Ibn al-Raqqām's Notes on Practical Geometry

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Key Words: note (*tanbīh*), measurement (*taksīr*), Ibn al-Raqqām.

Abstract
This paper deals with the single example of Ibn al-Raqqām's activity in the field of geometry that has reached us: his booklet on the measurement of surfaces.

1. Introduction

The following pages contain an edition, a translation and a commentary on the booklet *al-Tanbīh wa-l-tabṣīr fī qawānīn al-taksīr* by Ibn al-Raqqām (fl. Tunis and Granada, 1245-1315).[1]

In this section, I present some remarks on the terms *tanbīh* and *taksīr*, and a brief summary of the measurement tradition in medieval Arabic science.

Ibn al-Raqqām was a highly prolific writer of *taqyīd*, according to the list of his works recorded by Ibn al-Khaṭīb (*Al-Iḥāṭa*, 69-70). *Taqyīd* and *tanbīh* writings bear similarities, such as shortness and concision, and thus may be considered a genre.[2] Therefore, Ibn al-Raqqām's booklet may be translated

1 On Ibn al-Raqqām, see Vernet 1980; Carandell 1988; Lamrabet 1994, 35; ʻAbd al-Raḥmān 1996; Kennedy 1997; Díaz-Fajardo 2005; Samsó 2006; Díaz-Fajardo 2007; Samsó 2008; Samsó 2011.

2 It is worth mentioning that manuscript D2233/11 (pp. 217-219) at the Bibliothèque Nationale du Royaume du Maroc was believed to be *Notes on the use of the spherical astrolabe* (*Taqyīd fī-l-ʻamal bi-kurat al-asṭurlāb*) (al-Kattānī and al-Tādilī 1997, 176) or *Treatise on the astrolabe* (*Risāla fī-l-asṭurlāb*) (Lamrabet 1994, 35). However, it turned out to be an untitled and partial

as *Note and instruction on the rules for measurement.* However, a more literal translation would be *Remark and perceptiveness on the rules for measurement.*

As for the term *taksīr*, several meanings can be inferred from Ibn al-Raqqām's notes: the practice of measuring surfaces, see paragraph [2];[3] area, according to [6]; the volume or space occupied by a solid, see [27]; and, lastly, the computation of areas, in that *taksīr* is an infinitive that comes from the verb *kassara*, which means 'calculate the area' in [14] and [19].[4]

The extant works on measurement[5] produced by Eastern mathematicians in the 9th-13th centuries may be classified in two traditions.

The first tradition (9th-11th centuries) has been called 'scholarly geometry' and was headed by Muḥammad ibn Mūsà al-Khwārizmī (fl. Baghdad, d. *ca.* 846). Scholarly geometry draws on Greek mathematicians such as Euclid, Apollonius and Archimedes and makes use of algebraic procedures. Works of scholarly geometry were addressed to readers with a mathematical background.

The second tradition (10th-13th centuries) had a wider following. It included Thābit ibn Qurra (836-901), Ibn Ṭāhir al-Baghdādī (11th century) and al-Qāḍī Abū Bakr (fl. before the end of the 12th century). Their works were illustrative of a 'practical geometry', also named 'skill geometry', as they drew on the ancient practices of Babylon and the experience of artisans. In their treatises, formulas were applied in numerical examples and there were no geometrical demonstrations. They dealt with the area of plane figures, the area of a circle and the portions of a circle, and the area and volume of a variety of solids.

copy (the introduction and first and second chapters) of Ibn al-Raqqām's *Treatise on shadow science* (*Risāla fī 'ilm al-ẓilāl*). These chapters are edited in Carandell 1988, 256-259.

3 The number in brackets refers to corresponding paragraphs in the edition and the translation.

4 *Taksīr* was employed in surveying and law. Ibn al-'Aṭṭār (Cordoba, 941-1009) was an expert jurist in inheritance rules (*farā'iḍ*). One of the documents summarised in his book *Kitāb al-wathā'iq wa-l-sijillāt* (Chalmeta and Corriente 1983, 20) is entitled 'Muḥammad ibn Aḥmad's document on the purchase of two measured lands' (*Wathīqa bi-ibtiyā'* [*li-ibtiyā'*] *ḥaqlayn 'alà-l-taksīr li-Muḥammad ibn Aḥmad*).

5 This short summary is based on the following works: Busard 1968; Djebbar 2007; Moyon 2008; Moyon 2011. Due to briefness, this summary gives only the name of the authors used in the present paper and mentioned in section 3. For a detailed discussion, see the references *cit. supra.*

However, several of these treatises did feature quotations from Euclid, Archimedes and Ptolemy, and the mathematical propositions were given demonstrations or followed their procedures. Thus, these works pursued an 'intermediate geometry', mingling the scholarly and practical geometries. The purpose of this tradition was to offer accurate works on the rules and local practices of surface measurement.

In the Islamic West, Ibn 'Abdūn (Cordoba, 923-*ca.* 976) and Ibn al-Bannā' (Marrakech, 1256-1321) continued the tradition of practical geometry, whereas Abraham Bar Hiyya (fl. Zaragoza and Barcelona, *ca.* 1065-*ca.* 1136) and Ibn al-Yāsamīn (d. Marrakech, 1204), practiced an intermediate geometry by including materials from scholarly geometry in their works.

In al-Andalus, the practical geometry tradition shared a few practices such as the use of some unusual solids and the definition of figures.[6] The study of measurement was included in training for several professions, including surveyors, designers, bricklayers, farmers and merchants. Ibn 'Abdūn's treatise is the first extant example of measurement teaching practice in al-Andalus and it is illustrative of this local tradition using ancient and pre-algebraic procedures. On the other hand, surface measurement was a subject recommended by Ibn Hazm in elementary education in the second half of the 11th century.

2. Text

The text is extant in five manuscripts: the Bibliothèque al-Ḥasaniyya (Rabat) preserves manuscripts 4748, 4749 and 5415, and the Bibliothèque Nationale du Royaume du Maroc (Rabat) houses manuscripts D1588 and D2215/16.

During a research stay at the two libraries, I was able to study manuscripts 4749 (ff. 1r-1v), 5415 (ff. 15v-16v) and D1588 (ff. 146r-147v). At that time, I was unaware of Muḥammad al-'Arbī al-Khaṭṭābī's edition (1986, 39-42), which followed the three manuscripts in the Bibliothèque al-Ḥasaniyya.

