Remarks on two dimensional array tables in Latin astronomy: a case study in layout transmission

Matthieu Husson

Abstract: Several aspects of astronomical tables can be transmitted from one source to another: numerical parameters, underlying functions and theoretical models, computational methods, etc. The concern here will be on the transmission of layout. We will study how a certain family of layouts was used in European Latin astronomies after its transmission from Arabic sources. Three different stages of the circulation of this layout will be identified: first transmission, assimilation and innovation. Moreover we will see how two distinct uses of the layout will finally converge in the compilation of large trigonometric tables in the 15th and 16th centuries.

Keywords: Astronomy, transmission, tables, layout.

1. Introduction: Defining the object and issues of the case study

Numerical tables in general and astronomical tables in particular are complex and multilayered objects. Several aspects of astronomical tables can be transmitted from one source to another: numerical parameters, underlying functions and theoretical models, computational methods, degree of precision, etc. The concern here will be on the transmission of layout. We will study how a certain family of layouts was used in European Latin astronomies after its transmission from Arabic sources. Beyond this examination lie more general historical questions. Analysing layout transmission is an opportunity to understand the different meanings a table layout may have according to the

Suhayl 13 (2014), pp. 103-117

diverse users be they rulers, astronomers, or practitioners. In this study we will focus mainly on the astronomers' perspective. We will see how the layout of the tables is closely linked to a set of computational techniques and to certain representations of numbers. Thus, once layout transmission paths are identified we intend, in other publications, to deepen the study by looking at the possible transmission of these computation techniques along with tables' layout.

The occasion for this case study is given by a single set of tables for the latitude of the planets, found in the Mingshi edition of the *Huihui lifa* calendar system (1739)[1]. The point of interest in this set of tables is the characterisation of its layout as "European" by the Mingshi's compilers and editors. In fact, there is a historical puzzle here as the "European" layout is not an invention of European table makers but rather a creation of Islamic astronomers. In this paper we will focus on the transmission of this layout from Arabic to Latin sources. Another paper of this issue will examine the transmission to Chinese sources and thus solve this little historical puzzle[2].

Let's now describe with more precision the notion of layout in the tabular context. First of all it is important to distinguish between the numerical data and the layout which is used to display it. In the context of this study we will encounter two types of tables: single argument tables and double argument tables. Numerical tables were presented in many ways. For instance one can state an algorithm allowing the computation of any value in a table however the most common way to represent a table is to use an array (i.e. a grid of rows and columns in which the numerical data is written) and, where necessary, to associate this array to an interpolation scheme.

These arrays can be laid out in many ways. The layout which will be at the centre of our attention here has two characteristics. First the array is two dimensional: at least first row and first column are used to display the entries or arguments of the tables. Second the array is "double entry" or bidirectional: entries can be either read from left to right starting at the top row and from top to bottom starting from the first column, or from right to left starting from the last row and from bottom to top starting from the last column (see fig. 1). This type of layout was often used to display double argument tables. However single argument tables could be represented as well in ways we will expose below. Eventually we will not restrict ourselves strictly to this type of layout but will look more broadly at tables displayed in two dimensional arrays.

---

[1] See Yunli Shi (2014). "Islamic Astronomy in the Service of Yuan and Ming Monarchs", *Suhayl,* 103-117, for more precision about the Mingshi.

[2] Liang Li (2014). "The adoption of "European" layout in China: A case study in tabular layout transmission", *Suhayl*, 83-101.

