The Introduction to Geometry by Qusṭā ibn Lūqā: Translation and Commentary

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Abstract
The paper contains an English translation with commentary of the Introduction to Geometry by the Christian mathematician, astronomer and physician Qusṭā ibn Lūqā. This elementary work was written in Baghdad in the ninth century A.D. It consisted of circa 191 questions and answers, of which 186 are extant today. The Arabic text has been published in a previous volume of Suhayl by Youcef Guergour, on the basis of the two extant Arabic manuscripts. The Introduction to Geometry consists mainly of material which Qusṭā collected from Greek sources, some of which are now lost. Most of chapter 2 of the Jumal al-Falsafa by Abu Abdallah al-Hindi (12th century) was directly copied from Qusṭā's Introduction.

1. Introduction

Qusṭā ibn Lūqā was a Christian physician, philosopher and astronomer who was active in the second half of the ninth century AD. He was born in Baalbek in Lebanon, spent the middle part of his life in Baghdad, and then retired to Armenia, where he died. Qusṭā translated medical and scientific

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works from Greek into Arabic and in addition he authored a number of works of his own.[1]

The subject of this paper is Qusṭā's *Introduction to Geometry*, which we will call the *Introduction* from now on. In the previous issue of *Suhayl*, Youcef Guergour published a valuable critical edition of the Arabic text of the *Introduction* together with an introduction and a brief commentary. The purpose of the present paper is to make Qusṭā's *Introduction* available in an English translation. The extant text consists of 186 questions on geometry and their answers, and Qusṭā intended it to be a preparation for the study of the *Elements* of Euclid, which was available in several Arabic translations at the time.

Qusṭā addressed the *Introduction* to someone whose name is not mentioned in the extant Arabic manuscripts. The biographer Ibn Abī ʿUṣaybīʿa gives the complete title of Qusṭā's *Introduction* as "The Book on the Introduction to the Science of Geometry in the Way of Question and Answer. He (Qusṭā) composed it for Abu l-Ḥasan ʿAlī ibn Yaḥyā, Client the Caliph."[2] According to the *Fihrist*[3], this Abu l-Ḥasan ʿAlī ibn Yaḥyā was a specialist in literature, who authored a work on poetry, and who was a member of the courts of a succession of caliphs, from al-Mutawakkil until al-Muʿtamid. He was not a mathematician or astronomer himself, but he was the son of the famous astronomer Yaḥyā ibn Abī Manṣūr[4]. Because Abu l-Ḥasan ʿAlī ibn Yaḥyā died in 275 H. (A.D. 888-889), Qusṭā's work must have been written before that date, perhaps considerably.

Qusṭā's *Introduction* is interesting for several reasons. Qusṭā was widely read in Greek[5], and it is likely that almost all of the *Introduction*

[1] On the mathematical and astronomical works of Qusṭā ibn Lūqā see Sezgin vol. 5, p. 285-286, vol. 6, p. 181-182, Rosenfeld and İhsanoglu no. 118, p. 59.

[2] *Kitāb fi l-madkhal ilā ʿilm al-handasa ʿalā ṭarīq al-mas'ala wa-l jawāb allafahu li-Abi l-Ḥasan ʿAlī ibn Yaḥyā mawlā amīr al-mu'minīn*, see Gabrieli p. 346 following Ibn Abī ʿUṣaybīʿa.

[3] See Ibn al-Nadīm p. 143.

[4] On the family of astronomers Banu l-Munajjim, see Gabrieli, p. 365.

[5] Ibn al-Nadīm states that Qusṭa's Greek and Arabic was very good.

consists of material that he had collected from Greek sources, some of which may be lost today. The *Introduction to Geometry* is the probable place where some of this Greek material entered the Arabic tradition.

Because the *Introduction* is not a direct translation from Greek, the mathematical errors and infelicities in the work give us some insight in Qusṭā's limitations as a mathematician. Some examples: In Q 48, Qusṭā thinks that if two circles do not have the same center, they will intersect. According to Q 136, Qusṭā believed that an irregular tetrahedron cannot have a circumscribed sphere. As a matter of fact, any tetrahedron has a circumscribed sphere. In Q 175 Qusṭā incorrectly states that in a right cone, any straight line on the surface of the cone makes a right angle with the plane of the circular base. And so on. It seems that Qusṭā was not a creative geometer such as, e.g., his contemporaries Thābit ibn Qurra and Abū ʿAbdallāh al-Māhānī. Of course one should realize that mathematics was only one of Qusṭā's many fields of interest.

We will now proceed to a brief summary and analysis of Qusṭā's *Introduction to Geometry*, which extends the valuable commentary in Guergour's paper[6]. In Section 3 I discuss some Greek sources of the *Introduction* and its influence in the Arabic tradition. Section 4 is about the Arabic manuscripts and Guergour's edition. My translation is in Section 5. Section 6 contains a few explanatory notes to some of Qusṭā's questions and answers. Section 7 is an appendix containing a list of (mostly insignificant) notes to Guergour's Arabic edition of the *Introduction*.

2. Summary of the Introduction to Geometry

Qusṭā divided his *Introduction to Geometry* into a brief introduction and three chapters, on lines, surfaces, and solids respectively. For sake of convenience I have numbered the questions and answers. A notation such as Q 8 will refer to the question and answer to which I have assigned the number 8. In my notation, the introduction and the three chapters consist of Q 1 - 8, Q 9 - 57, Q 58 - 122, and Q 123 - 186, where the extant text breaks off. It is likely that Qusṭā's original contained five or six more questions and answers (see my note to Q 186 below), so the text we have is almost complete.

[6] Compare Guergour pp. 9-14.

In the introduction, Qusṭā first explains that geometry is about magnitudes and he then presents definitions of solid, surface, line and point. The definitions are similar to those in Euclid's *Elements*, but unlike Euclid, Qusṭā also discusses where the solid, surface, line and point are "found". According to Q 1, geometry includes the theory of ratio and proportion, but Qusṭā does not discuss this theory anywhere in the *Introduction*. He (rightly) considered the theory of proportions of Book V of Euclid's *Elements* as too difficult for a beginner.

In Chapter 1, Qusṭā first presents classifications of lines and angles in an Aristotelian vein. For lines, for example, the two "primary" species of lines are composed lines and incomposed lines. A composed line is a combination of incomposed lines. The incomposed lines are further subdivided into straight lines, circular lines (i.e., circumferences of circles and their arcs), and "curved" lines (such as conic sections). In Q 11 no less than six definitions of a straight line are presented. For Qusṭā, the circle itself is a plane surface, which belongs to Chapter 2.

Many questions and answers in Chapter 1 are devoted to explanations of geometrical terminology. Qusṭā does not provide figures anywhere in the *Introduction*. For example, the plane sine of an arc is simply introduced as "half the chord of twice the arc" (Q 46) without any further explanation. This was probably not very helpful for a beginning student of geometry who had never worked with chords and sines before. At some point, someone made an edited version of the text, which has been preserved in one of the manuscripts (L, see Section 4), and in which figures were added.

In Chapter 1, Qusṭā first discusses geometrical objects separately, and then in relation to one another. The division is not strict: Q 16 and Q 17 are on parallel and meeting straight lines, as a preliminary to the discussion of angles which starts in Q 18. Qusṭā continues the discussion of straight lines in relation to one another in Q 38.

In the end of Chapter 1, Qusṭā asks about the "properties" of certain geometrical figures. In the answers, he summarizes one or more theorems about the figure in question. For example in Q 54, the question is about the properties of parallel straight lines, and in the answer, Qusṭā summarizes several theorems on parallel lines which Euclid proved in Book I of his *Elements*. Qusṭā does not give any proofs.

In the last question Q 57 in Chapter 1, Qusṭā informs us that five "species" of curved lines are used in geometry: the parabola, hyperbola and ellipse, a spiralic line, and a mechanical line. Because the

Introduction is a work for beginners, he does not give further details. Thus we cannot decide whether he had any particular spiralic or mechanical line in mind.

In Chapter 2, Qusṭā first divides the surfaces into plane, convex and concave. In Q 60 he presents three definitions of a plane, and he then devotes Q 63 - 111 to what he calls *plane surfaces*, which are always finite figures. In his classification, *plane surfaces* include triangles, quadrilaterals, polygons, circles and their segments, the crescent-shaped and egg-shaped figure, and the sector. A large part of the discussion is devoted to the "properties" of these figures, and Qusṭā summarizes many theorems from the *Elements* of Euclid, of course without figures. He presents a classification of triangles into seven species and then (in Q 74 - 81) a primary classification of quadrilaterals into seven species which are then further subdivided into a total of 22 species. In Q 88 - 92 he explains the composition of polygons from triangles and the determination of the angle in a regular polygon in what seems to be archaic terminology. Q 93-99 are devoted to properties of regular polygons. In Q 100- Q 111 Qusṭā discusses the circle, the crescent- and egg-shaped figures, and circular segments and sectors. Finally, in Q 112 - 122 he proceeds to convex and concave surfaces, namely the surfaces of the sphere, the right cylinder and the right cone. He also discusses the way in which these three surfaces are produced by the rotation of a semicircle, a rectangle and a triangle around an axis.

In Chapter 3, Qusṭā distinguishes between solids contained by plane surfaces, solids contained by convex surfaces and solids contained by a combination of the two (such as the hemisphere). He discusses the five regular polyhedra and he informs the reader (in Q 132) that the ancients compared these polyhedra to the four elements and *falak* or celestial substance. In Q 133, Qusṭā introduces the well-known semiregular polyhedron of 8 triangles and 6 squares as well as a semiregular polyhedron of 6 triangles and 8 squares, which does not in fact exist. Then he discusses pyramids, rectangular parallelepipeds, and right prisms. The convex solids include the sphere, the ring (torus) and the "egg", which turns out to be the solid of revolution of a circular segment. Finally, in Q 159, Qusṭā distinguishes five solids contained by plane and convex surfaces: the cylinder, the cone, a segment of a sphere, a segment of a cylinder, and a segment of a cone. The extant text breaks off in Q 186, in the middle of the further classification of segments of a cylinder.

3. Sources and influences

In the *Introduction*, Qusṭā mentions Euclid (Q 11, 82, 133, 166, 168, 171-174), Archimedes (Q 11, 133), Plato (Q 11), Apollonius (Q 167, 175) and unspecified "others" (Q 11) and "ancients" (Q 132). Using this and other evidence, I have put together the following incomplete list of Greek authors and works which were somehow related to Qusṭā's *Introduction*.

1. Euclid's *Elements* was Qusṭā's most important source. It is the only work which he explicitly mentions in the *Introduction* as "the Book of Euclid" (before Q 1). The *Elements* was the basic Arabic textbook on geometry and Qusṭā took most of his definitions and theorems from Books I-IV, XI and XIII of that work, usually without reference. In Q 82, Qusṭā explicitly discusses the "quadrilaterals which Euclid mentioned" in agreement with definition 22 of Book I of the *Elements*[7] Qusṭā's references to Euclid in Q 166, 168, 171-174 agree with definitions 18-19 of Book XI of the *Elements*[8]. In Q 133 Qusṭā correctly says that Euclid did not mention semiregular solids.

2. The *Definitions* of Heron of Alexandria are similar in scope to Qusṭā's *Introduction*. Heron also presented his material in the form of question and answer. I now discuss what is to my mind the most striking similarity between the two works.

In Q 133, Qusṭā says the following about semi-regular polyhedra: "Euclid did not mention this, but Archimedes mentioned that a sphere can contain two solids which are both contained by triangles and squares, and each of these possesses fourteen faces. One of them is contained by eight equilateral and equiangular triangles and six equilateral and equiangular quadrilaterals, so this solid can be composed of air and earth. The other, inversely, is contained by six equilateral and equiangular triangles and eight equilateral and equiangular quadrilaterals." The latter polyhedron cannot exist, see my notes to Q 133.

Qusṭā's so-called quotation of Archimedes is incorrect because we know from Pappus of Alexandria that Archimedes knew all thirteen semi-regular polyhedra[9]. We can explain Qusṭā's mistake by the assumption

[7] Euclid vol. 1, p. 154.

[8] Euclid vol. 3, p. 261.

[9] See, e.g., Heath, *History of Greek Mathematics*, vol. 2, pp. 98-101.

that he summarized Heron's definition no. 104. Heron says: "Euclid proved in Book XIII of the *Elements* how these five (regular) solids can be circumscribed by a sphere; he only considers the Platonic (solids). Archimedes says that there are thirteen solids which can be inscribed in a sphere, since he adds eight to the five that had been found before; of these, Plato also knew the polyhedron with 14 faces, but this is a double, which is composed from eight triangles and six squares, from water and air. This was also known by some of the ancients. The other polyhedron is (composed) from eight squares and 6 triangles, but this one seems to be more difficult."[10] Qusṭā's Q 133 can be obtained from Heron's definition no. 104 by a process of truncation. It is of course also possible that Qusṭā possessed the text of Heron's definition 104 in a corrupted form.

The reader of Heron's *Definitions* will find many more similarities with Qusṭā's *Introduction*. Although Qusṭā does not mention Heron anywhere, I think that there are good grounds for believing that Heron's *Definitions* were an important source of inspiration for him. It should be noted that the *Introduction* is by no means a translation or edited version of Heron's work. Heron's work is strictly confined to definitions; unlike Qusṭā, Heron almost never discusses theorems in the form of "properties" of figures (an exception is the isoperimetric property of the sphere in Definition 82[11], also mentioned by Qusṭā in Q 147.) Heron is not really concerned with a classification of geometrical objects into Aristotelian "species". I note that Heron gives some details about the theory of proportion, which is missing in Qusṭā's *Introduction*.