6 Muḥammad al-Mursī (fl. *ca.* the first half of the 13th century) and Ibn al-Jayyāb (fl. *ca.* 1236-1281) wrote treatises on *taksīr* that also dealt with the unusual solids treated by Ibn 'Abdūn. Ibn al-Jayyāb proposed alternative practices as, for example, a squared drawing board to measure plane figures because he wanted a practical rule so that items were convincing with the naked eye and by experience. For books written in the Andalusian tradition of surface measurement and the geometrical terminology, see Djebbar 2007, 18-31; Moyon 2008, vol. I, 115-125.

Nevertheless, due to interest in the geometrical vocabulary, I present a new edition of the Arabic original.

2.1 Edition

I use the symbols [حس أ] for manuscript 4749, [حس ب] for 5415, and [وط] for D1588.

Manuscript 5415 contains plenty of gaps and its binding complicates the reading of the inner margins. I kept the foliation as found in the original, although the second and third pages were bound in inverse order: folio 16 verso should be folio 16 recto, while folio 16 recto should be folio 16 verso.

Manuscript 5415 mentions its copyist's name, Abū-l-Qāsim ibn Aḥmad al-Ghūl, and one date: before evening prayer on the night of Friday the seventh of the year one thousand and forty. The copyist, Abū-l-Qāsim ibn Aḥmad ibn Muḥammad ibn ʿĪsà, was a follower of Ibn al-Raqqām. Called al-Ghūl al-Fashtālī (d. 1649), he produced extant works on mathematics and medicine.[7] If the year is 1040 of the Hijra (the unit is illegible in the manuscript), the copy was made either in A.D. 1630 or 1631.[8]

The basis for my edition is no. 4749, the clearest copy. I present the original text in small paragraphs to make reading and commentary easier. A number in brackets precedes each paragraph to provide a cross-reference between the original text and the commentary. I have added the punctuation in the edition. The symbol [...] stands for a gap in the original.

// [حس أ. 1 و] // [حس ب. 15 ظ] // [وط. 146 و] بسم الله الرحمن الرحيم وصلّى الله على سيّدنا محمّد وآله وصحبه وسلّم.

[1] التنبيه والتبصير[9] في قوانين التكسير للشيخ الأجلّ الفقيه أبي عبد الله محمّد بن ابراهيم الأوسي عرف بابن الرقّام.[10]

[7] Regarding Abū-l-Qāsim, see al-Khaṭṭābī 1983, 465; Lamrabet 1994, 147-148; al-Kattānī and al-Tādilī 1997, 254; Allouche and Regragui 2001, 354.

[8] 7 Friday, 1040 is in the month either of *Muḥarram* (7 *Muḥarram*, 1040 corresponds to Thursday, 5 August, 1630) or of *Shawwāl* (7 *Shawwāl*, 1040 corresponds to Thursday, 28 April, 1631).

[9] والتبصير: والتبصر.

[2] التكسير صناعة ينظر فيها في مساحة الأشكال و[11]حدودها.

[3] في السطوح.[12]

[4] وأوّلا في المربّعات:[13]

[5] المربّع المتساوي الأضلاع القائم الزوايا: اضرب ضلعه[14] في نفسه، يكن مساحته وهو تكسيره. وأضعف[15] التكسير وخذ جذر المجتمع، يكن القطر. وربّع القطر ونصّف المجتمع، يكن التكسير. وخذ جذر التكسير، يكن الضلع.[16]

[6] في المستطيل القائم الزوايا: اضرب طوله في عرضه، يكن التكسير. وربّع الطول والعرض واجمع المربّعين وخذ جذره، يكن القطر.

[7] في المثلّثات:

[8] وأوّلا في المتساوي الأضلاع: اطرح مربّع نصف ضلعه من مربّع ضلعه وخذ جذر الباقي، يكن العمود. فاضربه في نصف الضلع، يكن التكسير.

[9] في المتساوي الساقين: أسقط مربّع نصف القاعدة من مربّع أحد الضلعين المتساويين وخذ جذر الباقي، يكن العمود. فاضربه في نصف // [وط. 146 ظ] القاعدة أو نصف العمود في القاعدة، يكن التكسير.

[10] بسم الله... بابن الرقّام:

[في م وط:] قال الإمام العالم المتفنّن أبو عبد الله محمّد ابن ابرا هيم بن علي بن محمّد بن الرقّام الأوسي رحمه الله.

[في م حس ب:] بسم الله الرحمن الرحيم صلى الله على سيّدنا محمّد وآله و[...] التنبيه والتبصير في قوانين التكسير للشيخ الأجلّ الفقيه أبي عبد الله [...] ابراهيم عرف بابن الرقّام.

[11] مساحة الأشكال و: [في م وط:] مساحة الأشكال ومساحة.

[12] في السطوح: [في م وط:] في السطوح. [في م حس أ، وحس ب:] في تكسير السطوح.

[13] المربّعات: [الكلمة ناقصة في م وط].

[14] ضلعه: [في م وط:] الضلع.

[15] وأضعف: [في م وط:] واضف.

[16] وخذ جذر... الضلع: [الجملة ناقصة في م حس ب].

[10] في القائم الزاوية: اضرب أحد الضلعين المحيطين بالزاوية القائمة في نصف الآخر المحيط بها، يكن التكسير.[17]

[11] في المختلف الأضلاع: اجعل أحد أضلاعه قاعدة. وانقص[18] مربّع أحد الضلعين الباقيين من مربّع الآخر.[19] واقسم نصف الباقي على القاعدة. واطرح الخارج من نصف القاعدة، يكن المسقط الأقصر أو احمله عليه، يكن المسقط الأطول.

[12] ثمّ أسقط مربّع المسقط الأقصر من مربّع الضلع الأصغر أو مربّع المسقط[20] الأطول من مربّع الضلع // [حس ب. 16 ظ] الأكبر. وخذ جذر الباقي، يكن العمود. فاضربه في نصف القاعدة أو نصفه في القاعدة، يكن التكسير.

[13] وفي جميع المثلّثات[21] وجه عامّ: وهو أن تجمع أضلاع المثلّث وتحفظ نصف المجتمع. وتعلم زيادته على كلّ واحد من أضلاعه. واضرب[22] الزيادة الأولى في الثانية وما اجتمع في الثالثة وما اجتمع في المحفوظ. وخذ جذر المجتمع، يكن التكسير. فاقسمه على نصف القاعدة يخرج العمود. فاطرح مربّعه من مربّع أحد الضلعين. وخذ جذر الباقي، يكن المسقط الذي يليه.

[14] في المعيّن والشبيه به: ولا بدّ من تحديد قطريهما.[23] فيَنْقَسِم // [وط. 147 و] كلّ واحد منهما إلى مثلّثين.[24] فتكسّرهما كما تقدّم وتجمع التكسيرين.