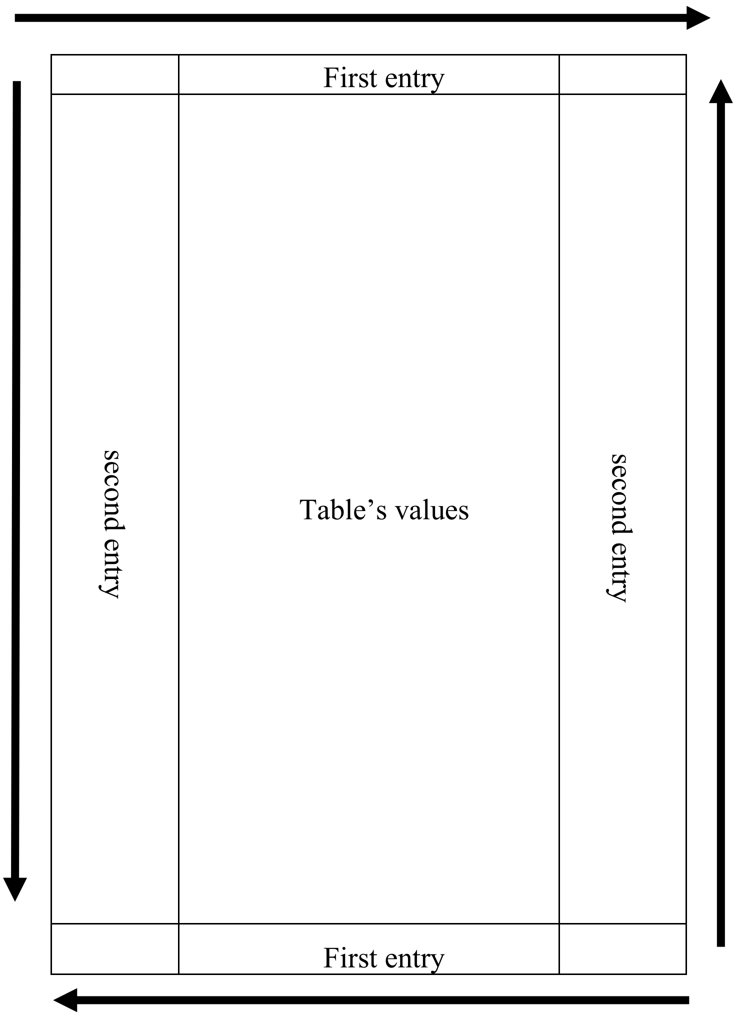


Figure 1: schematic presentation of the "European" Layout: Arrows represent reading directions

We begin this paper by pointing at the first known uses of these layouts in Arabic sources and trace briefly their diffusion inside the Arabic context. We then turn to Latin and related vernacular sources and survey how these sorts of layout were transmitted and used up until the 16th century.

2. The *Ḥabṭaq* tables

The main goal of this first part is to show that the specific layout here under consideration was present, with its two characteristics, in the Arabic traditions of astronomical tables. We also need to show that, although not mainstream, this layout was not uncommon and could be found in various sort of tables from the Iberian Peninsula to Persia. Of course completeness is not required and not pursued there.

According to D. A. King[3], in 10th century Cairo, Ibn Yūnus designed a two argument table for the lunar equation with the precise layout we are studying here. This is the earliest acknowledged instance of this type of layout for tables in mathematical astronomy. In the Arabic sources this layout, when used with two argument tables for planetary equations either in longitude or latitude, was later known as *ḥabṭaq* types of tables. Thus, following this denomination, we will now use the expression "*ḥabṭaq* layout" in order to refer to the type of layout which is at the centre of inquiry here and we will use the expression "pseudo-*ḥabṭaq* layout" when at least one of the two characteristics of the "*ḥabṭaq* layout" is present in a numerical table [4].

Since the 10th century, although it never became a dominant way to organise astronomical data , the *ḥabṭaq* layout was used in several Arabic tables. As Yunli Shi and José Chabás' contributions to this issue clearly establish[5], the 12th and 13th centuries are key periods for the transmission of Arabic astronomical tables both in Latin and in Chinese. Thus the two following instances will be from the 12th century and taken both from western and eastern traditions of Arabic astronomies.

Apparently the *ḥabṭaq* layout was first used to simplify complex astronomical computations like those of the planetary equations. However this layout can also be applied to much simpler astronomical questions. Such is the case, for

---

[3] D. A. King (1974). "A double argument table for the lunar equation attributed to Ibn Yūnus", *Centaurus*, 18:29-146.

[4] It is important here to note that we are creating a new term for the purpose of our analysis: "*ḥabṭaq* layout" does not designate a type of table but a specific type of array.

[5] José Chabás (2014). "Aspects of Arabic Influence on Astronomical Tables in Medieval Europe", *Suhayl,* 23-40, and Yunli Shi (2014). "Islamic Astronomy in the Service of Yuan and Ming Monarchs", *Suhayl,* 41-61.