3. We cannot be sure whether Qusṭā actually had access to Proclus' *Commentary to Book I of the Elements (of Euclid)*, but on the other hand, the *Introduction* and Proclus' commentary were not completely independent. In Q 9, Qusṭā presents a classification of lines into "composed" and "incomposed" (i.e., simple) lines. The same classification into *syntheton* and *asyntheton* lines is given by Proclus[12], who attributes this classification to Geminus. Thus it is also possible that

[10] My translation from Heron ed. Schmidt p. 64-67.

[11] Heron ed. Schmidt p. 54-55.

[12] Proclus ed. Friedlein p. 111, tr. Morrow p. 90.

Qusṭā had access to the work of Geminus rather than Proclus. Qusṭā's classification of lines is different from that of Heron, but unlike Qusṭā, Heron distinguishes between "composed" and "incomposed" surfaces.[13]

4. Qusṭā's classification of mathematical objects into "species" was evidently inspired by Aristotelian ideas. We will see below that Simplicius' commentary on Euclid's *Elements* is a likely source of inspiration for Qusṭā. I have not investigated whether Qusṭā's *Introduction* was directly inspired by works of the master himself.

5. In Q 167, Qusṭā mentions the construction of the cone in the *Conics* of Apollonius. In Q 176-177 Qusṭā renders definitions of right and oblique cones which he attributes to "Apollonius". The definitions are wrong since Qusṭā confuses the axis of the cone with the straight lines on its surface. In Q 170 Qusṭā mentions as "property of the cone" the fact that no two equal circles can be located on its surface. This property is correct only for a right cone, for an oblique cone it is false, as is proved in *Conics* I:5. These errors show that Qusṭā's knowledge of the *Conics* was superficial.[14]

6. In Q 11, Qusṭā presents a definition of the straight line which he attributes to "Archimedes". The same definition is found in Archimedes *On the Sphere and Cylinder*.[15]

7. The definition of a straight line, which Qusṭā attributes to "Plato" in Q 11, is not found in precisely the same way in the Greek literature. Qusṭā's source is probably not a work by Plato, but rather, some unidentified ancient commentary to definition 4 of book 1 of Euclid's *Elements*.

8. We now turn to the commentary to Book I of Euclid's *Elements* by Simplicius. This commentary is lost in Greek and only a few fragments have come down in the Arabic commentary to Euclid's *Elements* by al-Nayrīzī. In Q 21, 26-27, Qusṭā presents the same threefold classification of angles and (what is more important) the same subdivision of angles of circular lines as Simplicius. Simplicius says (in al-Nayrīzī's translation): "The species of angles which are contained by two circular lines are three,

[13] Heron ed. Schmidt p. 50-51.

[14] See Heath, *History of Greek mathematics* vol. 2, p. 133-135.

[15] See Heath, *History of Greek mathematics* vol. 2, p. 35.

I mean: (angles) such that the two convexities (*al-muḥdawdibān*) are opposite, and (such that) the two concavities (*al-muqaʿʿarān*) are opposite, and (such that) in it the concave is opposite to the convex. And the species (of angles) which are from a circular line and a straight line are two: I mean the species which is called horn-angle, and it is produced by the joining of the circular line to the straight line at the convexity, and the species which is produced by the joining of the circular line to the straight line at the concave side, such as the segment of a circle."[16] We note that Simplicius uses the same Aristotelian term "species" as Qusṭā. Thus Simplicius' commentary emerges as a likely source of Qusṭā's *Introduction*. If one compares the above quotation with Q 26-27, one can see how Qusṭā summarized his source, making things less clear in the process.

9. Lost Greek works. See also no. 7 above. In Q 74-81, Qusṭā divides quadrilaterals into 22 species. Proclus presents a much less complicated classification of quadrilaterals into seven species.[17] In Q 80, Qusṭā forgot to mention one category in the classification (which should therefore consist of 23 categories). The error can easily be corrected but it cannot be explained as a scribal error, so it was probably in Qusṭā's original. If Qusṭā were the author of the classification it is difficult to see how he forgot one obvious species in Q 80. I conclude that he did not invent the classification but adapted it from an (imperfectly transmitted) Greek source which has not come down to us otherwise.

10. Non-Greek sources. The concepts of plane sine and versed sine (Q 46, 47) were transmitted from India to the Islamic world in the late eight century A.D. They were well known so it is impossible to define the specific source(s) from which Qusṭā took them.

We now turn to influences of the *Introduction* on later authors. Guergour mentions (p. 14) three Arabic works which were influenced by Qusṭā's *Introduction*:

1. In the tenth century A.D. Abū ʿAbdallah al-Khwārizmī[18] composed a lexicographic work *Mafātiḥ al-ʿUlūm* (Keys to the Sciences). He copied

[16] My translation from the Arabic text in Arnzen p. 12.

[17] Proclus ed. Friedlein p. 170-171, tr. Morrow p. 133-134.

[18] He was not the same as the famous mathematician Muḥammad al-Khwārizmī who lived in the early ninth century A.D.

most of the material in the fifth Chapter[19] of the second Treatise of his work from Qusṭā's *Introduction* without reference. This fifth Chapter consists of four sections. Al-Khwārizmī used Q 2-8 in the first introductory section. In the second section on lines, he adapted Q 10, 16-25, 39-47, adding a figure for the plane sine and the versed sine. The fact that al-Khwārizmī presented the questions and answers in Q 39-47 in the same order proves the dependence. In the third section on surfaces, al-Khwārizmī adapted Q 59, 64, 65, 82, 84, 102, 109 (108), 113, 116, 119, and in the fourth section on solids Q 127-131, 136, 137, 139, 142, 143, 144, 145, 146, 150, 156, 157, 160, 165. At the end, al-Khwārizmī added two more solids of rotation. The *halīlajī* or myrobalan-shaped, is evidently a solid of rotation of segment greater than half a circle around its base, so the result resembles an apple. The *ʿadasī* or lentil-shaped solid is the same as Qusṭā's egg, which is also mentioned by al-Khwārizmī. He does not seem to have realized that Qusṭā's egg and his lentil are the same.

2. Guergour mentions Qusṭā's *Introduction* as one of the sources of the Letters (*Rasā'il*) of the Brethren of Purity (*Ikhwān al-Ṣafā'*), an anonymous tenth-century mystical group in Basra. I believe that the influence is likely but not certain. Here are some details.

In the second *Letter* on geometry, the Brethren cover part of the same ground as Qusṭā's *Introduction*, They also used other sources and they often understood what they were writing, so there was no need for them to copy their sources literally. In the section on circular lines[20], for example, the authors mention the four species of circular lines (Q 33), the semicircle (Q 34), the arc greater than a semicircle (Q 36), the arc less than a semicircle (Q 35), the center of the circle (Q 34), the diameter (Q 42), the chord (Q 44), the arrow (Q 45), the versed sine (Q 47) the plane sine (Q 46), parallel circular lines (Q 49), intersecting circular lines (Q 50), tangent circular lines (cf. Q 56), and "curved" lines (Q 57). Here the *Brethren* may have used Qusṭā's *Introduction* as a source of inspiration.

[19] Al-Khwârizmī ed. van Vloten, p. 202-209. Note on p. 202 line 1 the misprint *al-maqāla al-thālitha* for *al-maqāla al-thāniya*.

[20] Cairo ed. p. 53-55.

Their section on "species of solids"[21] is clearly independent of Qusṭā. The Brethren first divide the solids into categories depending on the number of surfaces by which they are bounded: by one (sphere), two (hemisphere), three (quarter of a sphere), four (tetrahedron), five (no example is given), or six surfaces. Here they mention the cube and the three other rectangular parallelepipeds in Q 137. This does of course not prove that Qusṭā was the source because the parallelepipeds are also mentioned elsewhere, e.g. in the *Definitions* of Heron.

For later use I will discuss here the section on "species of surfaces"[22]. In this section the Brethren distinguish three types of surfaces, namely plane, convex and concave (Q 59). Then, as examples of (which?) surfaces they mention the egg-shaped (Q 105), the crescent-shaped (Q 104), the pine-cone, the myrobalan-shaped, the dome-shaped (*nīmkhānijī*), the drum-shaped (*ṭabalī*), and the olive-shaped (*zaytūnī*). The last five "surfaces" are not mentioned by Qusṭā, and unfortunately the Brethren give imperfect figures without further specifications. So we cannot determine what exactly they meant by these five figures.

3. In the 12th century, Abū Abdallāh al-Hindī compiled his *Jumal al-Falsafa* ("Sentences on Philosophy") in seven Books. In these books he explained the terminology of different branches of "philosophy" in the form of questions and answers. He does not claim originality, and it turns out that the first part of his second book, dealing with geometry, is almost a literal copy of Qusṭā's *Introduction*. In my article on al-Hindī's section on geometry,[23] I was unaware of the connection with Qusṭā's *Introduction*, so my article is now out of date. For example, I stated in my article that the curved line called *nīmkhānijī*, which al-Hindī mentions as one of the five species of curved lines, was taken by him from the *Letters* of the Brethren of Purity. However, it is now clear that al-Hindī's *nīmkhānijī* has nothing to do with the Brethren but is a misspelling of the "mechanical" (*mīkhānijī*) curve in Q 57 of Qusṭā's *Introduction*.

[21] Cairo ed. p. 57-58.

[22] Cairo ed. p. 56-57.

[23] Hogendijk 1987-88, p. 11.

4. The manuscripts and Guergour's edition

Quṣṭā's *Introduction* is extant in two manuscripts, which will be indicated by the letters R and L[24]

R is MS. Rabat, Ḥasaniyya, 5829, 1b-10b. This undated manuscript is written in a Maghribi hand. The name of the scribe is not mentioned anywhere. The scribe was not competent in geometry, and the quality of the text as a whole is poor, so several passages in Quṣṭā's original cannot be reconstructed from R.

L is MS. Istanbul, Süleymaniye Library, Aya Sofia 4843, 1b, 32a-37b, 2a-11a. This manuscript is also undated and there are many scribal errors in it, sometimes of the most grotesque kind. L is actually a copy of an edited version of Quṣṭā's text. The anonymous editor (who more or less understood the subject and who therefore cannot have been identical to the scribe of L) deleted the introductory text before Q 1 and also Q 26-32. 48-51, 57. He interchanged the order of Q 53 and 52. He changed the beginning of every answer by systematically repeating the text of the question. In some places he "corrected" Quṣṭā's archaic terminology or he adapted Quṣṭā's answers to the standard Arabic versions of Euclid. He added examples and figures for Q 23-25, 34-37, 39-47, 53, 52, 54-56, 69, 71-73, 75-82, 103, 105. These figures and the accompanying text are duly rendered by Guergour in an appendix on pp. 57-69 of his paper.

H is Book 2 of al-Hindī's *Jumal al-Falsafa*, which has been published in facsimile by the Frankfurt Institute for the History of Arabic-Islamic Science on the basis of Ms. Istanbul, Süleymaniye Library, Esat Efendi 1918. Book 2 of the *Jumal al-Falsafa* includes an almost literal copy of Quṣṭā's *Introduction*, so H can sometimes be used to restore Quṣṭā's text where this is impossible on the basis of L and R. Al-Hindī only did some slight and superficial editing and it is clear that his understanding of the material was very limited. Thus we can be sure that he made no non-trivial changes to the text. I have referred to the pages in H according to the numbering of the facsimile-edition, in which the part corresponding to Quṣṭā's text is printed on pp. 16, 33-34, 17-32, 35. The editors of the facsimile did not realize that the leaf 33-34 is misplaced in the manuscript. Al-Hindī's treatise on geometry continues on p. 35-44 with a section on

[24] For more information on these manuscripts see Guergour p. 16-17.

practical mensuration which is unrelated to Qusṭā and written in a different style.

On the poor quality of the manuscripts and the lack of understanding of their scribes, compare Section 6, my notes to Q 56, 57, 126, 164 (R); Q 1, 38, 78, 126, 134 (L); Q 74, 154, 164 (H). More examples could easily be found.

G is Guergour's edition of Qusṭā's text in the previous issue of Suhayl. Guergour decided, to my mind correctly, that manuscript R is closest to Qusṭā's original and hence his edition is mainly based on R, although he also consulted L. Because L is not a direct copy but an edited version of Qusṭā's text is it natural that Guergour did not render all variants of L in his critical apparatus. Guergour did an excellent job in reconstructing Qusṭā's text from the two poor manuscripts R and L. Guergour did not use H.

My translation is essentially based on Guergour's critical edition. I have checked the whole edition with the three manuscripts R, L and H, and there are a few cases where my reconstruction of Qusṭā's original differs from Guergour's text. In such cases, I render in pointed brackets <...> translations of Arabic words which do not occur in Guergour's edited text. In Section 7 I print the Arabic text which I have translated, in so far as it is different from Guergour's editions, with references to page and line of Guergour's edition. Here is an example.

