited in this book have not been calculated, but very likely they were observed directly so it was not needed to use any mathematical device such as a sine table. A survey of the values shows that they have been rounded off to multiples of 0.50 foot. Furthermore, since we have not found any trace about a professional group, institution or work for determining Zoroastrian prayer times based on mathematical and astronomical formulae, and even nowadays it is the duty of their clergies to do it based on an oral tradition, we can refer the values which are presented in the Shāyest Nāshāyest to folk astronomy.

Supposing the length of a person's foot is equal to $\frac{1}{7}$ of his height ${ }^{45}$ and on the basis of the declination of the sun, if we calculate the corresponding latitude for each shadow length which is cited in the Shāyest Nāshāyest by using modern mathematical methods, we find that the computed latitudes corresponding to the various shadow lengths are unequal, though it is plausible to assume that all of which were initially observed in a specific latitude. In the following, we will show that all of the computed latitudes result in a mean value equal to $31 ; 30^{\circ}$.

It seems that the variation of the latitudes would be a consequence of the author's rounding off the shadow lengths. ${ }^{46}$ In any case, it should be kept in mind that this is a religious text and is not written for astronomical, mathematical, geographical, etc. purposes, so the author would not have felt any need of highly accurate values.

[^9]
## B) Recomputation of the Most Compatible Values in the Shāyest Nāshāyest

In this section, we develop further computations to find the latitude best agreeing with the shadow lengths recorded in the text. For this purpose we have prepared a computer program which computes shadow length values for noon of the beginning and middle of each zodiacal sign during a year in different latitudes. ${ }^{47}$ The inputs of the program are the declination of the sun ( $\delta$ ) and latitudes in steps of $0.1^{\circ}$ from $25^{\circ}$ to $40^{\circ}(\varphi)$. For any latitude, this program computes a series of values of noon shadow lengths when the sun is at the beginning and middle of each zodiacal sign. Then, we computed the sum of differences between every calculated absolute value for a given latitude and the corresponding value in the text. Then, we chose the latitude that corresponds to the series that has minimum total difference and its values almost coincide with the values which are given in the text. Based on this computation, the values of latitudes between $31 ; 18^{\circ}$ and $31 ; 42^{\circ}$ accord with the data. So we can introduce the mean latitude ( $31 ; 30^{\circ}$ ) which best fits our values. We have also used the least mean squares method for our recomputation and it confirmed the conclusion. The southern part of Zābulistān ${ }^{48}\left(31^{\circ}\right)$ and $\operatorname{Yazd}^{49}\left(32^{\circ}\right)$ are two main regions lying in this range in Iran. ${ }^{50}$ As we know, in the history of Zoroastrians, Yazd was one of their main cities and their chief temple is located near there. ${ }^{51}$ This historical information about Yazd supports our computations.
Moreover, we then recomputed by linear interpolation to find the shadow length values, based on the shadow lengths at the equinoxes and solstices as the basic values. We used this method two times; the first time, we chose the four basic shadow length values of the text, and the second time, we chose these basic values according to the best fitted latitude. Finally we compare them with the values of the text and concluded that it is more probable that the author has rounded off the values and he didn't use the linear interpolation method.

[^10]In the table $1, S_{\mathrm{Z}}(F)$ and $S_{\mathrm{Y}}(F)$ indicate the length of one's shadow in his feet for the latitudes of the southern part of Zābulistān, and Yazd. ${ }^{52}$ In addition, we have subtracted the values that were cited in the text from the recomputed values and shown the differences in the parentheses.

| $\lambda$ | $S_{\mathrm{Z}}(F)$ | $S_{\mathrm{Y}}(F)$ |
| :---: | :---: | :---: |
| $0^{\circ}$ | $4.21(-0.24)$ | $4(-0.50)$ |
| $15^{\circ}$ | $3.35(-0.15)$ | $3(-0.50)$ |
| $30^{\circ}$ | $2.46(-0.04)$ | $3(0.50)$ |
| $45^{\circ}$ | $1.87(-0.13)$ | $2(0.00)$ |
| $60^{\circ}$ | $1.33(-0.17)$ | $1(-0.50)$ |
| $75^{\circ}$ | $1.05(0.05)$ | $1(0.00)$ |
| $90^{\circ}$ | $0.93(-0.07)$ | $1(0.00)$ |
| $105^{\circ}$ | $1.01(0.01)$ | $1(0.00)$ |
| $120^{\circ}$ | $1.33(-0.17)$ | $1(-0.50)$ |
| $135^{\circ}$ | $1.76(-0.24)$ | $2(0.00)$ |
| $150^{\circ}$ | $2.45(-0.05)$ | $3(0.50)$ |
| $165^{\circ}$ | $3.16(-0.34)$ | $3(-0.50)$ |
| $180^{\circ}$ | $4.18(-0.32)$ | $4(-0.50)$ |
| $195^{\circ}$ | $5.13(-0.37)$ | $5(-0.50)$ |
| $210^{\circ}$ | $6.34(-0.16)$ | $7(0.50)$ |
| $225^{\circ}$ | $7.45(-0.05)$ | $8(0.50)$ |