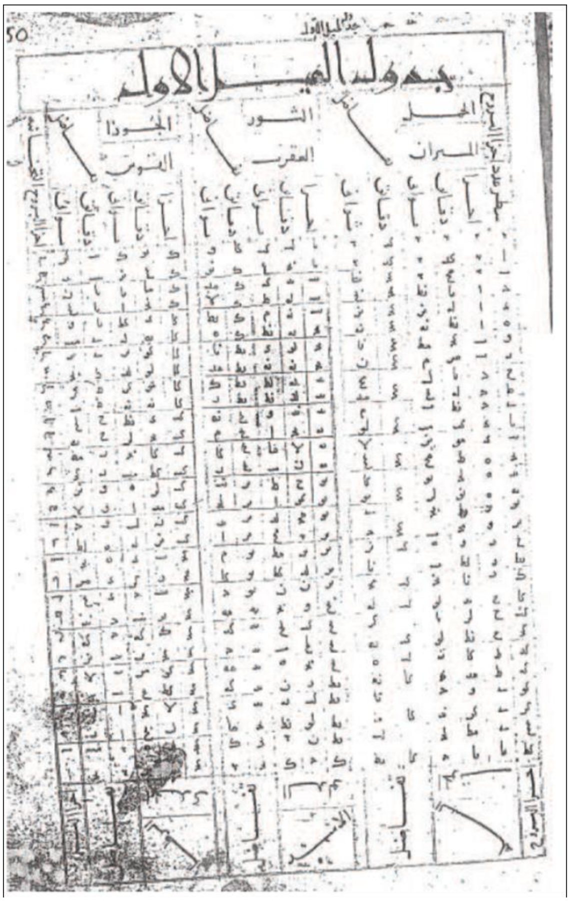
instance, when al-Khāzinī (12th century) in Marw (now Turkmenistan) compiled a declination table (see fig. 2) using exactly the *ḥabṭaq* layout but for a single argument function[6]. There the horizontal entries of the two dimensional array are used for zodiacal signs and the vertical entries are used for degrees. This type of use of the *ḥabṭaq* layout for single argument tables has at least two interesting features: first it takes advantage of and makes evident some of the symmetries of the numerical data; second it does this by noting the numbers used in the table in a specific way (presenting them with signs and degrees, rather than degrees only, in this case). These two features then have consequences on the way the table is used and on the way astronomical (if any) phenomena are depicted.

This use of the *ḥabṭaq* layout is slightly different from Ibn Yūnus. Ibn Yūnus used the layout to make a complex computation easier: instead of combining many different values from several single argument tables according to a complex algorithm one only needs to look up one value. However, in Ibn Yūnus type of uses of the layout, a large number of values need to be computed if the table compiler wants to avoid interpolation and obtain precision. Thus Ibn Yūnus use of the *ḥabṭaq* layout typically results in the production of big numerical tables. These tables exist in the eastern, central, and western traditions of Arabic mathematical astronomy and are a recurrent topic in modern scholarship on mathematical astronomy in Arabic sources[7]. In constrast, al-Khāzinī uses the *ḥabṭaq* layout to make a compact presentation of a simple table. It is important to keep in mind these two uses of the *ḥabṭaq* layout. The *ḥabṭaq* layout has an inner plasticity which allows it to be used in several ways.

---

[6] al-Khāzinī is probably not the first to use two dimensional arrays to represent single argument tables (See for instance the "table of the equinoctial circle" in al-Khwārizmī: table 59 and 59b in Suter, Heinrich (1914). *Die astronomischen Tafeln des Muḥammed ibn Mūsā al-Khwārizmī in der Bearbeitung des Maslama ibn Aḥmed al-Madjrītī und der lateinischen Übersetzung des Adelhard von Bath.* Copenhagen: Kongelige Danske Videnskabernes Selskab. (Reprinted in Suter, *Beiträge zur Geschichte der Mathematik und Astronomie im Islam.* Vol. 1, pp. 473–751. Frankfurt am Main, 1986.)).

[7] See among others the following Mark J. Tichenor. (1967). "Late medieval two argument tables for planetary longitudes", *JNES*, 26 (2): 126-8; Claus Jensen (1972), "The lunar theories of al-Baghdādī", *Archive for the history of exact sciences*, 8 (4): 321-6; George Saliba (1976). "The double-Argument lunar tables of Cyriacus", *JHA,* 7:41-6; George Saliba (1978). "The planetary tables of Cyriacus",*JHAS*, 2: 53-65; Glen van Brummelen (1998). "Mathematical methods in the tables of planetary motion in Kūshyār ibn Labbān's Jāmi Zīj", *Historia Mathematica*, 25 (3): 265-80; Benno van Dalen (1999). "Tables of planetary Latitude in the "Huihuili" in Yung Sik Kim and Francesca Bray (eds.). *Current Perspectives in the History of Science in East Asia*, Seoul National University Press, 315-29. Julio Samsó (2003). "On the Lunar tables in Sanjaq Dār's Zıj al-Sharīf", in Jan P. Hogendijk and A. I. Sabra (eds.), *The enterprise of Science in Islam, New perspectives*, MIT press, 285-308.



جدول الميل الاول

الحمل الثور الجوزا

الميزان العقرب القوس

Figure 2: al-Khāzinī declination table from the *Wajīz al-Zīj al-muʿtabar al-sulṭānī*; MS Istanbul Suleymaniye Library 859, f. 39.

Now looking toward the West the first known example of a two dimensional array for the layout of a table is to be found in Ibn al-Kammād's (12th century) syzygy table[8]. In this particular case we can't say that the *ḥabṭaq* layout was used as the double argument function underlying the table does not have the appropriate symmetries. It's interesting to note that Ibn al-Kammād's work is

---

[8] Chabás, J. and Goldstein, B.R. (1997). "Computational astronomy: Five centuries of finding true syzygy". *JHA*, 28:93-105 and Chabás, J. and Goldstein, B.R. (2012). *A Survey of European Astronomical Tables in the late Middle Ages*, Leiden:146

among the first to address the syzygy problem by means of a table. From this initiation many double argument tables were developed from the 13th to the 17th centuries in the Maghreb and al-Andalus[9].