In Q 51 G reads, following R: "What is a finite arc? It is (such) that if from the center which is common to them two straight lines are drawn to them, they pass through all their extremities." The condition actually defines two or more finite arcs which are similar. The passage does not occur in L, but H has the reading "finite similar arcs" (*al-qisī al-mutanāhiya al-mutashābiha*) instead of "finite arc" (*al-qaws al-mutanāhiya*). I have therefore adopted H's reading, and the word < similar > appears in pointed brackets in my translation. In the translation I do not draw attention to the difference between arc (in G's text) and arcs (in my translation). In Section 7 I print "Q 51: G 27:4 change *al-qaws* (R) to *al-qisī* (H). To make mathematical sense, I have added *al-mutashābiha* with H."

Most of my changes to Guergour's edition are in cases where he followed the reading of R, and where L or H have another reading which is preferable from a mathematical point of view, or more in agreement with the traditional mathematical terminology. The differences in Section 7 are on the whole insignificant and they in no way detract from my positive judgment of Guergour's edition. The reader will note that H confirms many of Guergour's emendations to the text.

In my translation I also use parentheses (...). These include my numbers of the questions and answers, my own explanatory additions, and also the beginning of pages in the three manuscripts L, R and H and in the edition G.

5. Translation of the text

The book of Qusṭā ibn Lūqā from Baalbek, on the Introduction to the Art of Geometry.

He said: My wish to strengthen my good relationship to you, may God elevate you, and my desire to come closer to your heart, urge me to present you with everything to which I have found a way. I have found you, may God elevate you, attracted to the art of arithmetic and geometry, and inclined to the study of it. Thus I decided to compose for you a book on geometry, with which you can practice before studying the Book of Euclid (i.e., the *Elements*). It is similar to an introduction to this art. I have composed this book in the form of question and answer, because this is easy to understand, simplifies the notions, and facilitates memorization. I have divided it into three chapters (*maqālāt*):

The first chapter: on lines and angles, their species and their subdivisions.
The second chapter: on surfaces, their species and their properties.
The third chapter: on solids, their species and their angles.

Before this, I have presented the definition of geometry and the clarification of it (this definition), and I have made my presentation in this in the natural order. In God I put my trust. He is my certainty and my hope.

The discussion of geometry, (1) (H 16) (L 1b) and what is geometry? It is an art dealing with (a) the magnitudes and (b) the knowledge of their nature, their species and their properties, and (c) the size of those (magnitudes) which are of the same kind, with respect to one another.

(2) What are magnitudes? Magnitudes are (mathematical objects) possessing dimensions/distances (same word in Arabic).

(3) How many are the magnitudes? Three: lines, surfaces and solids.

(4) How many are the dimensions? Three: length, breadth and depth. It is the same if you say depth or if you say height, but the custom is that this dimension is called “depth” if its beginning is conceived from the higher of the two endpoints of it (i.e., the magnitude), (G 19) and “height” if its beginning is conceived from the lower endpoint.

(5) What is a solid? A magnitude possessing three dimensions: length, breadth and height. Its extremity consists of surfaces.

(6) What is a surface? A magnitude possessing two dimensions: length and breadth, without height. It is conceived in isolation (R 2a) by the mind and by the imagination, not by sensory perception. It is found by sensory perception (only) in the solid, since it is the extremity of it. For if the height, that is the depth, is removed from the solid, only length and breadth remain, and this is the surface. The extremity of the surface consists of lines.

(7) What is a line? < A line is > magnitude possessing only one dimension, namely length, without breadth and without height. In isolation, it is conceived only by the mind and the imagination, not by sensory perception. It is found by sensory perception in the surface, since it is the extremity of it. For if the breadth is removed from the surface, only the length remains, and that is the line. The extremity of the line is two points. (G 20)

(8) What is a point? A point is a thing without dimension, that is to say, without length, without breadth and without height. In isolation it is found by the mind and the imagination, not by sensory perception. It is found by sensory perception in the line, since the line is length without breadth, so if the length is removed form it, its extremity remains, which is the point, without length, breadth and height, that is to say, with no dimension at all. What has no dimension cannot be subdivided, since what is subdivided has dimensions. What cannot be subdivided does not have any part, since the parts of the whole are its subdivisions. So the point necessarily does not have any part.

(9) How many are the (L 32a) primary species of lines? Two: composed and uncomposed.

(10) How many are the species of uncomposed lines? Three: straight, circular, and curved.

(11) What is a straight line? (11.1) It is a line < whose distance/dimension, (same word in Arabic) is equal to the distance/dimension between the two points which are its extremity. (11.2) And it is the (line) stretched out > in the straightness of the two points

which are its extremities. This is how Euclid defined it. (11.3) And Archimedes defined it by saying: it is the shortest line joining between two points. (11.4) Plato defined it by saying: A straight line is a (line) such that each point which is assumed on it is in one direction. (11.5) Others defined it by saying: A straight line is a line which, if its two extremities are fixed and it is rotated, it is not moved from its place. (11.6) Others defined it by saying: A straight line is a (line) such that its parts fit on one another from (G 21) all directions.

(12) What is a circular line? It is (a line) such that no three points can be assumed on it in one direction, and there can be found a point such that the straight lines issuing from it to it (the line) are all equal.

(13) What is a curved line? It is (a line) such that no three points can be assumed on it in one direction, and there cannot be found a point such that the straight lines issuing from it to it (the line) are all equal.

(14) What is a composed line? It is (a line) composed of two or more (H 33) uncomposed lines of one species (R 2b) or more than one species.

(15) How many are the primary positions of straight lines? Two, the position of parallelism and the position of meeting.

(16) What are parallel lines? They are (lines) such that, if they are in one plane and if they are both extended continuously in both directions, they do not meet in any direction at all.

(17) What are meeting lines? They are (lines) which meet and contain angles. (G 22)

(18) How many are the primary species of angles? Two: plane and solid.

(19) What is a plane angle? It is the touching of two lines and their meeting, not in a straight line.

(20) What is a solid angle? It is the touching of three lines, and their meeting, such that each pair of them is not in a straight line, and such that all (three) of them are not in one plane.

(21) How many are the species of plane angle? Three: one of them is such that the two lines containing it are straight, and the second is such that the two lines which contain it are circular, and the third (species) is such that of the lines containing it, one is straight and the other is circular.

(22) How many are the species of angles contained by two straight lines? Three: right, obtuse and acute.

(23) What is a right angle? It is an (angle) such that if one of the lines containing (it) is extended in a straight line, the angle (L 32b) which is produced by the extended line and the other line is equal to it.

(24) What is an acute angle? It is an (angle) such that if one of the lines containing (it) is extended, then the angle produced by it (the extended line) and by the other line is greater than it (the first angle). Also: the acute (angle) is the angle less than a right (angle). That is to say: (G 23) if from any right angle < some thing > is subtracted, the subtracted (thing) is an acute angle and the remainder is an acute angle.

(25) What is an obtuse angle? It is such that if one of the lines containing it is extended, then the angle produced by the extended line and by the other line is an angle less than it, that is to say that it is an acute angle. And also: an obtuse (angle) is greater than a right (angle): If to any right angle some acute angle is added, the angle composed by the addition of them is an obtuse angle. (L 33a)

(26) How many are the species of angles which are contained by two circular lines? Their species are three: one of them such that the concavities of the two arcs which contain it (the angle) are opposite to one another, the other is < convex >, such that the convexities of the two arcs which contain it are opposite to one another, and the third is such that the convexity of one of the arcs which contain it is adjacent to the concavity of the other arc.

(27) How many are the species of angles which are contained by a circular line and a straight line? Two: one of them is such that the straight line is opposite to the convexity of the arc which contains the angle together with it (the straight line), and the other is such that the straight line is opposite to the concavity of the arc which contains the angle together with it (the straight line).

(28) How many are the names of the angles with respect to position? The names of the angles with respect to their positions are two: opposite and (R 3a) alternate.

(29) What is an opposite angle? It is such that if (H 34) a straight line meets two straight lines, it (the opposite angle) is on one side (i.e., on the same side of the first line as the angle to which it is opposite), and the two of them (i.e., the angle and its opposite angle) are contained by the meeting line together with the two remaining lines.

(30) What is an alternate angle? It is such that if a straight line meets two straight lines, then the two of them (i.e. the alternate angles) are contained by the meeting line and the two remaining lines, on the two opposite sides (of the meeting line).

(31) What are intersecting angles? They are (the angles) which are produced by the intersection of two straight lines at their point of intersection. (G 24)

(32) What are two adjoining angles? They are the two (angles) which are produced on one side between two straight lines which are met by one straight line. (L 33a)

(33) How many are the species of circular lines? Four: the circumference of the circle, the semicircle, the arc less than a semicircle, and the arc greater than a semicircle.

(34) What is a circular line which is the circumference of a circle? It is a line which begins at a point and ends at it, and which contains a surface by itself, and that surface is called a circle, and in its interior is a point such that all straight lines issuing from it to it (the line) are equal, and that point is called the center of the circle.

(35) What is an arc of a semicircle? It is a segment of the circumference of a circle such that if its extremity is joined by a straight line, it (that line) passes through the center of the circle.

(36) What is an arc less than a semicircle? It is a segment of the circumference of a circle such that if its extremity is joined by a straight line, the center of the circle falls outside it. (L 33b)

(37) What is an arc greater than a semicircle? It is a segment of the circumference of a circle such that if its extremity is joined by a straight line, the center of the circle falls inside it.

(38) How many are the names of the straight lines which meet one another? Nine: side, arm, base, diameter/diagonal (same word in Arabic), perpendicular/altitude (same word in Arabic), chord, arrow, plane sine, (G 25) versed sine.

(39) What are straight lines which are called sides? They are (the lines) which contain (plane) figures, because every straight line which contains with other straight lines a (plane) figure is called a side of that figure.

(40) What are straight lines which are called arms? They are two plane lines which contain an angle. If two straight lines contain an angle, each of these lines is called (L 34a) < an arm >.

(41) What is a straight line which is called base? It is a line which joins the two endpoints of two straight lines which contain an angle. If two lines contain any angle and a line joins their two endpoints, that line is called base.

(42) What is a straight line which is called diagonal/diameter (same word in Arabic)? It is (R 3b) (a line) which issues from one angle and

ends at another angle, and divides both angles in < some > division. And this figure is a quadrilateral. Diameter is also the name of the straight line which bisects the surface of a circle and passes through its center, and it is the longest straight line which falls inside the circle. (G 26)

(43) What is a straight line which is called perpendicular/altitude (same word in Arabic)? It is a (line) such that if it stands on another line, it contains with it a (L 34b) right angle. Also, if one draws from an angle to < its > base, I mean its chord, a line and if it contains with the two parts of its base two right angles, that line is called altitude (of the resulting triangle).

(44) What is a straight line which is called chord? It is (a line) (H 17) which joins the two endpoints of circular lines or curved lines. Also, any line which joins the two endpoints of two lines which contain an angle is called a chord, because it subtends that angle. The difference between it (chord) and a base is that a base only subtends an angle, but it (the chord) subtends angles < and > arcs, and therefore it has the special name "chord".

(45) What is a straight line which is called arrow? It is (the line) which is drawn from the point which bisects the chord of an arc and contains (L 35a) with the half chord a right angle, and it ends at the arc.

(46) What is a straight line which is called a plane sine? It is half the chord of twice the arc < of > which < it is the > sine.

(47) What is a straight line which is called versed sine? The < versed > sine is the arrow of twice < the > arc of which it is the (versed) sine. (G 27)

(48) How many are the positions of circular lines? Two: parallel and intersecting.

(49) What are parallel circular lines? They are (circular lines) drawn at the same center.

(50) What are intersecting circular lines? They are (circular lines) which contain angles.

(51) What are finite < similar > arcs? They are (arcs) such that if from the center which is common to them two straight lines are drawn to them, they pass through all their extremities.

(52) (L 35b) What are < the properties of > straight lines which intersect? That by the intersection of each pair of lines of them four angles are produced, and that these four angles are either four right angles, or equal to four right angles, and that each angle of them is equal to the angle opposite to it.

(53) (L 35a) How many are the species of parallel lines? Two: straight and circular. Straight (parallel lines) are such that if they are continuously extended in both directions, they will never meet. If they are three or four (lines), all of them are called parallel. Parallel circular (lines) are circular (lines) for all of which a common point can be found such that the lines drawn from it to each of them are equal, and such that the parts of them (i.e, of the lines drawn from the center) < between > the circular lines are also equal. (L 35b)

(54) What are the properties (R 4a) of parallel straight lines? They are (as follows): If a line meets them, the angle which is contained by that line and one of the parallel lines is equal to the angle (G 28) contained by that meeting line and the other parallel line, on that (same) side. The alternate angles - they are the (angles), one of which is contained by the meeting line and one of the two (parallel) lines on one side, and the other by the side of the meeting line with the second of the parallel lines on the other of the two sides - are equal. The opposite angles are equal to two right angles. The interior angles are equal to two right angles. (L 36a)

(55) What are the properties of similar circular parallel lines? Their properties are: that the angles (at the common center) which fall in them are equal, and that the two angles which subtend them (and whose angular points are) in the remainders (complements) of their circles are equal.

(56) Which lines are called tangent? Tangent lines are lines which meet but do not intersect, and that is only possible for two lines of which one is circular and the second one is straight <, or in two circles such that the convexity of one of them is adjacent to the convexity of the other, or in two circles such that the concavity of one of them is adjacent to the convexity of the other >. For every pair of tangent lines, their tangency is at one point.