[^11]| $240^{\circ}$ | $8.63(0.13)$ | $9(0.50)$ |
| :---: | :---: | :---: |
| $255^{\circ}$ | $9.42(-0.03)$ | $10(0.50)$ |
| $270^{\circ}$ | $9.79(-0.21)$ | $10(0.00)$ |
| $285^{\circ}$ | $9.52(0.02)$ | $10(0.50)$ |
| $300^{\circ}$ | $8.66(0.16)$ | $9(0.50)$ |
| $315^{\circ}$ | $7.62(0.12)$ | $8(0.50)$ |
| $330^{\circ}$ | $6.34(-0.16)$ | $6(-0.50)$ |
| $345^{\circ}$ | $5.26(-0.24)$ | $5(-0.50)$ |

Table 1. Recomputed amounts of the Shāyest Nāshāyest shadow length values for the southern part of Zābulistān and Yazd

In another manuscript of the Shāyest Nāshāyest, preserved in the Meherjirana Library (Navsari, India), some values of noon shadow lengths in the beginning and middle of zodiacal signs differ from that of the text published in Iran in 1990 (In the following, we will use "IND" for the manuscript which is preserved in the Meherjirana Library and "IR" for the text which was published in Iran). One of these differences concerns the shadow length at noon of the middle of Libra. The length of this shadow was recorded as 5 feet in IND, ${ }^{53}$ but based on our calculations for the latitudes of Yazd and the southern part of Zābulistān the value given as 5.50 feet which appears in IR seems to be more accurate. Shadow length values written in the English translation of the Shāyest Nāshāyest are similar to those of IR, except for the shadow length of the beginning of Cancer. ${ }^{54}$

Other differences between IND and IR, relate to the shadow lengths of the middle of Capricorn and the beginning of Aquarius. In IND, these values are recorded 6.5 and 7.5 feet, ${ }^{55}$ but in IR they are recorded as 9.5 and 8.5 feet. Moreover, in both of them, shadow lengths for the beginning of Capricorn and middle of Aquarius are recorded as 10 and 7.5 feet. But since the shadow length continuously decreases from the beginning of Capricorn to the beginning of Cancer and it is also based on the recalculated values of shadow length for Yazd

[^12]and the southern part of Zābulistān, the two values which are recorded in IND are inaccurate.

The way the author of our text cites the shadow length values is very similar to that of Tiberius. Both of them have cited the shadow lengths for midday of the first and fifteenth degree of each zodiacal sign (of course Tiberius' text includes shadow lengths values for the morning and the ninth hour of these dates too). ${ }^{56}$

## The Values of Shadow Length for the Uzerīn Prayer Time

In the next part of chapter 21 , our author presents the way by which, one can determine the beginning of the uzerīn prayer time (from three hours after noon to the beginning of night and the appearance of stars) ${ }^{57}$ based on the length of one's shadow. ${ }^{58}$ According to this part, it seems that Zoroastrians determined their prayer times based on the shadow length of a gnomon (or one's shadow length) much as Muslims did.

In this part, our author has written that the values of one's shadow length at the uzerīn prayer time on the summer solstice, the beginning of Leo, and the winter solstice are 6 feet and " 2 bahrs", 7.50 feet, and 14 feet and " 2 bahrs" respectively. ${ }^{59}$ Then, he states that from the summer solstice to the winter solstice, the shadow length increases $1 \frac{1}{3}$ feet per 30 days, and 0.5 feet per 10 days, and when the sun returns to the summer solstice, shadow length decreases at the same rate.

According to the text, the difference of shadow lengths between the solstices is $14 \frac{2}{3}-6 \frac{2}{3}=8$. So the mean rate of increasing the shadow length is $\frac{8}{6}=\frac{4}{3}=1 \frac{1}{3}$ for each month and ${ }_{1} \frac{1}{3} \div 3=\frac{4}{9} \cong \frac{1}{2}$ for each 10 days. However we know that on the days near the solstices, the change of the shadow length value is small. So, during the other months, the increase of the shadow length is more than on those days,

[^13]which compensates for this depletion. As we see, our author has recorded $7 \frac{1}{2}$ feet instead of $6 \frac{2}{3}+1 \frac{1}{3}=8$ feet for the beginning of Leo. ${ }^{60}$ This difference ( $\left(\frac{1}{2}\right.$ foot) will be compensated for in the other months. Furthermore, the decrease of the shadow length is cited as $\frac{1}{3}$ feet instead of ${ }_{1 \frac{1}{3}}$ feet for Capricorn. This difference (1 foot), too, will be compensated for in the other months. ${ }^{61}$ In the table $2, S$ shows the values of shadow length in terms of feet for the uzerīn prayer time on the three days mentioned above, according to the text.