[17] في القائم الزاوية... يكن التكسير: [الجملة ناقصة في م وط].

[18] وانقص: [في م وط، وفي م حس ب:] وانقص. [في م حس أ:] والقطر.

[19] الآخر: [في م وط:] الباقي.

[20] المسقط: [في م حس ب:] الضلع.

[21] وفي جميع المثلّثات: [في م وط:] وفي المثلّثات كلّها.

[22] واضرب: [في م وط:] أو اضرب.

[23] قطريهما: [في م وط:] قطرهما.

[24] مثلّثين: [في م وط:] مثلين.

[15] في العرائض:[25] اطرح الرأس من القاعدة، يبق[26] مثلّث أضلاعه ضلعا العريضة وفضل[27] القاعدة على الرأس. فاستخرج عموده.[28] واضرب العمود في نصف مجموع الرأس والقاعدة، يكن التكسير.

[16] في المنحرف: ولا بدّ من تحديد قطره.[29] فينقسم إلى // [حس أ. 1 ظ] مثلّثين.[30] وتكسيرهما كما تقدّم. وتجمع التكسيرين، فيكون[31] تكسير المنحرف.

[17] في ذوات الأضلاع الكثيرة[32] كالمخمّس والمسدّس وما فوقهما:

[18] إذا كانت هذه متساوية الأضلاع والزوايا فاضرب نصف إحاطة الشكل في العمود الخارج من مركزه إلى[33] نصف ضلع من أضلاعه، يكن التكسير.

[19] وإن كان مختلفا فلا بدّ من تحديد أقطاره. واقسمه[34] إلى مثلّثات وكسّر كلّ مثلّث على ما تقدّم. واجمع الجميع، يكن التكسير. وقد يصنع هذا[35] في المتساوي.

[20] في الدائرة: اضرب القطر في نفسه وانقص من مربّعه سبعه ونصف سبعه، يبق[36] التكسير. واضرب القطر في ثلاثة وسبع، يكن المحيط. واقسم المحيط على

[25] العرائض: [في م وط:] الفرائض.

[26] يبق: [في م وط:] يبقى.

[27] العريضة وفضل: [في م وط:] الفريضة وفصّل.

[28] عموده: [في م وط:] عمودا.

[29] ولا بدّ... قطره: [كذا في م وط. في حس أ، وحس ب:] ولا بدّ من قطره أن يكون معلوما.

[30] مثلّثين: [في م وط:] مثلين.

[31] فيكون: [في م وط:] كما تقدّم يكن.

[32] في ذوات... الكثيرة: [في م وط:] في الكثير الأضلاع.

[33] إلى: [الكلمة مكرّرة في م حس ب].

[34] واقسمه: [في م وط:] فاقسمه.

[35] وقد يصنع هذا: [كذا في م وط. في حس أ، وحس ب:] ويصنع ذلك.

[36] يبق: [في م وط:] يبقى.

ثلاثة وسبع، يكن القطر. واضرب نصف القطر في نصف // [وط. 147 ظ] المحيط، يكن التكسير.

[21] في تكسير القطّاع من الدائرة:[37] // [حس ب. 16 و] اضرب الضلع في نصف[38] القوس، يكن التكسير.

[22] في القطعة منها:[39] اضرب نصف الوتر في نفسه. واقسم المجتمع على السهم. واحمل[40] على الخارج السهم، يكن[41] القطر. فاضرب نصفه في نصف القوس، يكن تكسير القطّاع.[42] ثمّ انقص مربّع نصف الوتر من مربّع نصف القطر. وخذ جذر الباقي، يكن العمود. فاضربه في نصف الوتر، يكن[43] تكسير المثلّث. فانقصه من تكسير القطّاع إن كانت[44] القطعة أصغر من نصف دائرة أو احمله[45] عليه إن كانت[46] أكبر من نصف دائرة، يحصل تكسير القطعة.

[23] في المجسّمات.

[24] وأوّلا في المكعّب: اضرب ضلعه في نفسه وما اجتمع في الضلع،[47] يكن التكسير.[48]

[37] من الدائرة: [العبارة ناقصة في م وط].

[38] نصف: [كذا في م وط، وحس ب. في م حس أ:] نفس.

[39] في القطعة منها: [في م وط:] في تكسير القطعة.

[40] واحمل: [في م وط:] واقسم.

[41] السهم، يكن: [كذا في م وط]. [في م حس أ:] يكن السهم [الحرف خ (لخطأ) مكتوب فوق الكلمة يكن، والحرف ق (لقبل) مكتوب فوق الكلمة السهم]. [في م حس ب:] يكن.

[42] القطّاع: [في م وط:] القطعة.

[43] يكن: [في م وط:] تكن.

[44] كانت: [في م حس ب:] كان.

[45] أو احمله: [في م وط:] واحمله.

[46] كانت: [في م وط:] كان.

[47] وما... الضلع: [الجملة ناقصة في م وط].

[48] التكسير: [في م وط:] تكسيره.

[25] في المجسّم المستوي السطوح، القائم الزوايا المختلف الأضلاع: اضرب طوله في عرضه وما اجتمع في عمقه،[49] يكن التكسير.

[26] في الأسطوانة: كسّر قاعدتها على أيّ شكل كانت. واضرب المجتمع في ارتفاعها، يكن التكسير. ولو كان رأسها مخالفا لقاعدتها، كسّرت القاعدة والرأس. وأخذت النصف منهما وضربته في ارتفاعها.[50]

[27] في المخروط: كسّر قاعدته. واضرب التكسير في ثلث ارتفاعه، يكن التكسير.

[28] في المجسّمات المختلفة الأضلاع والزوايا: فصّلها إلى مجسّمات يحيط بكلّ واحد منها[51] أربعة[52] مثلّثات.[53] واجعل أحدها قاعدة وكسّرها. واضرب تكسيرها[54] في ثلث ارتفاع الشكل القائم على القاعدة،[55] {يكن تكسيره. ثمّ اجمع تكسير المجسّمات المذكورة،[56] يكن تكسير المجسّم الأعظم المذكور.

[29] في الكرة: اضرب قطرها في ثلاثة وسبع،[57] يكن المحيط. واضرب نصف المحيط في نصف القطر،[58] يكن تكسير الدائرة. فتضربه[59] في ثلثي القطر، يكن التكسير.[60]

[49] عمقه: [كذا في م وط. في م حس أ، وحس ب:] غمقه.