Thus, as a conclusion for the first section of this paper we can say that in the 12th century the *ḥabṭaq* layout, or at least a use of two dimensional array layout, was present in the Arabic astronomical tables, in the Eastern, Centre and Western traditions at a time when transmission to Europe and China was about to become intensive. We can also say that the *ḥabṭaq* layout was well distributed in the astronomical corpus both in simple and complex astronomy: it was used in simple trigonometrical problems like the sun's declination, or in more intricate planetary equations and even in syzygies and eclipse computations.

3. *Ḥabṭaq* layout in Latin sources from the late Medieval and Early Modern periods

From this base we continue toward the West and we now look at the way these sorts of layouts were transmitted and used in the Latin European tradition during the Middle Ages. José Chabás' contribution to this issue forms the necessary background to the more specific information that we will present here. The aims and limits of this section are the same as those of the preceding one: completeness is not pursued nor necessary for our purpose which is to understand the main transmission and reception trends of the *ḥabṭaq* layout in late Medieval and Early Modern periods.

Complex uses of *ḥabṭaq* or pseudo *ḥabṭaq* layouts are not found in Adelard of Bath's translation of al-Khwārizmī's tables (1126)[10], nor in Raymond of Marseilles' adaptation of the Toledan tables (1141)[11], nor in Plato of Tivoli's translation of al-Battānī's canons which come with no tables at all'[12]. However we have seen that al-Khwārizmī uses the *ḥabṭaq* layout for single entries tables and al-Battānī does the same[13]. Similarly, in the very diverse traditions of Toledan tables which circulated in 13th century Europe, some use two

---

[9] Samsó. J. (1998). "An outline of the history of Maghribī zījes from the end of the thirteenth century.", *JHA*, 29:93-102

[10] Neugebauer, O.(1962). *The astronomical tables of al-Khwārizmī*, Copenhagen

[11] D'Alverny, M.-T., Poulle, E., and Burnett, C. (2009). *Raymond de Marseille*, *Opera omnia*, *t. I*, Paris

[12] Dalen, B. van and Pedersen, F.S. (2008). "Re-editing the tables in the *Ṣabi' Zīj* by al-Battānī", in J. Dauben et al.(eds.). *Mathematics Celestial and Terrestrial: Festschrift für Menso Folkerts zum 65. Geburtstag*, Halle, 405-428. The canons of the tables usually contain enough information for us to know whether or not a *ḥabṭaq* layout is used.

[13] C. A. Nallino (1903-1907). *Al- Battānī sive Albatenii Opus astronomicum,* 2 vols, Milan, vol II: 61

dimensional arrays and the *ḥabṭaq* layout. We find a declination table[14] very similar to al-Khāzinī's table for the same purpose. It appears that in the Toledan traditions this layout was only used in mathematically simple cases like this one. Other instances concern multiplication and division tables[15]. The "simple"' uses of the two dimensional array type of layout are thus the first attested in the Latin sources and we will see that they will remain present until the end of the chronological frame of this study.

Along with the various adaptations of the Toledan tables, one more ambitious enterprise was done in Castillan ca. 1271 under the patronage of king Alfonso X of Castile (1221-1286) by two scholars, Isaac ben Sid and Judah ben Moses ha-Cohen[16]. The Castilian Alfonsine tables, of which only the canons are extent, are a part of a global endeavour which concerned all aspects of medieval astronomy with treatises on cosmology, astronomical instruments, other type of astronomical tables, astrology, and magical arts. From the Castilian canons of the Alfonsine tables we can infer that the type of layout we are considering was not used in the tables. However astronomical material under the name of Alfonso but different in many respects from the Castilian version appears in Paris some fifty years later. Part of this material is profoundly reworked by a handful of Parisian astronomers between 1320 and 1330 the most important being John of Murs (14th century), John of Lignières (14th century) and John of Saxony (14th century). Their work formed the core of the Parisian Alfonsine tables. Well into the 16th century, various versions of these tables were the standard astronomical computational tool in Europe.

In the first testimonies of the Parisian Alfonsine corpus we see a strong presence of two dimensional arrays and *ḥabṭaq* layout. In fact these types of layout are suddenly used in a wide variety of computations quickly gaining the same extension they had in the Arabic sources of the same period. We will give instances of such layout in syzygy computations and in planetary equations for longitude and latitude. The earliest extant Parisian Alfonsine set of tables, produced by John Vimond (14th century) in around 1320 reproduces Ibn al-Kammād's system and layout for the computation of syzygies[17]. He was followed in this respect by John of Lignières in his *Tabule magne*. The *Tabule*

---

[14] Pedersen, F. S. (2002). *The Toledan Tables : A Review of the manuscripts and the textual versions with an edition*, Copenhague: 965.