| $\boldsymbol{\lambda}$ | $\boldsymbol{S ( F )}$ | Hours since noon <br> (based on the shadow length in the text) |
| :---: | :---: | :---: |
| $90^{\circ}$ | $6+\frac{2}{3}=6.67$ | 3.25 |
| $120^{\circ}$ | 7.50 | 3.42 |
| $270^{\circ}$ | $14+\frac{2}{3}=14.67$ | 2.33 |

Table 2. The passed time since noon for the uzerīn prayer time for three important days of a year according to the Shāyest Nāshāyest

But, as we mentioned formerly, we know that the uzerīn prayer time coincides with three hours after noon, so if we recalculate those values for the best fitted latitude $\left(31 ; 30^{\circ}\right)$, we will have three new values (table 3 ):

| $\boldsymbol{\lambda}$ | $\boldsymbol{S}_{31 ; 30^{\circ}(\boldsymbol{F})}$ |
| :---: | :---: |
| $90^{\circ}$ | $5.97(0.70)$ |
| $120^{\circ}$ | $6.24(-1.24)$ |
| $270^{\circ}$ | $19.02(4.35)$ |

Table 3. Recomputed values of the shadow length for the tree uzerīn prayer times which are mentioned in the Shāyest Nāshāyest

[^14]
## C) Numerical Values of One's Shadow Length in the Yawāqīt al- 'Ulūm wa Darārī al-Nujūm

In this part, we shall survey the $27^{\text {th }}$ chapter of another Persian text entitled Yawäqīt al-'Ulūm wa Darārī al-Nujūm which includes the values of the length of one's midday shadow for the first, second and third ten-day periods of each zodiacal sign. Since the author of this work, Abū Muhammad al-Najjār, was adept in Islamic religious sciences and literature, ${ }^{62}$ and based on our current knowledge, except for the Shāyest Nāshāyest (which is authored around the $9^{\text {th }}$ c. A.D. as we mentioned formerly), there were no other Zoroastrian works including such information about determining the prayer times by shadow lengths, it is most probable that the $27^{\text {th }}$ chapter of the Yawāqīt al- 'Ulūm wa Darārī al-Nujūm was written influenced by other similar Islamic sources on this subject.

The framework of this text is similar to a short anonymous Syrian treatise on timekeeping. ${ }^{63}$ There is no numerical formula in this section and its purpose is informing the laity about the prayer times, just as the Shāyest Nāshāyest. Moreover, as we mentioned, al-Najjār was skilled in religious sciences and literature, and not in astronomy and mathematics, so we can view it as a text in folk astronomy in its time and not a professional work on mathematical astronomy. ${ }^{64}$

The author believes, based on the text, ${ }^{65}$ that the shadow lengths were observed directly in the latitude of Qazwīn $\left(37^{\circ}\right)$ or Rayy $\left(35 ; 35^{\circ}\right)$ and have been rounded off to multiples of $\frac{1}{4}$ and $\frac{1}{6}$ foot. ${ }^{66}$ We have tabulated these values and recomputed them for the mentioned latitudes. Moreover, we have also found the best fitted latitude $\left(34 ; 48^{\circ}\right)$ for the text values ${ }^{67}$ and compared them with the values which correspond to the latitudes of Qazwīn and Rayy. In the table $4, S_{\mathrm{Q}}$

[^15]$(F)$ and $S_{\mathrm{R}}(F)$ indicate the length of one's shadow in his feet for the latitudes of Qazwīn and Rayy.