[50] ولو كان رأسها... في ارتفاعها: [الجملة ناقصة في م وط].

[51] منها: [في م وط:] منهما.

[52] أربعة: [كذا في م وط، وفي حس ب]. [في م حس أ:] أربع.

[53] مثلّثات: [في م وط:] مجسّمات.

[54] تكسيرها: [في م وط:] تكسير.

[55] القاعدة: [الكلمة مكرّرة في م وط]. [الجمل بين القوسين {} مكتوبة على الهامش في م وط].

[56] المذكورة: [الكلمة ناقصة في م وط].

[57] وسبع: [في م وط:] وسبـ .

[58] القطر: [في م وط:] القـ[...]ـر.

[59] فتضربه: [في م وط:] فاضربه.

[60] القطر، يكن التكسير: [في م وط:] الـ[...] [...]ن تكسير الكرة.

انتهى بحمد الله وحسن عونه وتوفيقه الجميل ولا حول ولا قوّة إلاّ بالله العلّي العظيم.}[61]

2.2 Translation

In the name of God, the Compassionate, the Merciful. May God bless our Lord Muḥammad, his family and his companions and bring to them peace.
[1] *Note and instruction on the rules for measurement* by the master, the most honourable, the jurist Abū ʻAbd Allāh Muḥammad Ibn Ibrāhīm al-Awsī, known as Ibn al-Raqqām.
[2] Measurement is a practice that must be taken into account in the survey of shapes and their contours.
[3] In the surfaces.
[4] Turning first to quadrilaterals:
[5] For a square, which has equal sides and right angles: multiply its side by itself, [the product] will be its surface, that is to say, its area. Double the area and obtain the root of the total, [the result] will be the diagonal. Raise to the square the diagonal and divide the whole by two, [the result] will be the area. Obtain the root of the area, [the result] will be the side.
[6] In an oblong, which has right angles: multiply its length by its width, [the product] will be the area. Raise to the square the length and the width, add both squares and obtain the root, [the result] will be the diagonal.
[7] Turning to triangles:
[8] First, for the equilateral [triangle]: subtract the square of half of its side from the square of its side and obtain the root of the remainder, [the result] will be the height. Multiply it by half of the side, [the product] will be the area.
[9] For the isosceles [triangle]: subtract the square of half of the base from the square of one of the two equal sides and obtain the root of the remainder, [the result] will be the height. Multiply it by half of the base, or [multiply] half of the height by the base, [the product] will be the area.

[61] بحمد الله... العظيم: [في م وط:] والحمد لله كما هو أهله والصلاة [والسـ]ـلام على سيّدنا محمّد وآ[له] [وصـ]ـحبه. [في م حس ب:] بحمد الله وحسن عونه على يد أبي القاسم بن أحمد الغول [في المخطوط: القول] قبل عشاء ليلة الجمعة سابع [...] أربعين وألفا وصلّى الله على مولانا محمّد وآله وسلّم.

[10] For the right-angled [triangle]: multiply one of the two sides that surround the right angle by half of the other that surrounds it, [the product] will be the area.
[11] For the scalene [triangle]: settle on one of its sides as [the] base. Subtract the square of one of the two remaining sides from the square of the other [side]. Divide half the remainder by the base. Subtract the result from half of the base, [this] will be the shortest segment [of the base] or add it to half of the base, [this] will be the longest segment [of the base].
[12] Then, subtract the square of the shortest length [of the base] from the square of the shortest side, or the square of the longest length [of the base] from the square of the longest side. Obtain the root of the remainder, [the result] will be the height. Multiply it by half of the base, or [multiply] its half [of the height] by the base, [the product] will be the area.
[13] For all the triangles, there is a common method: add the sides of the triangle and keep half of the sum. Settle its excess for each one of its sides. Multiply the first excess by the second and this product by the third and the total result by the kept. Obtain the root of the total, [the result] will be the area. Divide it by half of the base to obtain the height. Subtract its square [the square of the height] from the square of one of the two sides. Obtain the root of the remainder, it will be the side adjacent to the place where the foot of the height drops.
[14] For the rhombus and the rhomboid: it is necessary to determine the diagonal of both [quadrilaterals]. In this way, each of them is divided into two triangles. Compute the area for both, as explained before, and add the two areas.
[15] For trapezia: subtract the cusp from the base, the remainder will be a triangle the sides of which are the two sides of the trapezium and the difference between the base and the cusp. Obtain its height. Multiply the height by half the sum of the cusp and the base, [the result] will be the area.
[16] For the trapezoid: it is necessary to determine its diagonal. In this way, it is divided into two triangles. The computation of their areas is as explained. Add the two areas, [it] will be the area of the trapezoid.
[17] For polygons with many sides, such as the pentagon, hexagon and polygons with a greater number of sides:
[18] If these have equal sides and angles, multiply half the perimeter of the figure by the straight line that goes from its centre to the middle of one of its sides, [the result] will be the area.
[19] If [the figure] is irregular, it is necessary to determine its diagonals. Divide it into triangles and compute the area of each triangle as explained. Add

the whole, [the result] will be the area. This is sometimes done for the regular [polygon].
[20] For the circle: multiply the diameter by itself and subtract from its square its seventh and half of its seventh, [the result] will be the area. Multiply the diameter by three and one seventh, [the result] will be the length [of the circumference]. Divide the length by three and one seventh, [the result] will be the diameter. Multiply half of the diameter by half of the length [of the circumference], [the result] will be the area.
[21] Computation of the area of the sector of the circle: multiply the side by half of the arc [of the circumference], [the result] will be the area.
[22] For the segment of a circle: multiply half of the chord by itself. Divide the total by the arrow. Add the arrow to the result, [the sum] will be the diameter. Multiply its half by half of the arc [of the circumference], [the product] will be the area of the sector. Then, subtract the square of half of the chord from the square of half of the diameter. Obtain the root of the remainder, [the result] will be the height. Multiply it by half of the chord, [the result] will be the area of the triangle. Subtract it from the area of the sector if the segment is smaller than half of the circle or add it if [the segment] is greater than half of the circle, it will be the area of the segment.
[23] Turning to solids.
[24] First, for a cube: multiply its side by itself and the product by the side, [the result] will be the volume.
[25] For a solid with equal bases, right angles and unequal sides: multiply its length by its width and the product by its depth, [the result] will be the volume.
[26] For a cylinder: compute the area of its base, according to the figure of the base. Multiply the total by its height, [the result] will be the volume. If its cusp is different from its base, compute the area of the base and the cusp. You take the half of both and multiply it by its height.
[27] For a cone: compute the area of its base. Multiply the area by a third of its height, [the result] will be the volume.
[28] For solids with unequal sides and angles: divide them into solids so that four triangles surround each one of them. Consider one of them as [the] base and compute its area. Multiply its area by a third of the height of the figure that rises on the base, [the result] will be the volume. Then, add the volume of the mentioned solids, [the result] will be the volume of the mentioned biggest solid.
[29] For a sphere: multiply its diameter by three and one seventh, [the result] will be the length. Multiply half of the length by half of the diameter, [the

result] will be the area of the circle. Multiply it by two thirds of the diameter, [the result] will be the volume.
This is the end by the grace of God, the kindness of His help and His beautiful blessing. There is no power and no faculty except with God, the Noble, the Great.