(57) How many are the species of curved lines? Their species are many, infinite in multitude. But five of them (H 18) are used in the art of geometry: three of them are the three sections of the cone which are called parabola, hyperbola and ellipse; and (the fourth line is) the line which is called the spiralic, and (the fifth line is) the line which is called < the mechanical >. Since these lines < are > made < by > instruments, they are only understood by someone who is accomplished in geometry. For us it is sufficient to (just) mention them in this book of ours, since it is an introduction for students. (G 29)

The second chapter, and the discussion in it will be about surfaces.

(58) (L 36b) What is a surface? It is length and breadth without depth, and its extremities are lines.

(59) How many are the primary species of surfaces? Three: plane, convex and concave.

(60) What is a plane surface? (60.1) It is (a surface) such that the distances/dimensions (same word in Arabic) of it are equal to the distances/dimensions of the lines of its extremity. (60.2) Others defined it by saying: a plane surface is such that if arbitrary < many > points are assumed (in it), it is possible to draw from any of these points to < any > (other) of them a straight line which is located in the surface. (60.3) And others defined it by saying: a plane surface is (a surface) such that if a straight line is placed (perpendicularly) on it at any position whatever on it, and if one rotates around it < a straight line >, it (the rotated line) touches the whole of it (the surface). A plane surface is called a plane.

(61) What is a convex surface? It is such that its extremities can be joined by another surface less than it, and the least surface (R 4b) which joins its extremities is < lower > than it. (G 30)

(62) What is a concave surface? It is such that its extremities can be joined by another surface less than it, and the least surface which joins its extremities is higher than it.

(63) How many are the < primary > species of plane surface? They are three: contained by straight lines, contained by circular lines, and contained by straight lines and circular lines together. Each of these surfaces is called a plane figure.

(64) How many are the species of plane figures which are contained by straight lines? They are many, infinite in multitude. They are named after the number of sides which contain them. They begin with three and increase (in) < always > ascending (order). The first of them is the triangle, < which is contained > by three lines. Then the quadrilateral, which is contained by four lines. Then the pentagon, which is contained by five lines, and similarly it (the number of lines) increases. The triangle is the first of them because two straight lines do not contain a plane figure. Of these plane figures, the ones which are contained by more than four lines are called polygons, and therefore the pentagon is the first of the polygons.

(65) How many are the primary species of triangles which are contained by straight lines? Three: right-angled, obtuse-angled, and acute-angled (literally: with an acute angle).

(66) How many are the species of the right-angled triangle? Two species: isosceles, and this is the one with two equal sides, and scalene, (L 37a) and this is the one of which all sides are different.

(67) How many are the species of the obtuse-angled triangle? Two species: isosceles, and this is the one with two equal sides, and scalene, and this is the one of which all sides are different. (G 31)

(68) How many are the species of the acute-angled triangle? Three: equilateral, (H 19) isosceles and scalene. So the division is like this.

(69) How many must the particular species of triangles be, which (species) cannot be subdivided? Their particular species which cannot be subdivided must be seven: the equilateral triangle, and this can always be acute-angled only; the isosceles acute-angled; the isosceles right-angled; the isosceles obtuse-angled; the scalene acute-angled; < the scalene right-angled, and the scalene obtuse-angled.> If the isosceles triangle is right-angled or obtuse-angled, the third side must necessarily be greater than the two remaining sides. If it is acute-angled, the (third) side can be greater than the two remaining sides or less than them.

(70) What is the property of the absolute triangle? Its property is that the sum of < each pair of > its sides is longer than the third, that its greater side subtends the greater angle, and that if one of its sides is extended, the exterior angle which is produced is equal to the (sum of the) two < interior > angles opposite to it; and that if < its three > angles are added, they are equal to two right angles. Therefore there can never be < in one triangle > two (R 5a) right angles, nor can there be one right angle and one obtuse angle. But it is necessary that two of the angles of each triangle are acute, and the remaining angle can be acute, right, (G 32) or obtuse.

(71) What is the property of the right-angled triangle? That the product of each of the two sides which contain the right angle times itself, if (L 37b) the two (products) are taken together, (the sum) is equal to the product of the side which subtends the right angle times itself; and that the product of the two sides which contain the right angle is equal to the product of the third side times the perpendicular drawn from the right angle to its chord (i.e., the hypotenuse), and that the product of the two parts of the chord (i.e. the two parts of the hypotenuse divided by the altitude) is equal to the product of the altitude times itself; and that the product of the whole chord (hypotenuse) times < one of its two parts (of the hypotenuse divided by the foot of the altitude) > is equal to the product of the side which is adjacent to that part times itself.

(72) What is the property of the obtuse-angled triangle? Its property is that the product of each of its two sides containing its obtuse angle times itself, if the two (products) are taken together, (the sum) is less than the product of the third (side) times itself, and that (side) is the chord of the obtuse angle. And if (H 20) one of the sides containing the obtuse angle is extended in the direction of the obtuse angle, and from the angle of the triangle (i.e., the angular point not on that side) a perpendicular is drawn (to the extended side), then the product of the side which subtends the obtuse angle times itself exceeds the (sum of the) product(s) of the two sides which contain the angle, each times itself, by (an amount) equal to the product of the side (to) which (the perpendicular) is drawn times twice the part of it on which the perpendicular falls. (L 2a)

(73) What is the property of the acute-angled triangle? That the (sum of the) product(s) of the two sides which contain the acute angle, each (side) times itself, is more than the product of the chord of that acute angle times itself. < And if from another angle of the triangle to one of the two sides containing its (above-mentioned) acute angle a perpendicular is drawn which meets one of these two (sides), then the (sum of the) product(s) of the two sides which contain the acute angle, each (side) times itself, is greater than the product of the third side times itself > by (an amount) equal to the product of the line on which the perpendicular falls (G 33) times twice the part of it (i.e., of that line) which is adjacent to the acute angle. (L 2b)

(74) How many are the < primary > species of quadrilaterals which are contained by straight lines? They are seven: equilateral (75), having three sides equal (76), having two pairs of equal opposite sides (77), having two pairs of equal meeting sides (78), < having two opposite sides equal and the two remaining (sides) different > (79), having two meeting sides equal and the two remaining (sides) different (80), and with all sides different (81).

(75) How many are the species of quadrilaterals of which four sides are equal? Two: one of them is right-angled and the other has < different > (angles) but two pairs of equal opposite angles. The equilateral right-angled one has the special name “square”, and the equilateral one with different angles has the special name “rhombus”. These two species of quadrilaterals have two pairs of parallel sides.

(76) How many are the species of quadrilaterals three of whose sides are equal? Three: in one of them the two unequal sides are parallel, and this (species of) quadrilateral cannot have a right angle; and the other (i.e.,

second) has no parallel sides but a right angle, and there cannot be more than one right angle in this (species); and the third (species) has neither parallel sides (R 5b) nor a right angle.

(77) How many are the species of quadrilaterals having two pairs of equal opposite sides? Two: rectangular and non-rectangular. The rectangular (species) is called “oblong square” (i.e., rectangle). The non-rectangular (species) (G 34) has two pairs of equal opposite angles, and it is called the rhomboid. These two quadrilaterals have (H 21) two pairs of opposite parallel sides.

(78) How many are the species of quadrilaterals having two pairs of equal meeting (i.e., adjacent) sides? Three: in one of them, two opposite angles are right, and the other angles are different; in the other (i.e., the second), one angle is a right angle, and the two (remaining) opposite angles are equal; and in the third, there is no < right > angle at all, and two opposite angles are equal. In these quadrilaterals it is not possible that there is parallelism (of the sides) at all.

(79) How many are the species of quadrilaterals of which two opposite sides are equal and the remaining two sides are different? Three: in one of them, the two different sides are parallel, and there cannot be a right angle in it; in the other (i.e., the second), there are < no >parallel sides and one of its angles is (a) right (angle), and there cannot be more than one right angle, and in the third (species) there are neither (L 3b) right angles nor parallel sides. (G 35)

(80) How many are the species of quadrilaterals of which two meeting (i.e., adjacent) sides are equal and the two remaining sides are different? Four: with two < adjacent > right angles and two of its sides parallel; with two alternate right angles but without parallel sides; with one right angle but also without parallel sides; and without right angle and without < parallelism > in it.

(81) How many are the species of quadrilaterals with different sides? Five: with two adjacent right angles and two parallel sides; with two opposite right angles and no parallelism (of sides) in it; with one right angle and no parallelism (of sides) in it either; (with) parallel (sides) < without > right angle; and with no parallelism and no right angle.

(82) How many are the quadrilaterals which Euclid mentioned? Five: right-angled with equal sides, and he called it the true square; right-angled with two pairs of equal opposite sides, and he called it an oblong square; equilateral without right angles, but with two pairs of equal opposite angles, and he called it the rhombus; only with two pairs of equal opposite

sides < and two pairs of equal opposite angles, > and he called it the rhomboid. And what went beyond these definitions he called trapezium (L 4a).

(83) Which are the quadrilaterals around which a circle can be circumscribed? Each quadrilateral with equal sides and angles, each quadrilateral with two opposite right angles; and each quadrilateral of which two opposite (H 22) angles (G 36) are equal to two right angles.

(84) Which plane figures are called polygons? < They are > (the figures) which are contained by more than four sides. The first of them is the pentagon, then the hexagon, then the heptagon, (R 6a) then the octagon, then the nonagon, then the decagon, then the (polygon) having eleven sides, then the (polygon) having twelve sides, then the figures which follow (this figure) by adding sides <one by one >, without end.

(85) How many are the species of each polygonal figure? Three: equilateral and equiangular, equilateral having different angles, and having different sides and angles.

(86) How can it be determined which of the polygons are equilateral and equiangular? This is determined by circumscribing a circle around it, and inscribing a circle in it, and circumscribing (it) around a circle, and inscribing (it) in a circle.

(87) What is the meaning of: a figure inscribed in a circle and a figure circumscribed around a circle? The (figure) circumscribed in a circle is a figure such that each of its angles is tangent to the circumference of a circle, and the figure circumscribed around a circle is a (figure) such that each of its sides is tangent to the circumference of the circle.

(88) Of what can the pentagon be composed, and into what can it be divided? The pentagon (L 4b) is composed of three triangles if there is made for each pair of them a common side, and the angles of all of them are combined at a point. And it is divided into three triangles if two < non-intersecting > lines are drawn inside it from one angle to the two remaining angles which are opposite to it (G 37).

(89) Of what can the hexagon be composed and into what can it be divided? It can be composed of four triangles < if there is made for each pair of them a common side, and the angles of all of them are combined at one point. It is divided into four triangles > if three non-intersecting lines are drawn inside it from one of its angles to the angles which are opposite to it.

(90) How can it be known for any polygonal figure of how many triangles it is composed and into how many triangles it can be divided? If

a polygon is assumed for us, in order that we know of how many triangles < it can be composed > and into how many triangles it can be divided, then this is done if you count how many figures are between it and the triangle. Then you look at that and you say: that figure is composed of triangles equal in number to that amount, if for each pair of these triangles one side is made common. And it is divided into them (i.e., triangles) if from one of its angles lines are drawn inside it < according to > that number, which (lines) end at the remaining angles.
The example of this is, as if someone says to us: of how many triangles is the decagon composed, and into how many triangles can it be divided? Then you count the figures from the triangle to the decagon, and they are the triangle, the square, the pentagon, the hexagon, the heptagon, the octagon, the nonagon and the decagon. The total of them is eight, so you say: < The decagon is composed from eight triangles, > if there is for each pair of them a common side, and it is divided into eight triangles if from one of its angles non-intersecting lines are drawn to the remaining angles. It is like this (H 23) for the remaining figures; the operation is the same.

(91) How do you know < how many are > (the angles) in each of the polygons inscribed in a circle, (expressed in terms) of a right angle? If we want this, then let us look < how many > are the plane figures from the triangle to that figure (G 38). We multiply it by two, and the result is the number (R 6b) of right angles in that figure. Example of this: if we want to know how many right angles are in the heptagonal figure, then you look how many figures are between the triangle and the heptagon, and they are five, I mean: the triangle, the square, (L 5a) the pentagon, the hexagon and the heptagon. So you multiply five by two, then the product is ten. Then you say: the right angles in the heptagon are ten angles, that is to say: if all angles of the heptagon are taken together, they are ten right angles.

(92) How do you know in each of the polygons inscribed in a circle, I mean the equilateral and equiangular, the magnitude of < its > angle (expressed in terms) of a right angle? If we want that, we look by the sentence which we have (just) described, how many right angles fall in it. Then we divide the number of (right) angles of the figure by the number of its sides, and the result of the division is the magnitude of the angle of that < figure > in terms of the right angle.
The example of this is as if we want to know the ratio < of the angle > of the equilateral heptagon with respect to a < right > angle. We know that all the angles of the heptagon, if they are added, are equal to ten right angles. And the sides of the heptagon are seven. So we divide ten by

seven, then one angle is, in the ratio of the right (angle), one and three-sevenths. (G 39)

(93) What are the properties of a quadrilateral inscribed in a circle? As for the equilateral (quadrilateral), that the product of its side times itself is equal to the power (i.e., the square) of the two halves of the diagonal, if each of them (the two halves) is multiplied by itself (and the products are added). Also for quadrilaterals with different sides, (the property is) that the product of its four sides, if each pair of opposite sides is multiplied, and (these two products are) taken together, the whole of that is equal to the product of its two diagonals.

(94) What are the properties of the pentagon inscribed in a circle? That the product of its side times itself is equal to the product of the side of the hexagon inscribed in the same circle times itself and (the product) of the side of the decagon, also inscribed in that circle, times itself, if these products are taken together.