| $S_{\text {R }}(F)$ | $S_{\mathrm{Q}}(F)$ | $\lambda$ |
| :---: | :---: | :---: |
| 4.70 (0.45) | 4.95 (0.70) | $0^{\circ}-10^{\circ}$ |
| 4.08 (-0.09) | 4.27 (0.10) | $11^{\circ}-20^{\circ}$ |
| 3.44 (-0.01) | 3.65 (0.15) | $21^{\circ}-31^{\circ}$ |
| 2.90 (0.07) | 3.10 (0.27) | $32^{\circ}-41^{\circ}$ |
| 2.49 (-0.18) | 2.66 (-0.01) | $42^{\circ}-51^{\circ}$ |
| 2.11 (-0.06) | 2.29 (0.12) | $52^{\circ}-62^{\circ}$ |
| 1.82 (0.15) | 2.00 (0.33) | $63^{\circ}-72^{\circ}$ |
| 1.64 (-0.03) | 1.81 (0.14) | $73^{\circ}-82^{\circ}$ |
| 1.54 (0.04) | 1.71 (0.21) | $83^{\circ}-93^{\circ}$ |
| 1.52 (0.19) | 1.71 (0.38) | $94^{\circ}-103^{\circ}$ |
| 1.60 (0.10) | 1.80 (0.30) | $104^{\circ}-113^{\circ}$ |
| 1.80 (-0.37) | 1.99 (-0.18) | $114^{\circ}-124^{\circ}$ |
| 2.06 (-0.19) | 2.27 (0.02) | $125^{\circ}-134^{\circ}$ |
| 2.38 (-0.79) | 2.65 (-0.52) | $135^{\circ}-143^{\circ}$ |
| 2.83 (-0.67) | 3.12 (-0.38) | $144^{\circ}-155^{\circ}$ |
| 3.34 (-0.83) | 3.66 (-0.51) | $156^{\circ}-165^{\circ}$ |
| 3.87 (-0.80) | 4.25 (-0.42) | $166^{\circ}-175^{\circ}$ |
| 4.57 (-0.60) | 4.95 (-0.22) | $176^{\circ}-186^{\circ}$ |
| 5.31 (-0.69) | 5.74 (-0.26) | $187^{\circ}-196^{\circ}$ |
| 6.05 (-0.70) | 6.57 (-0.18) | $197^{\circ}-206^{\circ}$ |
| 6.95 (-0.30) | 7.48 (0.23) | $207^{\circ}-216^{\circ}$ |
| 7.86 (-0.31) | 8.46 (0.29) | $217^{\circ}-226^{\circ}$ |
| 8.75 (-0.42) | 9.46 (0.29) | $227^{\circ}-236^{\circ}$ |
| 9.71 (-0.04) | 10.44 (0.69) | $237^{\circ}-246^{\circ}$ |
| 10.54 (0.54) | 11.31 (1.31) | $247^{\circ}-256^{\circ}$ |
| 11.16 (0.49) | 11.96 (1.29) | $257^{\circ}-266^{\circ}$ |
| 11.54 (0.54) | 12.29 (1.29) | $267^{\circ}-276^{\circ}$ |
| 11.59 (0.92) | 12.24 (1.57) | $277^{\circ}-286^{\circ}$ |
| 11.28 (1.28) | 11.87 (1.87) | $287^{\circ}-296^{\circ}$ |
| 10.64 (1.14) | 11.26 (1.76) | $297^{\circ}-306^{\circ}$ |


| $9.83(0.83)$ | $10.37(1.37)$ | $307^{\circ}-316^{\circ}$ |
| :---: | :---: | :---: |
| $8.95(0.45)$ | $9.38(0.88)$ | $317^{\circ}-326^{\circ}$ |
| $7.96(0.46)$ | $8.36(0.86)$ | $327^{\circ}-336^{\circ}$ |
| $6.94(-0.56)$ | $7.38(-0.12)$ | $337^{\circ}-346^{\circ}$ |
| $6.11(0.11)$ | $6.47(0.47)$ | $347^{\circ}-356^{\circ}$ |
| $5.40(0.15)$ | $5.67(0.42)$ | $357^{\circ}-365^{\circ}$ |

Table 4. Recomputed amounts of the Yawāqīt al- ${ }^{\text {'Ulūm shadow length values for }}$ Qazwīn and Rayy

As we see, the latitude that gives values closest to those of the values of shadow lengths recorded in the Yawāqīt al- 'Ulūm, is closer to the latitude of Rayy than to that of Qazwīn. Moreover, the differences between the interpolated values and the values written in the text show that the author did not use the interpolation method. It seems that he measured the shadow length values and rounded off them to the mentioned fractions of a foot, but his measurements were not very accurate.
After indicating the midday shadow, the author explained how the muezzin can determine the cast prayer time. The author has also shown a figure of an instrument by which one can measure the shadow length in terms of feet for determining the ‘ass prayer time (Fig. 1). Unfortunately the author didn’t explain how to use it, but it seems that one should hang it from its top which is indicated by the name "Hanging Position" (Mowdi ${ }^{\circ}{ }^{\text {' }}$ Aläqeh) and use the graduated scale at the bottom, for measuring the shadow lengths. Presenting the picture of this instrument in the text, makes it more likely that the shadow lengths were written based on direct observations and not on calculations.

## 4. Conclusions

A survey of the two mentioned texts shows some similarities between their frameworks, both of which have two main parts, the first being allocated to the values of midday shadow lengths to determine the zuhr (rapīthwan) prayer time, and the latter showing how to determine the 'aṣr (uzerīn) prayer time. Another similarity is that both texts show a strong religious interest and so the values that are cited by their authors are not so accurate as would be required in an astronomical work. Thus, the values presented in the both works belong to the tradition of folk astronomy.

Al-Bīrūnī's reference to a part of Shāh $Z \overline{1} \bar{j}$, which is related to the use of a gnomon for determining some solar parameters, demonstrates that the Sassanid astronomers knew how to use a gnomon for some sophisticated astronomical purposes such as timekeeping. Hence, citation of the values of midday length of one's shadow in a chapter of a Zoroastrian religious book whose main parts were written in the Sassanid era is not surprising.