[15] Pedersen, F. S. (2002). *The Toledan Tables*: 1613-1615.

[16] Chabás, J. and Goldstein, B.R. (2004). "Early Alfonsine Astronomy in Paris: The Tables of John Vimond (1320)", *Suhayl*,4: 207-294

[17] Goldstein, B.R. and Chabás, J. (2004). "Ptolemy, Bianchini, and Copernicus: Tables for Planetary Latitude". *Archive for History of Exact Sciences*, 58:453-473

magne are a set of tables produced in Paris between 1320 and 1330. They make an extended use of the two dimensional array and double argument tables. Three manuscript copies of the tables are known[18], but none contain tables for the computation of syzygies. Yet we can infer the presence of Ibn al-Kammād types of table for this purpose from the instructions in the text itself. John of Lignières uses, like his Andalusian predecessors, elongation and the so-called *superatio*[19] as arguments at intervals of 1º and the algorithm proposed to use the table is similar.

Interestingly enough, one manuscript of the *Tabule magne*[20] does have a table of the time between mean and true syzygies but it does not correspond to John of Lignières' canons and is of a completely different type. This table uses the mean argument of the Moon and the mean centre of the Sun at intervals of 6º as arguments. It computes directly the time interval between mean and true syzygy and follows exactly the *ḥabṭaq* layout. Luckily we find this same table in another set compiled in Paris by John of Murs: the *Tabule permanentes* (see fig. 3). The mathematical structure and the history of the *Tabule permanentes* are very well known. The genesis of the tables in the early astronomical works of John of Murs was studied recently by R.L Kremer and the table's values were correctly recomputed according to a procedure described by John of Saxony in his 1327 canons for the Alfonsine tables[21]. The canons of the tables have been edited[22]. More significantly for us, it was shown earlier by Porres and Chabás that the *Tabule permanentes* are, through John of Gmunden (15th century), the initial source of Georg Peurbach's (1423-1461) *Tabula distantie vere coniunctionis aut oppositionis a media*[23]. This makes John of Murs' works an important element in the presence of the *ḥabṭaq* layout in central European astronomical traditions of the late Middle Ages and Early Modern period.

John of Lignières and John of Murs did not limit their use of double argument tables to the computation of syzygies: both compiled *ḥabṭaq* layout tables for planetary equations in longitude. John of Lignières did so in the *Tabule magne*

---

[18] Erfurt F 4º 388 ff. 1r-42v (first half of the 15th century); London, B.L. addit. 24070 ff.1-57v ; Lisbon Ajuda 52-XII-35 ff. 66v-93r (only the equations)

[19] *Superatio* is the difference between the Sun's and the Moon's velocities.

[20] Erfurt F 4º 388 ff. 1r-42v

[21] Kremer, R. L. (2008). "John of Murs, Wenzel Faber and the computation of true syzygy in the fourteenth and fifteenth centuries", in J. W. Dauben et al. (eds.). *Mathematics celestial and terrestrial: Festschrift für Menso Folkerts zum 65. Geburtstag*, Halle: 147-160.

[22] Porres. B. And Chabás J. (2001). "John of Murs's *Tabulae permanentes* for finding true syzygies", *JHA*, 32:63-72

[23] Porres. B. And Chabás J. (2001). "John of Murs's *Tabulae permanentes* for finding true syzygies", *JHA*, 32:63-72

and John of Murs in the *Tabule principales*[24] (see fig. 4). John of Lignières is also responsible for double argument tables for the latitude of Venus and Mercury[25]. His works were transmitted very early to England where they are the sources of the Oxford or Batecombe tables (1348)[26]. These tables use the *ḥabṭaq* layout for latitude tables as well. Eventually the same type of double argument latitude tables are found in John of Gmunden's works (15th century)[27] and in Zacut's (15th century) *Almanach perpetuum*[28]. In Italy Giovani Bianchini (15th century) composed an important set of tables, extant in many manuscript copies and printed several times[29]: they contain double argument tables for the moon's and planets' equation, using the *ḥabṭaq* layout.

The simple uses of the two dimensional array were not eclipsed by the proliferation of *ḥabṭaq* layout tables in complex astronomical computations which followed the work of the early 14th century group of Parisian astronomers. Beginning with the most elementary, multiplication and proportion tables were copied from manuscript to manuscript in some cases accompanied by very interesting short instruction texts written by important astronomers like John of Murs in the 14th century or John of Gmunden in the 15th century.

---

[24] For a survey see Goldstein, B.R. and Chabás, J. (2009). "John of Murs's tables of 1321".*Journal for the History of Astronomy*, 40:297-320 and for manuscripts see Lisbonne, Ajuda 52-VI-25, fol. 26v-66r et Oxford, canon.misc.501, fol. 45r-106v.