(95) What are the properties of the hexagon inscribed in a circle? Its properties are that its side is equal to the radius of that circle.

(96) What are the properties of the heptagon inscribed in a circle? That if on its side an isosceles triangle is constructed in that (H 24) circle, each of < its > two angles above the base, I mean the two angles which are subtended by the equal arms, is three (L 5b) times the remaining angle. And therefore it is (true) that if it is possible for us to construct an isosceles triangle such that each of < its > two angles at the base is three times the < remaining > angle, it will be possible for us to construct a heptagon in a circle.

(97) What are the properties of the octagon inscribed in a circle? That if < on > its side an isosceles triangle is constructed in that circle, each of < its > two angles above the base is three (G 40) and a half times the remaining angle, I mean that if the angle subtended by the base is one, each of the two angles above the base is three and a half.

(98) What are the properties of the nonagon inscribed in a circle? That each of the two angles of its isosceles triangle inscribed in the circle, which (angles) are above the base, is four times the remaining angle.

(99) What are the properties of the decagon inscribed in the circle? That if the side of the pentagon inscribed in that circle is multiplied by itself and from that the product of the radius by itself is subtracted, and the root of the (R 7a) remainder is taken, it is equal to the side of it (the decagon). And if we construct on its side an isosceles triangle such that its angle is composed on the circumference of the circle, each of the two angles at the

base is four and a half times the remaining (angle). < Whenever the figure increases by one side, > and it is like this for the (polygon) having eleven sides, each of the two equal angles of the triangle is (in terms of the angle at the top) equal to a number whose ratio to the remaining... (text incomplete).

(100) How many are the species of plane figures which are contained by circular lines? (A figure) contained by one circular line, and it is called the circle; (a figure) contained by two circular lines; and (a figure) contained by more than two circular lines. (G 41)

(101) What is a circle? It is a plane figure which is contained by one line which is called the circumference, and in its interior is a point such that all the lines issuing from it to that circumference are equal, and that point is called the center for it.

(102) How many are the species of plane figures which are contained by two circular lines? Two; one is such that the convexity of one of the arcs < is adjacent to > the concavity of the other arc, and that figure is called crescent-shaped, and the other is such that the concavities of the two arcs are opposite, and this figure is called egg-shaped.

(103) How many are the species of crescent-shaped figures? Three: such that its exterior arc is equal to a semicircle, or less than a semicircle, or (L 6a) greater than a semicircle.

(104) What is the property of the crescent-shaped figure? That its interior arc, I mean the arc whose convexity is adjacent to the concavity of the other arc, < is less than the other arc, > and that its exterior arc, if it is (H 25) less than half (a circle), it is < (part) of > a circle which is smaller (in size) than the circle of which the other arc is (part). And similarly if it (the exterior arc) is a semicircle, it is < (part) of > a circle smaller (in size) than the circle of which the other arc is (part). And if it (the exterior arc) is greater than a semicircle, it can be < (part) of > a circle equal to the circle of the other arc, or < (part) of > a larger circle, or of a smaller circle.

(105) How many are the species of the egg-shaped figure? Two: one (species) such that the two arcs which contain it are equal, and the other such that the two arcs are different.

(106) What is the property of the egg-shaped figure? Its property is that if its two arcs are equal, they are all (part) of the same circle, and each of them is less than a semicircle. And if they are different, then (G 42) one of them can be a semicircle, or greater than a semicircle, or less than a semicircle, while the circle of which the great(er) arc is (part) is always smaller (in size) than the circle of which the small(er) arc is (part).

(107) How many are the plane figures which are contained by more than two circular lines? They are infinite in multitude. They begin with the triangle, then they continuously ascend (in number of sides), without limit. Each figure is < named > (L 6b) according to the number of arcs which contains it. So one says: a triangle of circular lines, and a quadrilateral of circular lines, and a pentagon and a hexagon, and in this way they continuously ascend without limit.

(108) How many are the species of plane figures which are contained by straight and circular lines? Their primary species (R 7b) are two: a (figure) contained by two lines, one straight and the other circular, and it is called a segment of a circle, and a (figure) contained by three lines: one of them is circular and the two others are straight, and it is called < a sector >. The (figures) which are not of these two species are infinite in multitude.

(109) < How many are the species of > segments of the circle? three (species): the semicircle, (the segment) less than a semicircle, and (the segment) greater than a semicircle.

(110) How many are the species of sectors? Two species: one is such that the vertex of the sector is at the center of the circle of the arc which subtends its angle, and the other such that the vertex of the sector is at the circumference of the circle of the arc which is its base. (G 43)

(111) What is the property of a sector? As for the sector whose angle is at the center of the circle of its arc, its property is that its arc is less than a semicircle; its arc can neither be a semicircle nor greater than a semicircle. As for the sector whose angle is at the circumference of the circle < of its arc >, it is possible that its arc is a semicircle, or greater than a semicircle, or less than a semicircle. Its property is (H 26) that the sector which is constructed at its arc and whose angle is at the center is (such) that the angle at the center is twice the angle at the circumference of that first sector. (L 7a)

(112) How many are the species of convex surface? Three: convex with spherical convexity, convex with cylindrical convexity, and convex with conical convexity.

(113) What is a convex surface with spherical convexity? It is a figure which is < homoeomeric (literally: similar in its parts) >, and such that in its interior is a point in such a way that all lines issuing from it (the point) to it (the surface) are equal. That point is the center of the spherical surface.

(114) By what is the spherical surface produced? It is produced by an arc of a semicircle, if its chord becomes < fixed > as the axis, and its arc is rotated until it returns in the position from where it began with its motion. Then the surface which that arc describes by its rotation is a spherical surface. (G 44)

(115) What is the property of the spherical surface? Its property is that no straight line can be < assumed > on it, and that every line which is < assumed > on it is the arc of a circle.

(116) What is the convex surface with cylindrical convexity? It is a figure which is < homoeomeric (similar in its parts) >, begins with a circle and ends at another circle equal to it.

(117) By what is the surface with cylindrical convexity produced? It is produced by a plane figure with parallel sides and right angles, if one of its sides is fixed as the axis, and the other side which is parallel to it is rotated around it, until it returns to the position from which it < began with its motion. > Then the surface which is described by that side by its rotation is a cylindrical surface.

(118) What is the property of the cylindrical surface? It is that a straight line can be < assumed > on it, and a circular line, and a curved line (i.e., an ellipse). The circles which are < assumed > on it are all parallel and equal. The straight lines which are < assumed > on it are all parallel.

(119) What is the convex surface with conical convexity? It is a figure which begins from a point (R 8a) and ends at the circumference of a circle.

(120) By what is the conical surface produced? It is produced by a right-angled triangle, if one of the lines containing the right angle is fixed as the axis, and the side subtending the right angle is rotated around it until it returns to the position from which it began with its motion. Then the surface which is described by the chord (G 45) of the right angle < in its rotation > is a conical surface.

(121) What is the property of the conical surface? Its property is that (L 7b) a straight line, a curved line and a circle can be located on it. The circles located on it are all parallel, and there cannot be two (H 27) equal (circles) on it. None of the straight lines which are assumed on it can be parallel, but they all end at one point.

(122) How many are the species of concave surfaces? < Their species > are the same in number as the species of convex surfaces, and their production is the same as the production of the convex surfaces, and their properties (of concave surfaces) are the same as their properties (of

convex surfaces). The reason is that in every surface which is convex according to < some > convexity, the exterior is convex according to this convexity and the interior is concave (in a way) < corresponding to > that convexity. Thus the discussion of the < concave > surfaces is < also > based on the < convex > surfaces. (G 46) The end of the second chapter.

The third chapter. The discussion in it is about solids.

(123) What is a solid? It is a magnitude possessing length, breadth and depth, and its extremities are surfaces.

(124) How many are the primary species of solids? Three: contained by plane surfaces, contained by convex surfaces, and contained by both < convex and plane > surfaces.

(125) How many are the species of solids which are contained by plane surfaces? Their primary species are two: (solids) which can be contained by a sphere, and (solids) which cannot be contained by a sphere.

(126) How many are the species of solids which can be contained (i.e., circumscribed) by a sphere? Five: having four triangular faces (tetrahedron), having six quadrilateral faces (cube), having eight triangular faces (octahedron), having twelve pentagonal faces (dodecahedron), and having twenty triangular faces (icosahedron). (G 47)

(127) What is (a solid) having four triangular faces (i.e., a tetrahedron)? It is a solid which is contained by four triangular equilateral faces, and it is called a pyramidal (literally: fiery) figure.

(128) What is (a solid) having six quadrilateral faces (i.e., a cube)? It is a solid contained by six quadrilateral equilateral and rectangular plane figures, and it is called a cube figure.

(129) What is (a solid) having eight triangular faces (i.e., an octahedron)? It is a solid contained by eight triangular equilateral and equiangular plane figures.

(130) What is (a solid) having twelve pentagonal faces (i.e., a dodecahedron)? It is a solid contained by twelve pentagonal equilateral and equiangular plane figures.

(131) What is (a solid) having twenty triangular faces (i.e., an icosahedron)? It is a solid contained by twenty (R 8b) equilateral (H 28) and equiangular triangles.

(132) To what did the ancients compare each of these < five > (L 8a) solids, which can each be contained by a sphere? They compared them to the < four > elements: earth, water, air, fire; and to celestial substance (*falak*). They compared (the solid) having four triangular faces to fire, and

similarly this figure is called “fiery”(i.e., pyramid), and they compared (G 48) (the solid) possessing six quadrilateral faces, I mean the cube, to earth, and they compared (the solid) having eight triangular faces < to air, and they compared (the solid) having twenty triangular faces > to water, and they compared (the solid) having twelve pentagonal faces to celestial substance. It is not possible that a sphere contains (i.e., circumscribes) equilateral and equiangular solids other than these five.

(133) Can a sphere contain (i.e., circumscribe) another solid, such that its plane figures (i.e., faces) are of two equilateral and equiangular species? Euclid did not mention < this >, but Archimedes mentioned that a sphere can contain two solids which are both contained by triangles and squares, and each of these possesses fourteen faces. One of them is contained by eight equilateral and equiangular triangles and six equilateral and equiangular quadrilaterals, so this solid can be composed of air and earth. The other, inversely, is contained by six equilateral and equiangular triangles and eight equilateral and equiangular quadrilaterals.

(134) How many are the species of solids which are contained by planes and cannot be contained by a sphere? Their species are infinite in multitude, but three of them are mentioned by the geometers: contained by triangles on different bases, and they are called pyramidal figures; contained by quadrilaterals, and they are called cubical and inclined; and contained by triangles and quadrilaterals < together >, and they are called prisms (literally: “sawed-off”). (G 49)

(135) What is a pyramidal figure? It is a figure which begins at a point and ends at a plane possessing sides (i.e., a polygon), which is called its base, so it is contained by that base and by triangular planes.

(136) How many are the pyramidal figures? They are infinite in multitude, < according to > the number of sides of the planes which are their bases. That is to say: they begin (L 8b) with a triangular base, then there is on < each > side of the base a triangle, so four triangles contain it (the pyramid). If these four triangles are equilateral, it is the pyramidal figure which is contained by a sphere. If the four are different triangles, it is the pyramid which is not contained by a sphere. So this is the first (species) of the pyramidal figures. Then after that is the figure whose base is a quadrilateral, so it is contained by four triangles together with the base. Then after that is a figure whose base is a pentagon, so it is contained by five triangles together with the base. Then a figure whose base is a hexagon, so it is contained by six triangles and the base. And in

this way always increasing, according to the increase of the sides of the base.

(137) How many are the species of solids which are contained by quadrilateral planes, and how many < quadrilaterals > contain < them >? The quadrilaterals which contain them are six and their species are < four >: one such that its three dimensions (or: distances) are equal, I mean its length, breadth and depth, and it is called (H 29) cube, and it belongs to the < five > (R 9a) figures which are contained by a sphere; and one such that two of its dimensions are equal, and the third is less than the two, and it is called "brick"; one such that two of its dimensions are equal and the third is greater than the two mentioned ones, and it is called < "beam" >; and one whose dimensions are different, and it is called "tablet." (G 50)

(138) What is a prism (literally: sawed-off solid)? It is a figure contained by two opposite triangles with parallel sides and three quadrilaterals with parallel sides and right angles. Each pair of parallel sides of the two triangles is joined by one of the three quadrilaterals.

(139) By what is the prism produced? It is produced by one of the solids (i.e., in 137) which are contained by quadrilaterals, if it is bisected along the diagonal of one of the quadrilaterals which contain it.

(140) How many are the species of prisms? Their species are the same number as the species of the (solids contained by) quadrilaterals of which they are the halves.

(141) How many are the species of solids which are contained by one surface? (L 9a) Three: the sphere, the egg and the ring.

(142) What is a sphere? It is a solid figure contained by one surface, in whose interior is a point such that all the lines which issue from that point to the surface of it (the sphere) are equal.

(143) What is the center of the sphere? It is the point in the interior of the sphere, such that all straight lines issuing from it to the surface of the sphere are equal.

(144) What is a diameter of the sphere? It is any line which passes through the center of the sphere and ends at its surface.

(145) What is the axis of the (celestial) sphere? It is the diameter of the sphere around which the sphere moves, while (the sphere) is fixed on it.