According to the $21^{\text {st }}$ chapter of the Shāyest Nāshāyest, we can conclude that Zoroastrians, like Muslims, used the shadow length of gnomon for determining their prayer times. And, on the basis of this Zoroastrian text, there is a similarity between the Zoroastrian and Muslim methods of determining the daily prayer times. But since we don't know whether this chapter was included in main parts of the book, we do not claim that the use of a gnomon for determining prayer times was in common in pre-Islamic Iran. Furthermore, we have to take account of one Indian source, inspired by Mesopotamian astronomy, which includes the values of midday shadow length of a gnomon for measuring the time. In fact in the lands around Iran it seems there was a custom of timekeeping which is very similar to the way that is used in the Shāyest Nāshāyest, and various Islamic sources such as Yawāqīt al- 'Ulūm wa Darārī al-Nujūm, Aḥsan al-Taqāsīm and al-Mughnī. All of these things lead us to conclude that it is quite possible the $21^{\text {st }}$ chapter of the Shāyest Nāshāyest was written under the Islamic domination in Iran, and that Zoroastrians were inspired by Muslims to use the shadow length of a gnomon for determining their prayer times, which were also influenced by Muslims.

Finally, our calculations and corrections for the sizes of shadow lengths which were written in the Shāyest Nāshāyest, shows that the original table was composed for a latitude between $31 ; 18^{\circ}$ and $31 ; 42^{\circ}$. The most likely regions in Iran are therefore, the southern part of Zābulistān (31 ) and Yazd (32 $)$. On the basis of the historical information we have about Zoroastrians, it is more probable that the city was Yazd. In addition, the latitude $\left(34 ; 48^{\circ}\right)$ yielding results that best fit the values of shadow lengths that are cited in the Yawāqīt al- ${ }^{〔}$ Ulūm
wa Darārī al-Nujūm. Though the author of this book wrote that the values correspond to the latitudes of the cities Rayy and Qazwīn, our calculations show that the values accord more closely with the latitude of Rayy $\left(35 ; 35^{\circ}\right)$ rather than that of Qazwin $\left(37^{\circ}\right)$. It is noteworthy that for finding the closest cities to the best fitted latitudes in both cases, we used medieval values of the latitudes. We have also corrected some errors in the shadow lengths cited by the authors.

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## Appendix

a. Original Text of the Shāyest Nāshhāyest (Shāyest Nāshhāyest, Mazdapour, pp. 251-253)

## 21

1. Nišān ī sāyag ī nēm-rōz nibēsēm, farrox bawād!
2. Xwaršēd pad karzang, panĵ-ēk < $>$, pāy $\overline{1}$ mard; pānzdahom $\overline{1}$ karzang, ēk pāy; xwaršēd pad šēr, ēk pāy ud nēm; pānzdahom ī šēr, dō pāy; xwaršēd pad hōšag, dō pāy ud nēm; pānzdahom ī hōšag, se pāy ud nēm; tarāzug, čahār pāy ud nēm; pānzdahom ī tarāzug, panĵ pāy ud nēm; gazdum, šaš pāy ud nēm; pānzdahom $\overline{1}$ gazdum, haft pāy ud nēm; nēmasp, hašs pāy ud nēm; pānzdahom ī nēmasp, nō pāy ud nēm; wahīg, dah pāy, pānzdahom $\overline{1}$ wahīg, nō pāy ud nēm; dōl, hašt pāy ud nēm, pānzdahom ī dōl, haft pāy ud nēm; māhig, šaš pāy ud nēm, pānzdahom ī māhig, pan̂̂ pāy ud nēm; warrag, čahār pāy ud nēm; pānzdahom $\overline{1}$ warrag, se pāy ud nēm; gāw, dō pāy ud nēm; pānzdahom $\overline{1}$ gāw, dō pāy; dō-pahīkar, ē pāy ud nēm; pānzdahom $\overline{1}$ dō-pahīkar, ē pāy.
Sāyag ī nēm-rōz wašt, xūb-frazām bawād!
3. Nišān $\overline{1}$ uzērin nibēsēm, xūb ud farrox bawād; pad yazadān ayārīh!
4. Ka rōz pad abzōn bawēd, xwaršēd pad sar ī karzang āyēd ud sāyag šaš pāy ud dō bahr bawēd; uzērin gāh gīrēd.
5. Har sīh rōz-ē pāy-ē ud se ēk-ē hamē abzāyēd, nūn čiyōn hard ah rōz, nēm pāy ōšmār hamē bawēd; xwaršēd pad sar $\overline{1}$ sēr, sāyag haft pāy ud nēm.
6. Pad ēn padisār, har axtar-ē ham-gōnag, ud māhīgān ham-gōnag; tā xwaršēd be sar $\overline{1}$ wahīg āyēd, sāyag čahārdah pāy ud dō bahr bawēd.
7. Andar wahīg, se ēk $\overline{1}$ pāy abāz kāhēd, az anōh abāz wardēd, čiyōn kāhišn $\overline{1}$ šab ud abzāyišn $\overline{1}$ rōz. har māhīgān-ē pāy-ē se ēk-ē hamē kāhēd, čiyōn har dah rōz, nēm pāy ōšmār hamē bawēd; tā abaz $\bar{o}$ šaš pāy ud dō bahr āyēd. har axtar-ē ham-gōnag ud māhīgān hamgōnag.
8. a. Nibišt ud frazāmēnīd hēm, man dēn-bandag mihr-āban $\overline{1}$ kayhusraw hērbed-zād.
9. b. Nibišt xwēš man awestād pešyōtan $\overline{1}$ rām, hērbed.
b. A Manuscript of the Shāyest Nāshāyest Preserved in the Meherjirana Library (Navsari, India), no. 3.3

c. Original Text of the Yawāqīt al- 'Ulūm wa Darārī al-Nujūm ( Yawāqīt al- 'Ulūm wa Darārī al-Nujūm, Mohammad-Taqi Daneshpazhooh, pp. 242-244)