[25] Goldstein, B.R. and Chabás, J. (2004). "Ptolemy, Bianchini, and Copernicus: Tables for Planetary Latitude". *Archive for History of Exact Sciences*, 58:453-473

[26] North, J. D. (1977). "The Alphonsine Tables in England", in Y. Maeyama et W. G. Salzer (eds.), *Prismata : Festschrift für Willy Hartner*, Wiesbaden, 269-301.

[27] Chabás, J. and Goldstein, B.R. (2004). "Early Alfonsine Astronomy in Paris: The Tables of John Vimond (1320)", *Suhayl*, 4: 207-294 and Porres, B. (2003). *Les tables astronomiques de Jean de Gmunden : édition et étude comparative*, thèse de l'EPHE. Paris

[28] Chabás, J. and Goldstein, B.R. (2000). *Astronomy in the Iberian Peninsula: Abraham Zacut and the Transition from Manuscript to Print*, Philadelphia

[29] Chabás, J. and Goldstein, B.R. (2009) *The Astronomical Tables of Giovanni Bianchini*, Leiden-New York: Brill

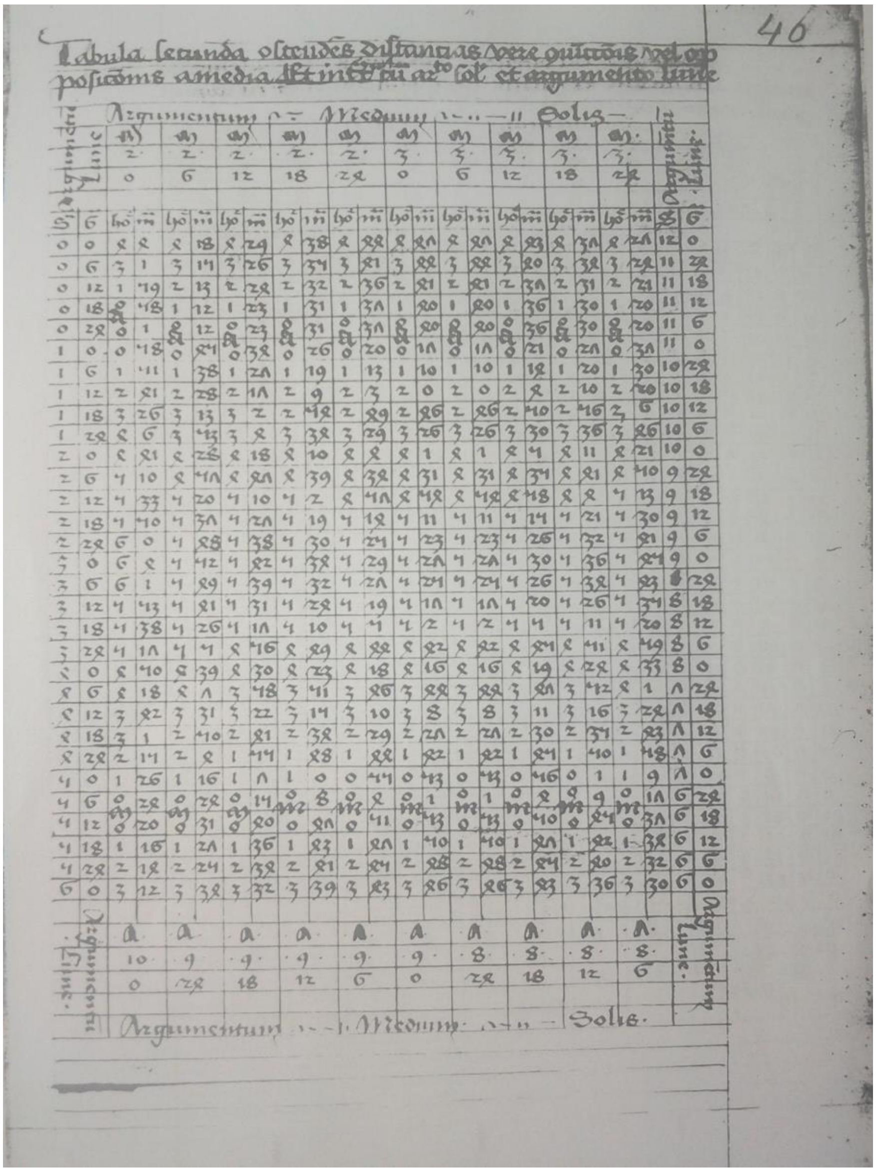


Figure 3: John of Murs syzyggy table, Erfurt F 4 388 f.40r

Ḥabṭaq layout declination tables also spread until the 16th century. The Oxford or Batecombe tables (1348), which were mentioned earlier regarding equation tables in longitude and latitude, also contain a declination table using the *ḥabṭaq* layout like those of al-Khāzinī. A copy of this table can be found in Florence Ms San Marco 185 at f.121v. The obliquity of the ecliptic used is 23;30º. This is the earliest known attestation of this value in the Latin sources. Later it is notably found in Regiomontanus' *Tabule directionum* first printed in 1490. In the *Tabule directionum* there are in fact two different declination tables. The first is a classical declination table but with values significantly diverging from those of the Oxford tables suggesting they were computed anew according to a different algorithm. The second, found on fol.1-7r, is an extended declination table with two arguments. Like in a classical declination table, the first argument is the ecliptic longitude and the second argument is the latitude ranging from 8º below to 8º above the ecliptic. When only objects on the ecliptic are considered (i.e. when the second argument is 0º) this declination table is closer, although not entirely identical, to the Oxford one.