3. Commentary and mathematical transcription

These symbols are used in the commentary that follows:

A	area	F	arrow
Ab	area of the base	h	height
Ac	area of the cusp	L, l	side
a	apothem	n	degrees of an angle
B, b	base	P	perimeter
C	chord	r	radius
D, d	diagonal	s	semi-perimeter
dm	diameter	V	volume

As mentioned before, the numbers in brackets refer to matching numbered paragraphs in the edition and the translation.

[3] SURFACES (*al-suṭūḥ*).

[4] Quadrilaterals (*al-murabbaʿāt*).

[5] The square (*al-murabbaʿ al-mutasāwī al-aḍlāʿ al-qā'im al-zawāyā*):

$A = L^2$
$A = D^2/2$
$D = \sqrt{A \times 2}$
$L = \sqrt{A}$

[6] The rectangle (*al-mustaṭīl al-qā'im al-zawāyā*):

$A = L \times l$

$D = \sqrt{L^2 + l^2}$

[7] Triangles (*al-muthallathāt*): equilateral (*al-mutasāwī al-aḍlāʿ*), isosceles (*al-mutasāwī al-sāqayn*), right-angled (*al-qā'im al-zāwiya*) and scalene (*al-mukhtalif al-aḍlāʿ*) triangles:

[8]-[10]; [12] The Pythagorean theorem:

Ibn al-Raqqām followed a common procedure with these triangles: he used the formula for the area involving the height, and applied the property of the right-angled triangle to find the height.

$h = \sqrt{L^2 - (B/2)^2}$

$A = h \times B/2$

$A = h/2 \times B$

[11] The foot of the height (*al-masqaṭ*):

First, Ibn al-Raqqām computed the sector MT (figure 1) of a scalene triangle. For triangle JKO, the height JT divides the triangle JKO into two unequal right-angled triangles, and two bases (E and e) of unknown length, E = OT and e = TK. The midline JM divides the base into two equal parts, OM and MK. Then, OT = OM + MT, and TK = MK – MT.

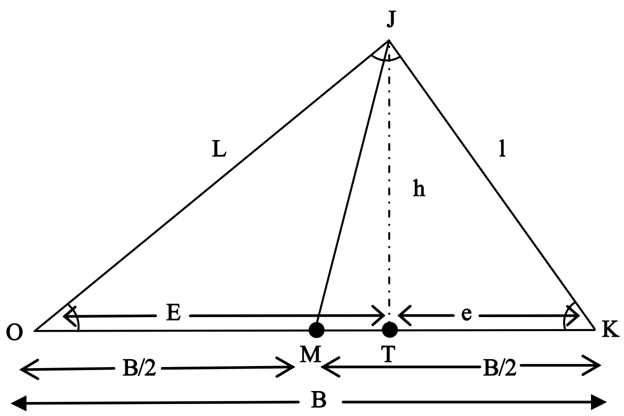

Figure 1

According to Ibn al-Raqqām, $MT = \frac{(L^2-l^2)/2}{B}$

Because:

$$L^2 = h^2 + OT^2 = h^2 + \left(\frac{B}{2} + MT\right)^2, \text{ and } l^2 = h^2 + TK^2 = h^2 + \left(\frac{B}{2} - MT\right)^2$$

$$L^2 - l^2 = [h^2 + \left(\frac{B}{2} + MT\right)^2] - [h^2 + \left(\frac{B}{2} - MT\right)^2] =$$

$$= \left(\frac{B^2}{4} + MT^2 + B \times MT\right) - \left(\frac{B^2}{4} + MT^2 - B \times MT\right) =$$

$$= \frac{B^2}{4} + MT^2 + B\times MT - \frac{B^2}{4} - MT^2 + B\times MT = 2B\times MT$$

$$MT = \frac{L^2-l^2}{2B} = \frac{(L^2-l^2)/2}{B}$$

Then, Ibn al-Raqqām subtracted or added the value of MT to half of the base:

$$e = \frac{B}{2} - \frac{(L^2-l^2)/2}{B} \text{ , and } E = \frac{B}{2} + \frac{(L^2-l^2)/2}{B}$$

The foot of the height was a familiar question among medieval geometricians. Muḥammad ibn Mūsà al-Khwārizmī[62] solved it with an algebraic calculation equal to: $l^2 - e^2 = L^2 - (B - e)^2$.

Other, such as Abraham Bar Hiyya,[63] al-Qāḍī Abū Bakr[64] and Ibn al-Bannā',[65] preferred a direct computation:

Bar Hiyya: $e = \frac{(l^2+B^2-L^2)/2}{B}$, and $E = \frac{(L^2+B^2-l^2)/2}{B}$

Abū Bakr and Ibn al-Bannā': $E = \frac{B^2+L^2-l^2}{2B}$

62 Youschkevitch 1976, 49.
63 Millàs i Vallicrosa 1931, 53.
64 Hogendijk 1990, 135.
65 Lamrabet 1994, 245.

It may be seen that both expressions are equivalent: $l^2 - e^2 = L^2 - (B - e)^2$, where:

$$l^2 - e^2 = L^2 - B^2 + 2\,B\times e - e^2, \text{ and } e = \frac{l^2 - L^2 + B^2}{2B}$$

In the same way, these procedures are equivalent to Ibn al-Raqqām's:

$$e = \frac{(l^2+B^2-L^2)/2}{B} = \frac{l^2+B^2-L^2}{2B} = \frac{B^2-(L^2-l^2)}{2B} = \frac{B^2}{2B} - \frac{L^2-l^2}{2B} = \frac{B}{2} - \frac{L^2-l^2}{2B}$$

Nevertheless, Ibn al-Raqqām's solution is the less symbolic. It was also used by Ibn 'Abdūn[66] and Abū Bakr. Abū Bakr gave both explanations: the sector MT = $\frac{L^2-l^2}{2B}$ and the direct computation.