مسئلهُ يب: شناختن زوال آقتاب به اقدام جپكونه باثشد؟


 ديگرباره در زيادت افتن. آن لحظه كه زيادت سايه در توان يافتن، وفت زووال باثشد. و در علم خذاى
 و اما ققر سايه در ازمان و بلدان بكرددد. و غايت طول سايه در عرض فز وزوين و رى يازده قام باثند. اما مثال چنان بود كه چون آقناب به حمل آيد در عشر اول جهار قام و جهار يكـ از قام بعردد، و در عشر
 و چجون به ثور آيد در عشر اول بر سه فقم كم شش يكى از فـدم بكردد، و در عشر دوم بر دو قام و

دو بهر از قام بگر دد، و در عشر سوم بر دو قاقم و شش يكـ از قام بكردد. و چجون به جوزا آيد در عشر اول بر يكـ قام و جهار شش از يكـ قام بگردد، و در عشر دوم بر

قامى و دو بهر از قـمى بعر دد، و در عشر سوم بر قام و نيم بـر دد. و چجون به سرطان آيد در عشر اول بر يكى قام و سه يكـ از قـمى بكرددد، و در عشر دوم بر يک
 و چون بهه اسد آيد در عشر اول بر دو قام و چهار ربع از قدمى بكرَدد، و در عشر دوم بر سه قام و
 و چجون به سنبله آيد در عشر اول بر چهار قام و سدس قـمى بعَردد، و در عشر دوم به جهار قام و
 و چون به ميزان آيد در عشر اول بر شش فـرم بكَردد، و در عشر دوم بر شش قام و سه ربع از قفمى بكردد، و در عشر سوم هفت قام و ربع قـمى بعردد. و چجون به عقرب آرد در عشر اول به هشت قام و سدس قدمى بكردد، و در عشر دوم بر نه قام و
سدس قفمى بكردد، وِ در عشر سوم بر نه قدم و سهِ ربع قدمى بكرددد. و چجون به ڤوس آيد در عشر اول به ده قام بكردد، و در عشر دوم به ده قام و چجهار سدس قامى بعردد، و در عشر سوم به يازده قام بكردد. و چجون به جـى آيد در عشر اول به ده قام و جهار سدس قدمى بگردد، و در عشر دوم به ده قام بكردد، و در عشر سوم به نه قا وم و نيم بكردد. و چجون به دلو آيد در عشر اول بهـ نه قام بڭردد، و در عشر دوم به ده قام و سه ربع بغردد، و در عشر سوم به نه قام و سدس قادمى بكر دد.
 عشر سوم به بينج قلم و ربـعى بعر دد. و اين به تحقيّ نزديكـتر است. و آن بهتر باثشد كه مؤذن چجون مقياس معتمد دارد روز به رووز ساية

[Fig. 1.]
و اين مشهور است نزديكى صوفيان و زهاد و عباد كه بدان مراعات سايه كننذ براي اوقات نماز و عبادات. مصطفى صلى اله عليه و سلم مىفرمايد: »خيار عبادالها الذين يحبون الشه ويحبون الى عباده

والذين ير اعون الثمس والنجومو والظلة لذكر الهن تعالى) در معرفت وقت نماز ديكر نيك احتياط كند. آن
 سر آن سايه زوال آن روز افز ايد. چون سايه اينجا رسد، وقت نماز ديكر باثشد. و الهّ اعلم بالصواب. [اين مغنى در اين فن بسنذه كئيم هرچند سخن درازتر مىشود.]


[^0]:    ${ }^{1}$ This paper develops a part of my M.A. thesis entitled: "Theoretical Foundations of the Sundials in Islamic Civilization, with Translation and Commentary of Ibn al-Haytham's Risāla fí alRukhāmāt al-Ufuqīya (Treatise on Horizontal Sundials)" under the supervision of Prof. M. Bagheri.

[^1]:    ${ }^{2}$ Neugebauer, pp. 736-740
    ${ }^{3}$ Pingree, pp. 1-12, especially pp. 3-6
    ${ }_{5}^{4}$ Harris, http://archive.org/stream/jstor-287167/287167_djvu.txt
    ${ }^{5}$ Tiberius was a monk who resided in a monastery situated in a valley of the Anglo-Saxon countryside at very nearly $53^{\circ}$ latitude and prepared the timekeeping table there around 850 A.D. Kellogg and Sullivan, pp. 8-9
    ${ }^{6}$ Kellogg and Sullivan, pp. 1-13
    ${ }^{7}$ On Muslims' method for determining their prayer times based on the shadow lengths see King, 1990, pp. 192-204; On the origin of the definitions see King, 2004-05, "On The Times of Muslim Prayer".