(146) What are the two poles of the (celestial) sphere? They are the two endpoints of the axis. (G 51)

(147) What is the property of the sphere? That it is the greatest (in volume) of the solids whose surfaces are equal to its surface, and that (for) each plane which intersects it, < its intersection >is a circle, and that every

great circle which is described on it bisects it, and that all circles which pass through its diameters bisect it, and that there cannot be located on it more than two equal and parallel circles.

(148) What is the pole of a parallel circle (i.e., a circle which is not a great circle) on the sphere? It is a point on the surface of the sphere such that all the straight lines issuing from it to the circumference of each circle of which this point is the pole, are equal.

(149) By what is the sphere produced? It is produced by a semicircle, if the chord of the arc (H 30) of a semicircle becomes the axis, and the semicircle is rotated around it until it returns to the position from which it began with the motion. Then the body which is produced by the motion of the plane of the semicircle is a sphere.

(150) What is an egg? It is a solid figure which is contained by one surface, in whose interior is a point such that the greatest straight line which passes through that point and arrives at the surface of it is such that it is not possible that through its interior passes another line equal to it.

(151) What is the axis of the egg? It is the longest straight line which is drawn from its interior and ends at its surface.

(152) What is the center of the egg? It is the line which bisects the axis of it.

(153) What are the two poles of the egg? They are the two endpoints of its axis. (G 52)

(154) What are the diameters of the egg? (L 9b) They are the straight lines which pass through the center of the egg and end at its surface.

(155) What is a shortest diameter of the egg? It is a straight line which issues from the center of the egg, and contains together with its axis a right angle, and ends at the surface of the egg.

(156) By what is the egg produced? (R 9b) It is produced by a segment less than a semicircle, if its chord is made the axis, and < the arc > is rotated around it until it returns to the position from which it began with the motion. Then the solid which is produced by that rotation is an egg.

(157) What is a ring? It is a solid which is contained by one round surface, and in its interior is < a space > in which a sphere can fit.

(158) By what is the ring produced? It is produced by two tangent circles, one of which is perpendicular to the other. If the perpendicular (circle) is rotated on the circumference of the other, until it ends on the circumference at the point at which it began with the motion, then the solid which is produced by the rotation of the perpendicular circle is a ring.

(159) How many are the < species of >solids which are contained by straight surfaces and convex surfaces? Five: cylinders, cones, segments < of spheres, segments of cylinders and segments of cones >.

(160) What is a cylinder? It is a solid which begins at a circle and ends at another circle equal to it, and which is contained by a cylindrical surface and the planes of two parallel circles (G 53).

(161) By what is a cylinder produced? It is produced by a quadrilateral with parallel sides and right angles, if one of its sides becomes the axis, and the quadrilateral is rotated around it until it returns to the position from where it began with the motion. Then the solid which it describes by its rotation is a cylinder.

(162) What is the axis of the cylinder? It is the side of the quadrilateral by which the cylinder is produced, which (side) is fixed in the rotation of the quadrilateral. It is also the straight line drawn from the center of one of the two circles containing the cylinder to the center of the other circle parallel to it.

(163) What is a side of the cylinder? (H 31) It is a straight line drawn from a point on the circumference of one of the two circles in the cylindrical surface to the circumference of the other circle, and it contains with both diameters of the two circles a right angle.

(164) What is the property of the cylinder? That the circles described on (L 10a) its surface are equal and parallel, and all straight lines < drawn on > its surface are parallel and equal.

(165) What is the conical solid? It is a figure which begins at a point and ends at the circumference of a circle, and which is contained by a conical surface and a circle.

(166) By what is the cone produced, according to the opinion of Euclid? It is produced by a right-angled triangle, if one of the two sides containing the right angle is fixed as the axis, and the plane figure of the triangle is rotated until it returns to the position at which it began with the motion. Then the plane figure of the triangle describes a conical solid in its rotation. (G 54)

(167) By what is the cone produced, according to the opinion of Apollonius? It is produced by a circle in a plane and a point < above > that plane. The point is joined to the circumference of the circle by a straight line. That line is rotated on the circumference of the circle, while the point remains fixed, until the line returns < to the point > on the circumference of the circle at which it began with the rotation.

(168) What is the axis of the cone? According to the opinion of Euclid, (R 10a) it is the side of the triangle which is fixed in the rotation of the triangle (used) in the description of the cone. According to the opinion of Apollonius, it is the line drawn from the point of the apex of the cone to the center of the circle of its base.

(169) What is a side of the cone? It is a straight line which is drawn from the point of the apex of the cone and ends at the circumference of its base.

(170) What is the property of the cone? Its property is that no two equal circles can be located on < its > surface, nor two parallel < straight > lines. But it is possible that on < its > surface are located straight lines, circular lines, and curved lines.

(171) How many are the species of cones according to the opinion of Euclid? Three: right-angled, acute-angled and obtuse-angled.

(172) What is a right-angled cone? According to the opinion of Euclid, it is (the cone) whose angle at the point of the apex is a right angle, I mean that the triangle < by > which (the) right-angled (cone) is produced, is isosceles. (G 55)

(173) What is an acute-angled cone? According to the opinion of Euclid, it is (the cone) whose angle at the point of the apex is acute, I mean that one of the two sides containing the right angle of < its > triangle, (namely the side) which was fixed as the axis, is greater than the other (side containing the right angle).

(174) What is an obtuse-angled cone? According to the opinion of Euclid, it is (the cone) whose angle at the point (H 32) of the apex is obtuse, I mean (L 10b) that the side containing the right angle of < its > triangle, (namely the side) which was fixed as the axis, is less than the other (side containing the right angle).

(175) How many are the species of cones in the opinion of Apollonius? Two, right and oblique.

(176) What is a right cone? It is (the cone) such that the line by which it is produced contains with the diameter of the circle of its base a right angle.

(177) What is an oblique cone? It is (the cone) such that the line by which it is produced contains with the diameter of < the circle of > its base an obtuse angle.

(178) How many are the species of segments of a sphere? Three: the hemisphere, (the segment) greater than a hemisphere, and (the segment) less than a hemisphere.

(179) What is a hemisphere? It is a solid which < is > contained < by > the plane figure of a great circle located on a sphere and half of the surface of the sphere, and the center of the < sphere > is in that plane.

(180) What is a segment of a sphere which is less than a hemisphere? It is a solid contained by less than half of the surface of a sphere and a circle less than a great circle located on the sphere. Inside that segment are neither the axis (G 56) of the sphere, nor < its > center, nor any of its diameters.

(181) What is a segment of a sphere which is greater than a hemisphere? It is a solid contained by a circle which is not a great circle on the sphere, and a surface greater than half of the surface of the sphere. It is possible that in the interior that segment are located the center of the sphere, its axis, and many of its diameters.

(182) What is a segment of a cylinder? It is a solid which is contained by a plane figure and a section of a cylindrical surface.

(183) How many are the species of segments (R 10b) of a cylinder? Their primary species are two; (segments) such that their (plane of) section is < through > the axis or parallel to the plane of the axis; and (segments) such that their (plane) of section intersects the axis either inside the cylinder < or > outside it.

(184) What is a segment of a cylinder such that its (plane of) section is through its axis? It is half a cylinder, contained by half a cylindrical surface, a quadrilateral plane figure, and the halves of two parallel < circles >.

(185) What is a segment of a cylinder such that its (plane of) section is parallel to the plane of its axis? It is a segment less than half a cylinder, < or greater than half a cylinder >, contained by a quadrilateral plane figure, a section of the cylindrical surface either less or greater than half a cylinder, and two segments of equal circles, (both) either less or greater than two semicircles.

(186) What is a segment of a cylinder whose (plane of) section intersects its axis inside the cylinder? It is a solid which is contained (H 35) by some part of a (G 57) (L 11a) cylindrical surface, and the plane figure of a circle, and a plane figure contained by a curved line. (The text breaks off here.)

6. Notes to the translation

I have deleted religious opening sentences, such as the following. The Rabat manuscript says: “In the name of God, the Merciful, the Compassionate. May God bless our lord and protector Muḥammad, his family and his companions, and grant them salvation”.

Q 0. The reference to “angles” in the title of the third chapter (G 18:6) is probably to Q 171, where Qusṭā discuses right-angled, obtuse-angled and acute-angled cones.

Q 1. In (c) Qusṭā probably means the theory of ratio and proportion in Book V of the *Elements*. He does not discuss this subject in the rest of his text. Instead of (c), manuscript L has: “the size of each species of it with respect to (the quantities) having its (i.e., the same) species.” Clearly the editor of L did not understand the meaning of the passage.

Q 9. See Section 3 of this paper, source no. 3 (Proclus).

Q 10. My translation “circular” for *muqawwas* (literally: arched) is inspired by the corresponding Greek term *kuklikos*, cf. Definition 5 of Heron (ed. Schmidt, pp 18-19). These lines are parts of the circumference of a circle. On “curved” lines see Q 57.

Q 11. Definition (11.1) is related to the commentaries by Proclus and by Simplicius to Euclid’s *Elements*. Proclus says: “Euclid gives the definition of the straight line ..., making clear by it that the straight line alone covers a distance equal to that between the points that lie on it” (Proclus ed. Friedlein p. 109, tr. Morrow p. 88.) Simplicius’ commentary is lost in Greek but has come down to us in Arabic in quotations in the commentary of al-Nayrīzī. The relevant passages exist in a medieval Latin translation and also in one imperfect Arabic manuscript which has recently been published by Arnzen. Simplicius says (Arnzen p. 1-2, translation mine, cf. also Curtze p. 5) “Euclid means ... the distance which is between the two points of its two extremities (i.e., of the straight line), for if we assume the two points which are the two extremities of the line ... and take the distance ... between them, that distance is equal to the straight line of which these two points are the two extremities.” Note that the Arabic *buʿd* means distance as well as dimension. Definition (11.2) is not exactly equivalent to, but related to the Greek text in *Elements* I def. 4: “a straight line is a line which lies evenly with the points on itself” (tr. Heath, vol. 1, p.165). Definion (11.3) appears in Archimedes, *On the Sphere and Cylinder* I, but also in the commentary of Proclus (ed. Friedlein p. 110, tr. Morrow p. 89). The definition (11.4) which Qusṭā attributes to Plato is not found in this wording in the extant Greek literature. Proclus (ed. Friedlein p. 109, tr. Morrow p. 89) and Simplicius (in the version of al-Nayrīzī, see Arnzen p. 3 line 7) attribute to Plato a definition of a straight line as “that of which the middle intercepts the view of the ends.” Thus Qusṭā cannot have quoted the definition from either Proclus or Simplicius. Definitions (11.5) and (11.6) are found in Heron’s *Definitions*, def. 4, and also in Simplicius’ commentary as preserved by al-Nayrīzī (Arnzen p.4:11-12, p. 5-6).

My reconstruction of the six definitions is based primarily on H. Definition (11.2) is omitted in R. Definitions (11.1) and (11.4)-(11.6) are omitted in L. H omits the references to Euclid, Archimedes and Plato.

Q 13. On curved lines see Q 57.

Q 20. L omits the statement that the three lines are not in the same plane.

Q 21. R has “not straight” instead of “circular”. The reading of R is inconsistent with the classification of Q 26-27.

Q 26. See Section 3 sub Simplicius. H changed Qusṭā’s terminology: for concavity H used *akhmaṣ* (“hollow side”) instead of *inḥinā’*, and for convexity *muḥdawdib* instead of *iḥdīdāb*.

Q 27. See Section 3 sub Simplicius.

Q 28-32 These problematic questions and answers are omitted in L.

Q 29. The idea seems to be that if a straight segment *AB* meets two straight lines *AC* and *BD*, and if *C* and *D* are on the same side of *AB*, angle *ABD* is called an angle opposite to angle *BAC*. The concept of “opposite angle” is used in Euclid’s *Elements*, I:16, see the same terminology in al-Ṭūsī p. 19. The word *mutaqābila* for opposite is used in a different sense in Q 52, also in this different sense in al-Ṭūsī p. 18.

Q 30. If a straight segment *AB* meets two straight lines *AC* and *BD*, and if *C* and *D* are on different sides of *AB*, angles *ABD* and *BAC* are called alternate angles. The concept is used in Euclid’s *Elements* I: 27, 29, see also al-Ṭūsī p. 28.

Q 32. The concept boils down to the same as in Q 29. “Adjoining” angles were not used elsewhere by Qusṭā (if one agrees with my change to Guergour’s text in Q 70). Q 32 is omitted in H.

Q 38. L changes “names” to “subdivisions,” but the change is incorrect because the following questions and answers concern explanation of terminology rather than classification of objects.

Q 42. The idea is that two angles *ABC* and *ADC* are joined by the diagonal *BD*, thus producing a quadrilateral *ABCD*.

Q 48. What Qusṭā says is false, because two circles can be non-concentric and at the same time not intersecting.

Q 51. The addition of the word < similar > is discussed in Section 4 as an example of my editorial policies. In the *Introduction to Astrology* (p. 22), al-Bīrūnī defines similar arcs in essentially the same way: "Similar arcs on a plane surface are those which are intercepted between two straight lines proceeding from the centre of a series of concentric circles to the largest of these." Of course it does not follow that al-Bīrūnī took this definition from Qusṭā. In *Elements* III, def. 11, Euclid defines similar segments in a vague way.

Q 52. Note that the word "opposite" is used here in a way different from the definition in Q 32. The properties which Qusṭā mentions are proved in *Elements* I:15.