Ibn 'Abdūn,[67] Bar Hiyya and Abū Bakr illustrated the foot of the height problem with the same standard example of a scalene triangle with sides of lengths 15 and 13 and a base of length 14.

Regarding the terminology, *al-masqaṭ* means the place to which something is dropped. Its signification is better shown in the variant of Abū Bakr[68] or Ibn al-Yāsamīn:[69] *nuqṭat masqaṭ al-ḥajar* (the point to which the stone drops), e.g. the place at the base where the foot of the height drops. At the end of paragraph [13], Ibn al-Raqqām used this meaning in the sentence 'the side adjacent to the *masqaṭ*'. However, he now says 'the longest *al-masqaṭ*' and 'the shortest *al-masqaṭ*', which implies a side of the base and not just a point on the base. We find this second meaning in *al-Īḍāḥ 'an uṣūl ṣinā'at al-massāḥ* of Ibn Ṭāhir al-Baghdādī: 'Each of the two segments of the base is called *masqaṭ al-ḥajar*'.[70]

[13] The 'common method':

With this name, Ibn al-Raqqām was referring to the computation of the area of any triangle in relation to Heron of Alexandria, although the scholars Y. 'Id and E. S. Kennedy (1983) believed that the computation may be associated with Archimedes. Ibn al-Raqqām's ordinary denomination differs from the more scientific one used by Bar Hiyya, who called the method the

66 Djebbar 2005, 51 [71].
67 Djebbar 2005, 51-53.
68 Hogendijk 1990, 147.
69 Lamrabet 1994, 245. For more on this mathematician, see Souissi, 1983.
70 Souissi 1968, 200; Sa'īdān 1985, 350-351.

'computation of the excesses'[71] because the semi-perimeter excess in the side lengths can be presented in the formula:

$$A = \sqrt{(s - L) \times (s - l) \times (s - B) \times s}$$

$$P = L + l + B$$

$$s = P/2$$

$$h = \frac{A}{B/2}$$

At the end of the paragraph, Ibn al-Raqqām dealt with the two bases of a scalene triangle, and he again applied the Pythagorean theorem:

$E = \sqrt{L^2 - h^2}$, and $e = \sqrt{l^2 - h^2}$

[14] The rhombus (*al-mu'ayyan*) and the rhomboid (*al-shabīh bi-l-mu'ayyan*):

In other *taksīr* treatises, such as the one by Ibn 'Abdūn,[72] the formula for the area of the rhombus ($D/2 \times d$) is found. Nevertheless, it seems that Ibn al-Raqqām preferred autonomy rather than new formulas since the reader has to integrate the recurring method of dissection into triangles for the area of both quadrilaterals.

This paragraph is somewhat confusing: first, Ibn al-Raqqām said that 'it is necessary to determine the diagonal' in spite of the fact that the diagonal value must be known to obtain the area, unless the word *taḥdīd* (determine) means 'distinguish'[73] or 'select' in this context. Later, he adds, 'Compute the area for both [that is, the triangles resulting from the dissection]', but the diagonal divides the rhombus and rhomboid into two equal triangles.

[15] Trapezia (*al-'arā'iḍ*):

In order to obtain a triangle from the trapezium in figure 2, we draw two straight lines perpendicular to the base, H-M and I-Q. According to Ibn al-Raqqām, the base of the resulting triangle is equal to JM + QK (= JK – HI).

[71] Millàs i Vallicrosa 1931, 59-60.

[72] Djebbar 2005, 39 [31].

[73] This also happens in paragraphs [16] and [19].

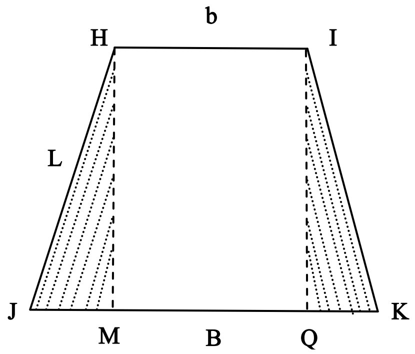

Figure 2

This paragraph represents Ibn al-Raqqām's methodology, featuring an enduring focus on the method of dissection into triangles combined with encouragement in the application of standard notions such as the Pythagorean theorem.[74]

The height of the triangle is equal to the height of the trapezium:

$$h = \sqrt{L^2 - \left(\frac{JM+QK}{2}\right)^2} = \sqrt{L^2 - JM^2}$$

where JM = QK, and $[(JM+QK)/2]^2 = JM^2 = QK^2$

As stated by Ibn al-Raqqām, $A = h \times \frac{b+B}{2}$

In this paragraph, Ibn al-Raqqām might mean both a regular and an irregular trapezium as the procedure for the height is useful for both. However, it is also possible that he is only referring to a regular trapezium since

[74] Ibn ʿAbdūn (Djebbar 2005, 43 [42]) follows somewhat formal steps: to obtain the height, he states that the cusp is subtracted from the base, then he explains the Pythagorean theorem with a worked example. However, he does not mention the dissection into a triangle. The more theoretical approach is the one explained by Bar Hiyya (Millàs i Vallicrosa 1931, 61-63), who (1) defines the trapezium, (2) gives a pictorial example of an isosceles trapezium, (3) explains how to find the height of the trapezium from the base of the triangle and the Pythagorean theorem, (4) finds the height by means of a worked example, (5) gives the formula for the area of the trapezium, and (6) proves this formula.

there was another formula for the irregular one used, for instance, by Ibn 'Abdūn[75] and Ibn Ṭāhir al-Baghdādī.[76]

As for the vocabulary, the trapezium is called *'arīḍa* (plural *'arā'iḍ*) by Ibn al-Raqqām as well as by other Western geometricians such as Ibn 'Abdūn,[77] Ibn al-Bannā'[78] and Ibn Haydūr (d. 1413).[79] Ibn 'Abdūn used this term along with the expressions *shakl 'arīḍ al-ra's* or *al-'arīḍa al-ra's*.[80] This archaic terminology appears in the practical treatises on measurement geometry of al-Andalus.[81] Abū Bakr[82] mentioned *munḥarif*, still in current use.

[16] The trapezoid (*al-munḥarif*):

In Arabic geometrical literature, *munḥarif* meant trapezium and any quadrilateral,[83] whereas *al-murabba' al-shabīh bi-l-munḥarif* stood for trapezoid.[84]

Ibn al-Raqqām used the term *munḥarif*, and that raises the question of which meaning he wanted to address.