[^2]:    ${ }^{8}$ This calendar included 365 days in twelve months and five extra days at the end of the twelfth month and one extra month per 120 years. Al-Bīr̄ū̄̄, 1923, pp. 43-45; Taqizadeh, p. 52-69
    ${ }^{9}$ Here is al-Bīrūnī's account of the $\underline{S h} \bar{a} h Z_{\bar{\jmath} j}$ method for finding the solar hour angle: "In the $\underline{S h} \bar{a} h$ $Z_{\bar{l}} \vec{j}$, for ascertaining the (part of the day) passed he directs division by the sine of the altitude at the time, of a thousand and eight hundred. There comes out the hypotenuse of the shadow for that time, and by it he divides the product of the length of the computed sine (i. e., the day sine) and the hypotenuse of the noon shadow. What comes out is subtracted from the length of the computed day sine, and the remainder he subtracts from a hundred and fifty, and the arc sine of the remainder is found. And so it will be the equation of the sine. If the altitude is easterly, subtract the equation of the sine from ninety, and if the altitude is westerly increase by it (the) ninety, and there results the arc of revolution of the sky". Kennedy, 1976, vol. 1, p. 199.
    ${ }^{10}$ Prof. M. Bagheri was first informed by the late Mr. H. San'ati Zadeh of the presence of this passage orally and he conveyed the idea to me.
    ${ }^{11}$ Shāyest Nāshāyest, p. 13 (intro)
    ${ }^{12}$ Ibid, p. 19-20 (intro)
    ${ }^{13}$ Boyce, p. 153
    ${ }^{14}$ Ibid, p. 1

[^3]:    ${ }^{15}$ Ibid, p. 32
    ${ }^{16}$ Shāyest Nāshhāyest, pp. 251-253
    ${ }^{17}$ As we know, the variation of the solar declination is not linear over the course of the year and it decreases very slowly from $90^{\circ}$ to $120^{\circ}$ of ecliptic longitude (from the beginning of Cancer to the beginning of Leo); see the third paragraph of the part "The Values of Shadow Length for the Uzerīn Prayer Time" of the present paper.
    ${ }^{18}$ In a manuscript of the Shāyest Nāshāyest which is preserved in Navsari (India), this chapter is written under the title "The Truth of the Rapīthwan Prayer Time and the Uzerīn Prayer Time", Meherjirana, f. 26 v
    ${ }^{19}$ King, 1990, pp. 191-249
    ${ }^{20}$ Ptolemy refers to seven climata in his Almagest, Book VI, Part 11, p. 315

[^4]:    ${ }^{21}$ al-Muqaddasī, pp. 59-61; al-Ḥamawī, Vol. 1, pp. 33-36; al- Urḍī, pp. 55-61; al-Bīrūnī, 1988, p. 190
    ${ }^{22}$ Berggren and Jones, p. 28
    ${ }^{23}$ King, 2004-05, vol. 1, pp. 476-477, 495, 507
    ${ }^{24}$ Ansari, p. 329-332; Mr. Daneshpazhooh stated that the author of this book is unknown [Yawāqīt al- 'Ulūm wa Darārī al-Nujūm, Edited by Daneshpazhooh, pp. 4, 6 (Intro)]
    ${ }^{25}$ Yawāqūt al- ‘Ulūm wa Darār̄̄̄ al-Nujū̀m, pp. 242-244; This work is published in Iran based on the three manuscripts: Tehran, Majlis, No. 5943; Tehran, Loghatnameh Dehkhoda Institute, No. 19; Aya Sofia Library, No. 4359
    ${ }^{26}$ Modarres Razavi, pp. 622-623; King, 2004-05, vol. 1, pp. 223-225; Noori, pp. 315-317

[^5]:    ${ }^{27}$ This English translation has been published in the book The Supplementary Texts to the Šäyest $n \bar{e}-$ Šäyest by Kotwal (Kotwal, pp. 87-89). In this translation, Kotwal has used modern numerals for the values, unlike the original text. So, we have replaced the numerals by the words here.
    ${ }^{28}$ This chapter has been written under the title "The Truth of the Rapīthwan Prayer Time and the Uzerīn Prayer Time" in the Meherjirana Library manuscript (f. 26 v).

[^6]:    ${ }^{29}$ For this translation, we have used two versions of this work; one of them is published in 1985 by the late Mohammad-Taqi Daneshpazhooh, and the other is a manuscript which is preserved in the Loghatnameh Dehkhoda Institute (no. 95). In the cases of having different records of values for a certain shadow length, we have chosen the correct one and cited the other in the footnote. In the following, we will use "LD" for the manuscript.
    ${ }^{30}$ Qazwīn (Qazvīn) is one of the ancient cities of Iran which is located in 150 km northwest of Tehran.
    ${ }^{31}$ Today, Rayy (Ray or Rey) is the oldest existing city in the province of Tehran.
    ${ }^{32}$ LD: Sixth a foot less than three feet
    ${ }^{33}$ LD: Two feet and two parts
    ${ }^{34}$ LD: Two feet and a sixth
    ${ }^{35} \mathrm{LD}$ : There is no information about the shadow lengths when the sun enters Taurus.