Q 54. In this answer, the definition of alternate angles in Q 30 is more or less repeated. The repetition may of course be a later addition, perhaps originally a marginal remark. The 'properties' are proved in *Elements* I:29. In L, the answer to this question is rephrased and illustrated by a figure, and the last property is omitted.

Q 55. In *Elements* III: 27, Euclid proves the properties for equal circular lines.

Q 56. The passage in angular brackets is inspired by H. L adds a figure illustrating external and internal tangency of arcs, see Guergour's appendix p. 66. H deletes the passage "for every pair of tangent lines, their tangency is at one point." Instead of "tangent line(s)" R has "proportional line(s)". These corruptions, which have been noted by Guergour in fn. 107, 108, 110, show that the scribe of R did not understand anything of what he was writing.

Q 57. The word "mechanical" *al-miḥānijī* in R was wrongly read as *nīmkhānijī*, half-dome shaped. by al-Hindī (H 18:2), by the present writer in a previous article (Hogendijk 1987-88), and in G 28:13. All these writers may have been misled by the *Letter on Geometry* of the *Ikhwān al-ṣafā'* which has been quoted at the end of Section 3 above. The scribe of R cannot have understood the meaning of the passage, because he says that the mechanical lines "make instruments" (instead of "are made by instruments"). Q 57 is omitted in L.

In L, the second chapter is called "the second treatise of the book of Qusṭā ibn Lūqā on the properties of Euclid and his propositions", *fī khawāṣṣ Uqlīdis wa-askhāl.*

Q 60. Definition (60.1) is not the same as Euclid's definition of plane surface in the Greek version of the *Elements*: "a plane surface is a surface which lies evenly with the straight lines on itself" (tr. Heath vol. 1, p. 171). Qusṭā's definition is related to the following statement in Simplicius' commentary to the *Elements*: "the distance of the (plane) surface is equal to the distance between the straight lines which contain it, and this is the well-defined distance which is the least of distances", see Arntzen p. 9 lines 1-3. Definition (60.2) and (60.3) do not occur in precisely the same form in Greek texts, see Euclid, *Elements*, tr. Heath, vol. 1 p. 171-172, although definition (60.2) is related to Simplicius's alternative definitions of plane surface, see Arnzen p. 9 line 8-11, Curtze p. 10. My interpretation of definition (60.3) is inspired by H. The idea is that a plane surface is the surface of rotation of a straight line around a perpendicular axis. L has a different version of definition (60.1) (see Guergour p. 29 fn 119): "The plane surface is (the surface which is) stretched rectilinearly together with the lines of its extremities, and it is such that all the lines which are assumed on it are all parallel to one of the lines of its extremities in one direction." The author of this definition probably thought of a rectangle or parallelogram. Note that in Greek mathematics, a plane surface is finite and therefore it always has a boundary. Definitions (60.2) and (60.3) are omitted in L.

Q 65. Here and elsewhere (Q 68, 69), Qusṭā used an inappropriate technical term for an acute-angled triangle, namely *ḥādd al-zāwiya*, i.e., with the acute angle. Right-angled and obtuse-angled triangles have two acute angles, as Qusṭā correctly remarks in Q 70. Other Arabic translators used the plural *ḥādd al-zāwāya*, meaning with (more than two) acute angles. Al-Hindī also used this more correct term. We can explain the error by assuming that Qusṭā mechanically translated the Greek words right-angled (*orthogōnia*), obtuse-angled (*amblygōnia*) and acute-angled (*oxygōnia*) in analogous ways as *qā'im al-zāwiya*, *munfarij al-zāwiya* and *ḥādd al-zāwiya* (with a right angle, with an obtuse angle and with an acute-angle).

Q 68 is omitted in L, probably by an oversight of the scribe.

Q 70. The properties are proved in Euclid's *Elements*, I:20, 18, 32.

Q 71. The first property is the Theorem of Pythagoras which is proved in Euclid's *Elements* I:47. The three other properties are consequences of *Elements* VI:8.

Q 72 The last property, which is mathematically equivalent to the cosine law for an obtuse-angled triangle, is proved in *Elements* II:12. L presents a figure with a numerical example (a combination of two Pythagorean triangles with sides 8, 6, 10 and 8, 15, 17, producing an obtuse-angled triangle with sides 15-6=9, 10, 17). The manuscript calls the example a “proof” (*burhān*). See for the text Guergour pp. 67-68 (the word “proof” occurs in G 67:13). Thus the author of the version in L and the scribe of L were not very familiar with the deductive structure of mathematics in the style of Euclid, although the example of course helps to illustrate the “property.”

Q 73. The property is proved in *Elements* II:13. Again L presents a figure with a numerical example: the combination of two Pythagorean triangles with sides (12, 5, 13) and (12, 9, 15) produces an acute-angled triangle with sides (9+5=14, 13, 15). See Guergour p. 68-69 for the text. Here the word “proof” is not used, but the ms. says “let us explain this by means of numbers” (G 68:13).

Q 74. In Q 74-81, Qusṭā presents a classification of quadrilaterals into seven categories. Following H I have added < primary >, because the classification into seven categories is further refined into 22 subcategories in Q 75-81. The text is the manuscript is partly corrupt, and none of the scribes seems to have understood the classification in detail. In Q 74, I have added the fifth category < having two opposite sides equal and the others different > ; my added species, which is to some extent inspired by the text in H, is further subdivided in Q 79, so it must have been in Qusṭā’s original. Al-Hindī apparently did not understand the classification and rendered the fifth category as: “having all pairs of opposite sides equal and the two others different”. This is a witness of his (lack of) understanding of the text.

Q 78: According to L, the number of species is “two: in one of them, two opposite angles are together equal to two right angles, and the two other angles are equal, and in the other there is no right angle at all, and all pairs of opposite angles are equal.” Thus the anonymous author of the version of the text in L did not understand the classification.

Q 79. The first of the three subcategories of this question consists of isosceles trapezia. In the second subcategory I have changed “(two) parallel sides” in Guergour’s edition into “no (two) parallel sides”, following L, and in order to make mathematical sense, because a quadrilateral with two parallel sides and one right angle must have two right angles. Guergour added, before the question mark in my translation, the words “with at most one right angle” to the text. I have deleted his addition because it is unnecessary. If a quadrilateral has at least three right angles, it must be a rectangle or a square, so it cannot have unequal opposite sides. If a quadrilateral has two adjacent right angles and two equal opposite sides, it must also be a rectangle or a square. If a quadrilateral has two opposite right angles and two equal opposite sides, the quadrilateral can be decomposed into two congruent triangles, so it belongs to the category of Q 77 or Q 78.

Q 80. Guergour added, before the question mark in my translation, the words "with at most two right angles" to the text. This addition is unnecessary because a quadrilateral with three right angles must be a rectangle or square and hence belong to one of the previous species. The first subcategory of Q 80 consists of quadrilaterals $ABCD$ with right angles A and B, $AB = BC$ and BC parallel to AD, with $AD \neq CD$, $CD \neq BC$, $AD \neq AB$. The second subcategory consists of quadrilaterals $ABCD$ with right angles A and C (hence the quadrilateral is concyclic and the diagonal BD is the diameter of the circumscribing circle), and $AB = AD$. (If AB were equal to BC, the quadrilateral could be decomposed into two congruent triangles BAD and BCD, hence it would belong to the category of Q 78.) Point C can be any point on the circumscribing circle of ABD, as long as C and A are on different sides of the diameter DB and AC is not itself a diameter of this circle. The classification in Q 80 is incomplete: there is a fifth subcategory consisting of quadrilaterals with two equal adjacent sides, without a right angle, and with two parallel sides. If one were to correct the classification, the number 'four' in the beginning of the question should also be changed to 'five'. Since the number four occurs in the text of Q 80 in R, L and H, I assume that the error was in Qusṭā's original text.

Q 86. The passage "and it is circumscribed by a circle, and it is inscribed in a circle" is mathematically useless. The passage is found in H, and is probably due to Qusṭā. What Qusṭā says is wrong: there are irregular polygons of arbitrary number of sides, having an inscribed and a circumscribed circle. See, e.g., Dörrie, problem no. 39.

Q 89. The passage in double angular brackets is one full line of text in R which was omitted in Guergour's edition by an oversight.

Q 91. Qusṭā's assumption that the polygons are inscribed in a circle is of course unnecessary. H omits the example at the end.

Q 92. In Guergour's edition, Q 92 ends at "we have described," and he considers the rest of my Q 92, beginning with "how many right angles fall in it" as a new question. Qusṭā used the word "ratio" in an uncommon (perhaps archaic) way in the expression "the ratio of the angle ...with respect to a right angle" (*nisba zāwiya ...min zāwiya qā'ima*). The terminology was changed in L (see G 38 fn 187) and also in H.

Q 93. An equilateral concyclic quadrilateral is a square, and the first property means that the square of the side is twice the square of the semi-diagonal. The second property is the so-called "theorem of Ptolemy" for a concyclic quadrilateral, which property is proved in Book I of the *Almagest* (for the Greek text and an English translation see, e.g., Bulmer-Thomas, vol. 2, pp. 422-425).

Q 94-99: Although the text has "properties," only one property is mentioned in R and L. H uses the singular "property."

Q 94. Qusṭā tacitly assumes the pentagon to be regular. The property is proved in Euclid, *Elements* XIII:10, see tr. Heath vol. 3, p. 457-460.

Q 96. The word (the) < remaining > (angle) has been restored from H; R has (the) right (angle). The passage shows that already in the time of Qusṭā, mathematicians were trying to construct the regular heptagon using the same triangle which was used in the late tenth century A.D. by Abū l-Jūd Muḥammad ibn al-Layth and Aḥmad ibn Muḥammad al-Sijzī in their constructions of the heptagon. See Hogendijk 1984.

Q 99. For the first property see Q 94. Both L and H omit the passage which begins with "< Whenever." The text in R is incomplete, but the meaning is clear: in a regular polygon with n angles, it is possible to inscribe an isosceles triangle whose angle at the base is $(n - 1)/2$ times the angle at the top.

Q 102. L calls the "egg" a "conic section", meaning an ellipse.

Q 106. A figure contained by two circular arcs with "opposite convexities" is called an egg-shaped figure, if at least one of the arcs is less than a semicircle; the other arc may be of any size as long as it is taken from a circle whose radius is less than the radius of the circle of which the first circular arc is part. The last condition ensures that the resulting egg-shaped figure is convex. According to the definition in Q 156, the egg itself is also a convex solid of revolution.

Q 112. From a modern point of view, this definition is too limited.

Q 113. Compare Q 142. To make mathematical sense, I have replaced the passage "which is contained by a circular line" in R by the term "similar in its parts", i.e. homeoemeric, found in L. The word homeomeric is a technical term which was used in Greek geometry, compare Proclus, ed. Friedlein 112-113, 120, tr. Morrow 91-92, 97. See also Q 116.

Q 115. Compare Q 147.

Q 116. Compare Q 160 and the note to Q 113 above. Here the Arabic edition has the word "of equal parts" (*mutasāwī al-ajzā'*) in R, which is a corruption of homoeomeric (*mutashābih al-ajzā'*) in L and H. According to Proclus, the sphere is a homoeomeric surface but the cylinder is not a homoeomeric surface. However, he calls the cylindrical helix one of the three homoeomeric lines (the others being a straight line and a circle), see Q 113 for the reference. There may be a misunderstanding involved here on the part of either Proclus or Qusṭā.

Q 117. Compare Q 161.

Q 118. Compare Q 164. The "curved line" is in this case an ellipse.

Q 119. Compare Q 165.

Q 120 Compare Q 166. The conical surface in Q 120 and Q 121 belongs to a right cone.

Q 121. Compare Q 170.

The chapter ends with religious sentences which I have omitted.

Q 126. "Having four triangular faces" etc., are technical terms for regular solids; Qusṭā does not mean any solid having four triangular faces. Note that the scribes of R and L consistently wrote *dhā* (having) as *idhā* (if), and the scribe of R wrote *mukhammasāt* (pentagonal) as *mujassamāt* (solids). Thus a dodecahedron is called a solid "if twelve solid faces" rather than "having twelve pentagonal faces". The errors show that the scribe of R did not understand the subject.

Al-Hindī combines the material in Q 126 and Q 132 to one question and answer. He deleted the reference to the four elements and to the ancients.

Q 132. Guergour's reading "five elements" is an emendation of "four elements" in the manuscripts R and L. I prefer the reading of the manuscripts because celestial substance is not an element according to ancient Greek philosophy. On the ancient (Platonic) comparison between regular solids and elements see Heath, *A History of Greek Mathematics* I, 296.

Q 133. For "triangles and squares", L has the interesting variant "triangles and pentagons." Thābit ibn Qurra wrote a text on the construction of a solid contained by six squares and eight equilateral triangles, see Bessel-Hagen and Spies. A solid with eight squares and six equilateral triangles is mentioned in the *Definitions* of Heron, see Section 3 of this paper.

We now show that such a solid cannot be inscribed in a sphere. For suppose that such a solid exists, and denote the number of faces, edges and angular points as F, E and P respectively. Then F - E + P = 2. For a solid with eight quadrangular and six triangular faces we have F = 14, E = ½ (8 × 4 + 6 × 3) = 25, hence P = 13. Now suppose that the solid can be circumscribed by a sphere, which passes through all these 13 angular points.