On first glance, *munḥarif* may be taken as a trapezoid[85] for a number of reasons: (a) it is the only quadrilateral still to be explained in which the dissection into triangles for the area is requisite and this is Ibn al-Raqqām's method for the figure in this paragraph; (b) he called the trapezium *'arīḍa* (pl. *'arā'iḍ*) in [15]; (c) it is unlikely that he was referring to any quadrilateral, because this generic sense would also involve the rhombus and the rhomboid, both already mentioned in [14], using the same dissection into triangles method.

However, it is also possible to think that Ibn al-Raqqām meant trapezium since the trapezium is called *'arīḍa* or *munḥarif* among the medieval geome-

75 Djebbar 2005, 58-59.

76 Sa'īdān 1985, 343.

77 Djebbar 2005, 43 [42] and 48 [58].

78 Lamrabet 1994, 246 note 42.

79 Souissi 1968, 251, 290.

80 *Shakl* means figure and *ra's* cusp. The term *'arīḍa* has the form of an adjective: *'arīḍ* (wide) but with the addition of the termination *a* (*tā' marbūṭa*). This was done to strengthen the idea of intensiveness in its signification. Wright 1896-1898, I, 139.

81 Djebbar 2005; Djebbar 2006; Djebbar 2007.

82 Hogendijk 1990, 148.

83 Souissi 1968, 132.

84 Expression used by Abū Bakr, see Hogendijk 1990, 137 and 148.

85 However, the use of *munḥarif* for a trapezoid is not recorded in M. Souissi's mathematical lexicon.

tricians, Ibn al-Raqqām might be drawing on both documented traditions. If so, in this paragraph he was setting out an alternative method to the method described in [15].

[17] Polygons: the pentagon (*al-mukhammas*), the hexagon (*al-musaddas*) or polygons with more sides:

[18] The regular polygon of five or more sides:

$$A = s \times a$$

The apothem of a polygon is not given a name. Rather, it is explained as 'the straight line that goes from its centre [the centre of a polygon] to the middle of one of its sides'.

The way to find the apothem is not mentioned. Therefore Ibn al-Raqqām might have understood the pairing of the dissection into triangles method and the Pythagorean theorem, although he did not directly express it.

[19] The irregular polygon of five or more sides:

Once more, the method in use for the area is the dissection into triangles.

[20] The circle (*al-dā'ira*):

$$A = dm^2 - \left(\frac{1}{7} dm^2\right) - \left(\frac{1/7\ dm^2}{2}\right)$$

$$A = \frac{dm}{2} \times \frac{P}{2}$$

These formulas for the area are found in other geometry treatises (see footnote 86). The second formula appears, it seems for the first time, in the Banū Mūsà brothers' (9th century) *Kitāb misāḥat al-ashkāl al-basīṭa wa-l-kuriyya* (*Book on the measurement of plane and spherical figures*).

$$P = dm \times 3\frac{1}{7}$$

$$dm = \frac{P}{3\frac{1}{7}}$$

Ibn al-Raqqām used 3 plus 1/7, that is, an approximation of the number π and the usual value in the Arabic geometrical tradition. For instance, it appears in al-Khwārizmī, Ibn 'Abdūn, Bar Hiyya, Abū Bakr and Ibn al-Bannā':[86]

$$A = dm^2 - \frac{dm^2}{7} - \frac{dm^2/7}{2} = dm^2 \times \left(1 - \frac{1}{7} - \frac{1}{14}\right) =$$

$$= dm^2\left(\frac{14-2-1}{14}\right) = dm^2\left(\frac{11}{14}\right) = 4r^2\left(\frac{11}{14}\right) = \frac{22}{7}r^2 \cong \pi r^2$$

where $\frac{22}{7} = 3\frac{1}{7}$

[21] The sector (*al-qaṭṭā'*) of a circle:

Ibn al-Raqqām computed the area with the same expression as al-Khwārizmī:[87] $A = r \times arc/2$

Ibn al-Raqqām does not mention the formula for the arc. Probably, he assumed some basic mathematical knowledge among his readers as the length of an arc of the circumference may be obtained by a rule of three ($arc = n\times P/360^o$).

The radius of the circumference is called 'the side of the sector' by Ibn al-Raqqām.

In medieval times, the sector was named *qaṭṭā'* as mentioned by the linguist al-Khwārizmī (fl. *ca.* 975) in *Mafātīḥ al-'ulūm* (p. 187):[88] 'the circular sector, *qaṭṭā'*... is a portion of the circle whose vertex is in the centre or on the circumference'.

86 Al-Khwārizmī: see Youschkevitch 1976, 49; Ibn 'Abdūn: Djebbar 2005, 66-67 [114]-[121]; Bar Hiyya: Millàs i Vallicrosa 1931, 71-73; Abū Bakr: Hogendijk 1990, 138; and Ibn al-Bannā': Lamrabet 1994, 246.

87 Youschkevitch 1976, 50.

88 See also Souissi 1968, 287-288.

[22] The segment (*al-qiṭ'a*) of a circle (figure 3):

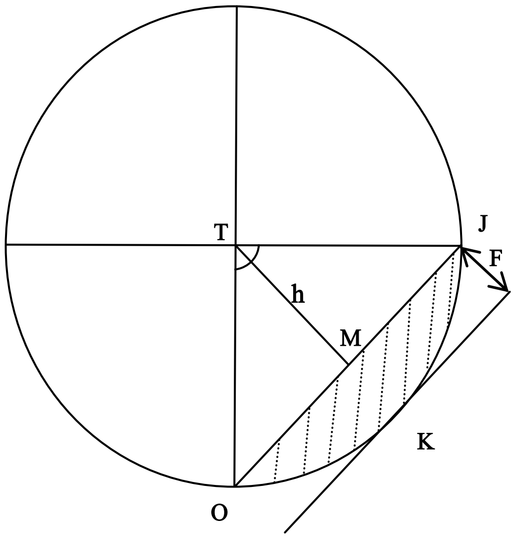

Figure 3

1) *Area of the sector JKOT of the circle*

$$dm = \frac{(C/2)^2}{F} + F$$

A = (dm/2)× (arc/2). See [20].

Ibn al-Raqqām's first formula (which was also used by Ibn 'Abdūn)[89] derives, clearly, from al-Khwārizmī's[90] formula: $dm = (C^2/4F) + F$.

2) *Area of the triangle JOT of the circle*

Again, the procedure combines the Pythagorean theorem (in the right-angled triangle JMT, $h = TM = \sqrt{TJ^2 - JM^2}$) and the formula for the area of the triangle JOT.