[^7]:    ${ }^{36}$ Although this value is cited in both of our sources, it is incorrect literarily and theoretically. According to the theoretical foundations of sundials, we will use two feet and "a quarter" of a foot for our calculations in this case.
    ${ }^{37}$ LD: Three feet and a sixth and a half
    ${ }^{38}$ LD: There is no information about the third ten days
    ${ }^{39}$ LD: There is no information about the second ten days

[^8]:    ${ }^{40}$ LD: Eight feet and a half
    ${ }^{41}$ LD: Seven feet and a half
    ${ }^{42}$ LD: Eight feet and a half

[^9]:    ${ }^{43}$ King, 2004-05, vol. I, p. 476
    ${ }^{44}$ Ibid, pp. 473, 495-497, 502, 514-517
    ${ }^{45}$ al-Bīrūnī, 1948, p. 36
    ${ }^{46}$ In the following, we will show that the rounding off the values is the author's most probable method to cite them. For this purpose, we will test the interpolation method too.

[^10]:    ${ }^{47}$ I would like to express my sincere gratitude to my dear friend Mr. Moeen Esghaei for his help in preparing this computer program.
    ${ }^{48}$ Zābulistān is a historical region based around today's Zābul province in southern Afghanistan and includes a part of today's Sīstān in Iran. Al-Bīrūnī (2002, p. 52) indicated the latitude of Zābulistān between $31^{\circ}$ and $33^{\circ}$.
    ${ }^{49}$ Yazd is a historical city located in central Iran. Its latitude was indicated $32^{\circ}$ in medieval Islamic sources (Kennedy, 1987, p. 379) which is very close to the modern value $31 ; 53^{\circ}$
    ${ }^{50}$ E. W. West states that the mean result derived from the calculated latitudes (based on each individual shadow length value), is in accordance with the latitude of Yazd. West, p. 399
    ${ }^{51}$ Boyce, pp. 162-164, 175

[^11]:    ${ }^{52}$ For calculating these shadow lengths, we have used al-Bīrūn̄̄'s value for number of feet in one's height (7). Furthermore, choosing the value $23 ; 26^{\circ}$ or $24^{\circ}$ for the declination of ecliptic doesn't affect our final results about determining the cities. Since the variation of the obliquity of the ecliptic is negligible, we can assume it as a constant equal to $23 ; 26^{\circ}$. Meeus, pp. 131-136

[^12]:    ${ }^{53}$ Shāyest Nāshāyest, Meherjirana, f. 26 v
    ${ }^{54}$ Kotwal, pp. 86-88
    ${ }^{55}$ Shāyest Nāshhāyest, Meherjirana, f. 27 r

[^13]:    ${ }^{56}$ Kellogg and Sullivan, pp. 10, 11; Shāyest Nāshāyest, pp. 251-253
    ${ }^{57}$ Oshidari, p. 140; According to Zoroastrian clergies, the Zoroastrians' prayer times are delineated based on an oral tradition (and not mathematical or astronomical background) by their clergies.
    ${ }_{59}^{58}$ Shāyest Nāshāyest, pp. 252, 253
    ${ }^{59}$ In the English translations of the Shāyest Nāshāyest and Yawāqīt al- 'Ulūm wa Darārī al-Nujūm (parts 2a. and 2b. of the present paper), "Bahr" was translated to "Part", but in the old Persian language "two bahr" was equal to $\frac{2}{3}$ (Dehkhoda, Vol. 3, p. 4439).

[^14]:    ${ }^{60}$ In IND, this value is written 8.50, Meherjirana, f. 27 r
    ${ }^{61}$ Shāyest Nāshāyest, p. 253

[^15]:    ${ }^{62}$ Ansari, p. 329-332
    ${ }^{63}$ King, 2004-05, vol. I, pp. 508-509
    ${ }^{64}$ In this part of the mentioned book, al-Najjār tried to answer a common person who asked him how one can determine the midday [for praying] simply.
    ${ }^{65}$ Yawāqīt al- 'Ulūm wa Darārī al-Nujūm, p. 242
    ${ }^{66}$ We chose the mentioned values of latitude for Qazwīn and Rayy according to al-Qānūn alMas ${ }^{〔} u ̄ d \overline{1}$ and some other medieval Islamic sources. Al-Bīrūnī, 2002, p. 60; Kennedy, 1987, p. 270, 284
    ${ }^{67}$ For this purpose we have used the mathematical methods which are discussed in the beginning of the part B. Similar to the calculations for the Shāyest Nāshāyest, choosing the value $23 ; 26^{\circ}$ or $24^{\circ}$ for the ecliptic declination doesn't affect our final results about determining the cities for the Yawāqīt al- 'Ulūm.