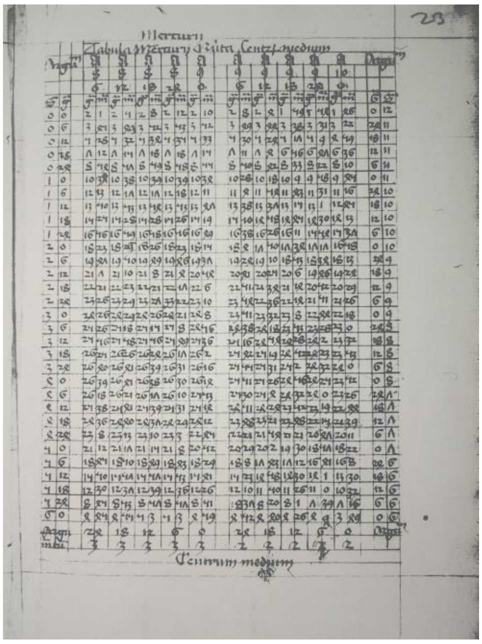


Figure 4: John of Lignières' Mercury equation table Erfurt F 4º 388 f.23

Regiomontanus' choice to make a simple table more complex by extending the number of situations in which it can be used is important for us because it creates a link between the simple, epitomised here by al-Khāzinī's declination table, and the complex, epitomised here by Ibn Yūnus lunar equation tables, uses of the *ḥabṭaq* layout. But before describing other instances where Regiomontanus made similar choices, let us finish the description of the diffusion of the declination table. In 1537, the famous Portuguese cosmographer Pedro Nunes (1502-1578) published *Tratado da Sphera*, a translation of Sacrobosco's *Tractatus de Sphaera* in which he included Peuerbach's *Theoricae novae planetarum* as well as Book I of Ptolemy's *Geographia*. However, the book also contained two very important contributions of Nunes, the *Tratado que ho doutor Pero nunez fez sobre certas duuidas da nauegacio* and *Tratado que ho doutor Pero nunez Cosmographo del Rey ... fez em defensam da carta de marear* .... In the second contribution Nunes gave a declination table which is only an excerpt of the complex Regiomontanus tables where only ecliptic values are conserved[30]. This table was soon inserted in Cortés' (1510-1582) *Breve compendio de la esfera y del arte de navegar* first printed in 1551[31]. The English translation of this almanac (1561)[32] quickly developed into the most widely used navigational compendiums in the second half of the 16th century, by 1630 there were at least nine editions.

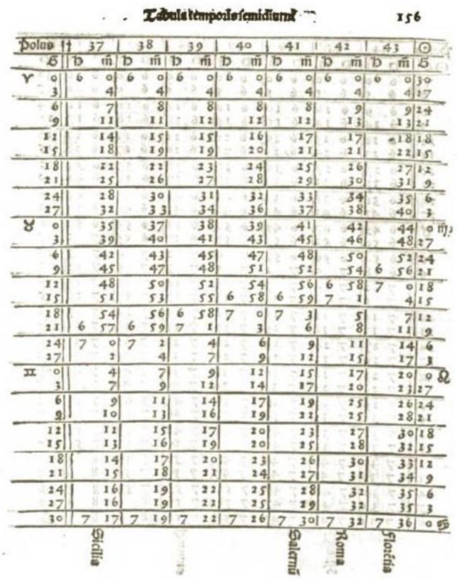
In the *Tabule directionum* Regiomontanus made the same sort of choices in many other "simple" astronomical and spherical tables. For instance, tables for half the length of daylight are computed simultaneously for several latitudes (see fig.5) and are displayed with a pseudo-*ḥabṭaq* layout. This is a new alteration of the *ḥabṭaq* layout where two reading directions are used for the vertical entry, but only one direction of reading is used horizontally in the first row.