89 Djebbar 2006, 81 [123].
90 Youschkevitch 1976, 50.

3) *Area of the segment of the circle*

A = A sector ∓ A triangle

Subtract when the segment is smaller than half of the circle and add when it is greater than half of the circle.

Al-Khwārizmī,[91] Ibn 'Abdūn[92] and Ibn al-Bannā'[93] followed the same procedure, but with different formulations. For a segment greater than half of the circle:

Al-Khwārizmī and Ibn 'Abdūn: $A = \left[\frac{\text{arc}}{2} \times \frac{\text{dm}}{2}\right] + \left[\left(F - \frac{\text{dm}}{2}\right) \times \frac{C}{2}\right]$

Ibn al-Bannā': $A = \frac{\text{dm}\times\text{arc}}{4} + \left(F - \frac{\text{dm}}{2}\right) \times \frac{C}{2}$

Both formulas reflect the expression of Ibn al-Raqqām:

A sector $= \frac{\text{dm}\times\text{arc}}{4}$

A triangle $= \left(F - \frac{\text{dm}}{2}\right) \times \frac{C}{2} = (F - r) \times \frac{C}{2} = h\times\frac{C}{2}$

where C is equal to the base of the triangle.

The segment is called *qiṭ'a* by Ibn al-Raqqām. Ibn 'Abdūn used *qaws*, but since this term also means arc, elsewhere he had to distinguish between the two meanings: *al-muqawwasāt allatī hiya aqall/akthar min niṣf dā'ira* (arcs smaller/greater than half of a circle).[94]

[23] SOLIDS (*al-mujassamāt*).

[24] The cube (*al-muka''ab*):

$V = L^3$

[25] The quadrangular prism (*al-mujassam al-mustawī al-suṭūḥ al-qā'im al-zawāyā al-mukhtalif al-aḍlā'*):

$V = L\times L\times h$

91 Youschkevitch 1976, 50.
92 Djebbar 2006, 82 [124].
93 Lamrabet 1994, 246.
94 Djebbar 2005, 67; Djebbar 2006, 81-82 [123]-[124].

Ibn al-Raqqām mentioned neither *manshūr* (the usual term for prism) nor *usṭuwāna* (see [26]). Instead, he gave a definition. Perhaps, this was done in order to specify the character of this solid exactly.

[26] The cylinder (*al-usṭuwāna*):

Ibn al-Raqqām used two formulas:

V = Ab×h, where Ab depends on the figure that the base of the solid has.

V = [(Ab+Ac)/2]×h, for a solid with a cusp different from its base.

Therefore, in the first case, *usṭuwāna* means cylinder and prism. In the second, Ibn al-Raqqām might refer to a truncated cone or a truncated pyramid.

The generic use of the term *usṭuwāna* is even greater in Abū Bakr,[95] for whom it covers Ibn al-Raqqām's four meanings and an additional two: the cube (according to Abū Bakr: 'the square cylinder (*usṭuwāna*) of which the two bases and the sides are equal') and the pyramid ('the conical [cylindrical] (*usṭuwāniyya*) solid figure'; 'the figure contained by planes which become cone-shaped and converge on a single point').

The Eastern mathematician Thābit ibn Qurra[96] used and proved a more accurate formula for the volume of the truncated cone and pyramid. The very same formula might have been known by geometrician practitioners as it is found in Ibn Ṭāhir's practical treatise:[97]

$$V = \frac{1}{3} h \left[S_1 + \sqrt{(S_1 S_2)} + S_2 \right]$$ where S_1 S_2 = area of the bases.

However, we have seen that Ibn al-Raqqām gave an approximate calculation, in spite of the fact that, as a mathematician, he might have been aware of Thābit ibn Qurra's formulation.

[27] The cone (*al-makhrūṭ*):

V = Ab× (h/3).

95 Hogendijk 1990, 140-142, 148.

96 Rosenfeld and Youschkevitch 1996, 452.

97 Sa'īdān 1985, 363.

[28] The irregular solid (*al-mujassamāt al-mukhtalifa al-aḍlāʻ wa-l-zawāyā*):

$V = V_n + V_n + V_n + V_n$ where n = solid from the irregular solid dissection

$V_n = Ab\times (h/3)$.

The dissection of a solid as explained by Ibn al-Raqqām is only possible in the quadrangular prism, but not in a prism with a greater number of sides.

To obtain pyramids with four triangular faces from an irregular solid, we draw the diagonals of the solid. These new solids are dissected into two solids with four triangular faces.

[29] The sphere (*al-kura*):

$$V = \left[\frac{P}{2} \times \frac{dm}{2}\right] \times \frac{2}{3} dm$$

Ibn al-Raqqām once again mentions (see [20]) the formulas for the length of the circumference and for the area of a circle.

4. Concluding remarks

Ibn al-Raqqām's booklet on measuring surfaces follows in the skill geometry tradition, with certain features remaining close to the practice started by Ibn ʻAbdūn. However, his booklet also shows a strong tendency toward simplicity in its contents and procedures that is particularly apparent in the application of a small number of standard formulas as well as the dissection into triangles method. The booklet is significant in the context of Ibn al-Khaṭīb's account[98] of Ibn al-Raqqām's work: 'He taught Mathematics, Medicine and Principles of Islamic Law in Granada ... He collected all these disciplines, condensed [them] and did not let up in making notes, commentaries, abridgements and compendiums'. Ibn al-Raqqām's booklet adds clarity to Ibn al-Khaṭīb's broad characterisation as it stands as an example of the summaries to which Ibn al-Khaṭīb refers. As mentioned in the introduction, al-Andalus was involved in instruction in surfaces measurement and, in all likelihood, Ibn al-Raqqām's booklet should be regarded in this light. Indeed, he might have written it for his teaching activity in the city of Granada between 1282 and 1315. Ibn ʻAbdūn[99] avoided demonstrations in his treatise so that begin-

[98] Ibn al-Khaṭīb, *Al-Iḥāṭa*, 69-70.

[99] Djebbar 2005, 27; Djebbar 2005a, 80.

ners in the study of measurement would not become discouraged. It is also likely that the astronomer and mathematician Ibn al-Raqqām had a similar goal in mind. Ibn al-Raqqām's interest in teaching texts is not a unique instance as most of the extant works of his Maghribī contemporary Ibn al-Bannā'[100] are also an outgrowth of his educational efforts. Ibn al-Raqqām's booklet illustrates the engagement in practical geometry training in the Islamic West during the 13th and 14th centuries.

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[100] Djebbar and Aballagh 2001; Djebbar 2005a, 89; Samsó 2007.

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