---

[30] For Nunes works see the online edition at : http://pedronunes.fc.ul.pt/works.html

[31] Cortés, M. (1551). *Arte of navigation.* (reprinted: Scholar's Facsimiles & reprints, Delmar,New York, 1992)

[32] See reference in note 27.



Tabula temporis semidiurni 156

Polus		37		38		39		40		41		42		43		☉	
	G	h	m	h	m	h	m	h	m	h	m	h	m	h	m	G	
♈	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	30	
	3		4		4		4		4		4		4		4	27	
	6		7		8		8		8		8		9		9	24	
	9		11		11		12		12		12		13		13	21	
	12		14		15		15		16		17		17		18	18	
	15		18		19		19		20		21		21		22	15	
	18		22		22		23		24		25		26		27	12	
	21		25		26		27		28		29		30		31	9	
	24		28		30		31		32		33		34		35	6	
	27		32		33		34		36		37		38		40	3	
♉	0		35		37		38		39		41		42		44	0	♍
	3		39		40		41		43		45		46		48	27	
	6		42		43		45		47		48		50		52	24	
	9		45		47		48		51		52		54	6	56	21	
	12		48		50		52		54		56	6	58	7	0	18	
	15		51		53		55	6	58	6	59	7	1		4	15	
	18		54		56	6	58	7	0	7	3		5		7	12	
	21	6	57	6	59	7	1		3		6		8		11	9	
	24	7	0	7	2		4		6		9		11		14	6	
	27		2		4		7		9		12		15		17	3	
♊	0		4		7		9		12		15		17		20	0	♌
	3		7		9		12		14		17		20		23	27	
	6		9		11		14		17		19		25		26	24	
	9		10		13		16		19		22		25		28	21	
	12		11		15		17		20		23		27		30	18	
	15		13		16		19		20		25		28		32	15	
	18		14		17		20		23		26		30		33	12	
	21		15		18		21		24		27		31		34	9	
	24		16		19		22		25		28		32		35	6	
	27		16		19		22		25		29		32		35	3	
	30	7	17	7	19	7	22	7	26	7	30	7	32	7	36	0	♋

Sicilia
Salernũ
Roma
Florẽtia

Figure 5: Regiomontanus' length of daylight table, Regiomontanus, *Tabule directionum*.

Eventually this process culminates in sine tables. Of course Regiomontanus was instrumental there as he computed and printed no less than three different sine tables. The first is part of the same *Tabule directionum* and it is already a big table of 5400 entries: using a radius of 60 000, sines are computed for every minute of arc for the first quadrant. He later enhanced the precision of his figures using radii of 10 000 000 and 60 000 000 in his *De triangulis*. All these tables use a two dimensional array layout but no explicit symmetry. Yet, computing the sine of the first quadrant only is clearly a tacit use the symmetries of the sine function. They were the source for most 16th century sine tables. For instance, Copernicus (1473-1543) edited a big sine table in his *De lateribus et angulis triangularum*, which covers 36 pages[33]. In these tables, the full *ḥabṭaq* layout is used in a new fashion. The left column and the top row are arguments for the sine and, in opposite reading direction, the right column and bottom row are arguments for the cosine. This means that the *ḥabṭaq* layout is not used to underline the symmetry of a function only but also the relation of two trigonometric functions. Interestingly, in the *De revolutionibus* Copernicus only used the classical layout designed by Ptolemy in the *Handy Tables*. This was

---

[33] N. Copernic. *De lateribus et angulis triangulorum*, Wittenberg, impr. Johannes Lufft, 1542.

probably one of the several ways he used to make his work more acceptable by his contemporaries. Copernicus' sine tables were used by Christopher Clavius (1538-1612) as a source for the *Tabulae sinuum, tangentium et secantium* (1607)[34] his text book on trigonometry.

4. Concluding remarks

Summing up we can mark three different stages in the transmission and assimilation of the *ḥabṭaq* and related tables layout in Europe. In a first phase only simple uses of these layouts seem to have been transmitted sporadically from Arabic sources. The compilation of the Parisian Alfonsine tables' core versions at the beginning of the 14th century by John of Murs and John of Lignières changed this situation. With them the *ḥabṭaq* and related layouts were used to tabulate more complex astronomical phenomena like equations or syzygies computation and thus gained a situation very similar to what we found, in the same period, in the Arabic sources. We have seen some diffusion paths of the Alfonsine tables, in England, in Italy, and in central Europe. In the latter place at least, and under the impulse of Regiomontanus, a convergence of simple and complex uses of this layout appeared which led to big "simple" tables like the large sine tables of Copernicus. This last phase led also to new uses of the *ḥabṭaq* and related layouts where more than one function is represented by the same table.

---

[34] Christopher Clavius. (1607). *Tabulae sinuum, tangentium et secantium, Moguntia.* A digital version is available here : http://reader.digitale-sammlungen.de/de/fs1/object/display/bsb10053379_00001.html