In all these 13 angular points, three, four or five faces must come together, since 6 angles of 60 degrees add up to the full 360 degrees. If in x angular points 3 faces come together, in y angular points 4 faces come together and in z angular points 5 faces come together, we have $x + y + z = 13$, $3x + 4y + 5z = 50$. Hence $2x + y = 15$ and we find the solutions $(x,y,z) = (3,9,1), (4,7,2), (5,5,3), (6,3,4)$ and $(7,1,5)$.

From the center of the sphere we can project the 25 (equal) edges on the sphere, and we then find a covering of the sphere with 8 projections of squares and 6 projections of triangles. Call S and T the sizes of these projections on the sphere. Then $8S + 6T$ is equal to the surface of the sphere, and it is clear that S is greater than T.

Now choose an angular point A at which three faces come together. These faces can be either (a) three triangles, (b) two triangles and a square, (c) one triangle and two squares, (d) three squares. By the choice of these faces (equilateral triangles or squares), at least three more points of the solid are determined, and hence the radius of the circumscribed sphere, and S and T. Thus, in case (d), S is equal to the projection of the face of the inscribed cube, so $6S$ is equal to the surface of the sphere, which is supposed to be equal to $8S + 6T$. Contradiction. In the other three cases, the solid angle at A is less than the solid angle for case (d) (which is one-eighth of the sphere). Thus the value of S in the cases (a), (b) and (c) is more than the value for case (d), and we have a contradiction in the cases (a), (b), (c) as well. We conclude that the required solid cannot exist.

Q 134. Qusṭā does not mention the "inclined" figures (probably non-rectangular parallelepipeds) in his further classification. He incorrectly states that none of these solids can be contained by a sphere. A pyramid with triangular base and also a rectangular parallelepiped can always be circumscribed by a sphere. Instead of "prisms" (literally: sawed-off (figures)), L has the odd scribal error "circular" (figures). The error shows again that the scribe of L did not understand the subject.

Q 136. Qusṭā erroneously states that a pyramid with different triangular faces cannot be contained by a sphere.

Q 137. Qusṭā's terms for rectangular parallelepipeds with (a) two equal sides and a third side which is shorter, (b) two equal sides and a third side which is longer, seem to be translations of the corresponding Greek terms plinthis and dokos used in Heron's *Definitions*, nos. 112-113. In the *Letters of the Ikhwān al-Ṣafā'*, (b) is rendered incorrectly as *bi'r* (spring, well).

L presents figures with numerical examples of the sides: cube: 5, 5, 5; brick: 10, 10, 5; beam: 8, 8, 12; tablet: 5, 8, 12.

Q 138. In Euclid, *Elements* XI, def. 13, the basis of the prism is a polygon, not necessarily a triangle, and the quadrilaterals are parallelograms which need not be right-angled.

Q 140. Of course, the prisms are not halves of quadrilaterals, but of the solids described in Q 137. Since the erroneous text is found in all manuscripts, and is not easily explained as a mechanical scribal error, I have not corrected it.

Q 148. A "parallel circle" is a circle parallel to the equator, i.e. perpendicular to the axis of the sphere.

Q 150. The definition boils down to the requirement that there exists a unique maximal straight line segment whose two endpoints lie on the surface of the egg. The center will turn out to be the midpoint of this maximal segment. The passage "is such that ... equal to it." is superfluous. Yet the text is probably authentic because it is the same in all three manuscripts. In Q 156, the egg turns out to be a solid of revolution of a segment less than a semicircle.

Q 154. Instead of "what are the diameters," Al-Hindī says: "what are the longest diameters". This is erroneous because the axis has been defined in Q 152 as the (unique) longest diameter. The error shows al-Hindī's lack of understanding of the subject.

Q 159. The odd expression "straight surfaces" occurs in all manuscripts. Al-Hindī changes "five" to "four" and omits the "segments of spheres" in Q 159, although he discusses them in Q 178-181.

Q 163. The last passage, "and it contains... a right angle," is found in L only and may not be authentic.

Q 164. Compare Q 118. The lines "drawn on" the cylindrical surface are called "lines issuing from" the surface in R and "lines drawn towards" the surface in H. It is clear that the scribe of R and al-Hindī understood nothing of what they were writing. Note that the straight lines are "equal" because Qusṭā thinks of line segments on a finite cylindrical surface.

Q 166. Al-Hindī combines Q 166 and Q 167 into one question, and he deletes the names of Euclid and Apollonius. He begins the equivalent of 167 by saying: "It is said that it (the cone) is produced..."

Q 168. H. changes the opinion of Euclid into "the first opinion" and the opinion of Apollonius into "the second opinion". L deletes "according to the opinion of Euclid" as well as "according to the opinion of Apollonius" and presents the two definitions as if they were one.
On an oblique cone, as defined by Apollonius, one can in fact construct equal non-parallel circles, as is explicitly proved in Apollonius, *Conics*, Book I, prop. 5. The proposition in question was available in an excellent Arabic translation made in the early ninth century A.D. under supervision of the Banū Mūsā. The incorrect statement that no two equal circles can be located on the surface of a cone must be due to Qusṭā, and the error shows that he did not really understand the *Conics* of Apollonius.

Q 171 Al-H. omits the name and has "according to the first opinion". The definition is also found in Euclid, *Elements* XI, def. 18.

In Q 172-174 al-H. omits "according to the opinion of Euclid" altogether.

Q 176. Qusṭā's definition is wrong. The "line producing the cone" is a straight line through the apex of the cone on the conical surface. In any right cone, this line intersects the base of the cone in a unique diameter of the circle at the base, and the line and diameter form an acute angle, which is constant for all straight lines through the apex on the conical surface. In a right cone, the axis makes a right angle with all diameters of the circle of the base. Thus Qusṭā confused the "line producing the cone" with the axis.

Q 177. Qusṭā's definition is also wrong. In an oblique cone, the angle between the axis and the base is not a right angle but an acute angle. The angle between the "line producing the cone" and the corresponding diameter in the base plane is not constant and may be acute or right in some situations.

Q 182. Of course, the solid must always be finite and therefore it must be contained by one or two of the base planes of the cylinder.

Q 183. The two primary species are probably (1) planes of section through the axis or parallel to it, and (2) planes of section non-parallel to the axis. In Q 183 and Q 185, the three manuscripts say that "the section is parallel to the plane of the axis". This must be the text which Qusṭā wrote and which I have therefore translated accordingly. It would be correct to say that the section is parallel to "the axis."

Q 186. The text breaks off here. According to Q 183, one would expect one more question and answer about a segment of a cylinder produced by a plane which intersects the axis outside the cylinder (which is always finite). Such a segment should be contained by "some part" of a cylindrical surface, "some part" of the plane of a circle and "some part" of the plane contained by a curved line (i.e., an ellipse). Then, according to Q 159, there should be (probably five or six) more questions and answers about segments of cones.

R adds: "The whole introduction to the Book of Euclid the Wise has been completed." L adds: "The third Chapter has ended, and with this, the book has ended." The manuscripts R and L end with religious sentences.

7. Appendix. Notes to the Arabic text

In this appendix I have noted all instances where I have departed from Guergour's edition. The notes are arranged according to my numbering of Qusṭā's questions and answers. The following conventions will be used. Q 7 refers to Qusṭā's question and answer which I have numbered 7. G 19:5 means page 19 line 5 of Guergour's edition, and fn 36 means footnote 36 in his critical apparatus. I sometimes quote angular brackets < > from his edition.

By means of this appendix, to reader is able to determine exactly which Arabic text I have translated. The appendix is not a substitute for a critical edition, because most variant readings in the manuscripts will not be indicated.

The manuscripts will be abbreviated as R (Rabat manuscript) and L (Istanbul manuscript). The letter H refers to the edition of the text in al-Hindī's *Jumal al-Falsafa*, and H 16:14 refers to line 14 of page 16 of the Frankfurt facsimile edition. A notation such as *qaws* (RH) means that the reading *qaws* is found in R and H.

In L and H the text begins with Q 1, The introduction is only found in R; everything before Q 1 is omitted in L and H.

Q 5: G 19:2 change *nihāyātuhu* (L) to the singular *nihāyatuhu* (R) (cf. fn 36), H omits the word. Compare also the singular *peras* in Euclid's *Elements* XI def. 2.

Q 6: G 19:7 instead of *nihāyāt* (L) I prefer the singular *nihāya*, "extremity" (R) (cf. fn 41); earlier in Q 6, the singular is also used. Compare Q 7.

Q 7: G 19:8 change *al-khaṭṭ huwa* (H) to *al-khaṭṭ al-khaṭṭ huwa* (RL). Instead of *fa-bi-anna* in G 19:9 I prefer *fa-innahu* (LH) (cf. fn 42); R has *fa-bi-annahu*.

Q 8: In G 20:3 change *baqiya nihayatāhu* (R) to *baqiyat nihāyatuhu* (LH), cf. fn. 45. In G 20:3 change *allatāni humā* (L) to *allatī hiya* (R), compare fn 46.

Q 11: Before G 20:9 *ʿalā istiqāma* (L, missing in R) add: *alladhī buʿduhu musāw li l-buʿd alladhī bayna al-nuqṭatayn allatayn humā nihāyatuhu*, as in H 16:14 and R (cf. fn 48, where *bayna* is read as *min*), then add *huwa al-mamdūd ʿalā* from H 16:15, the words are missing in R.

Q 19: fn 69 is misplaced and belongs to G 22:2.

Q 21: Guergour's readings (compare fn 67, 68) are confirmed by H.

Q 24: G 23:1 Guergour's addition < *qā'ima* > is confirmed by L and H. G 23:1 change *minhā zāwiya* (R) to *minhā shay' mā* (LH).

Q 25: In G 23:3 delete *ʿanhu wa*, which is not in RLH. In G 23:4 change *yazīd* to *yuzād* (RLH).

Q 26: G 23:7 change *maḥdūd* to *muḥdawdiban* (RH) and *bi-iḥdīdāb* to *iḥdīdāb* (R), or perhaps *iḥdīdābā*. In G 23:8 change *al-mutaqābilayn* (R) to: *mutaqābilān* (H has *mutaqābilayn*)

Q 29: G 23:16 I have emended *bihā* (RH) to *bihimā* for mathematical sense, compare Q 30.

Q 33: G 24:4 change *aṣghar* (R) to *aqall* (LH).

Q 36: G 24:10 instead of *al-aṣghar* I prefer *al-aqall* (LH) (In R the word is *al-awwal*, cf. fn 78).

Q 39: G 25:4 delete *al-makhṭūṭ*, which is in L but not in RH (cf. fn 82).

Q 40: G 25:6 delete *wāḥida* (fn 83) which is in L but not in RH. To make mathematical sense, change *qā'iman* in R and G 25:7 to *sāqan* (L) (cf fn 84).

Q 41: In G 25:10 delete *mustaqīm*, which is not in R. The passage was rephrased in L.

Q 42: G 25:12 change *musāw dhālika* to *mā wa-dhālika* (RH).

Q 43: G 26:2 change *qāʿida bi-khaṭṭ* (R) to *qāʿidatihā* (LH), to make mathematical sense.

Q 44: G 26:4 change *al-qawsiyya* (L) to *al-muqawwasa* (RH). Change *zawāya al-qisī* in G 26:7 to *zawāyā wa l-qisī* (RL).

Q 46: G 26:10-11. Change *alladhī yusammā jayban* to *alladhī huwa jaybuhā* (LH). R has *alladhī huwa yusammā jayban.*

Q 47: G 26:12 change *al-jayb* to *al-jayb al-maʿkūs* (LH). In G 26:12-13 change *qaws mā wa-lladhī* (R) to *al-qaws alladhī* (LH).

Q 51: G 27:4 change *al-qaws* (R) to *al-qisī* (H). To make mathematical sense, I have added *al-mutashābiha* with H. Compare Q 55. G 27:5 change *kullayhā* (R) to *kullihā* (H).

Q 52: G 27:6, change *mā* to *mā khawāṣṣ* (RL). H has *mā khāṣṣiyya*, "what is the property".

Q 53: In G 27:10 change *al-jihatayn* to *al-jihatayn jamīʿan* (RL). In G 27:10-11 delete *aw akthar wa-lam taltaqi fī shay' minhā*. These words are in L but they are unnecessary and not extant in R (cf. fn 102). I emend the word *min* in R and G 27:13 to *bayna*; L and H omit the whole passage.

Q 55: G 28:7 change *al-zāwiyatān* (R) to *al-zawāyā* (LH). In G 28:7 also change *tu'aththir fīhā* (R reads *tuwattir fīhā*) to *tuwattiruhā* (LH) to make mathematical sense.

Q 56: Note that G's readings (compare fn 107, 108) are confirmed by H. In G 28:9 I have deleted the words *muqawwasayn aw-khaṭṭayn*, which are in L (cf. fn 109). In G 28:9 before *wa-kull* I have added the following passage inspired by H 17:19-20: *aw fī dā'iratayn iḥdīdāb iḥdāhumā yalī (ms. ilā) iḥdīdāb al-ukhrā aw fī dā'iratayn inḥinā' iḥdāhumā yalī (ms. ilā) iḥdīdāb al-ukhrā*. I assume that Qusṭā used the same terminology as in Q 26-27.

Q 57: G 28:13 change *al-nīmkhānijī* (H) to *al-mīkhānijī* (R has *al-miḥānijī*). In G 28:14 I have emended *taf'alu al-ālāt* (R) to *tufʿalu bi l-ālāt*, to make mathematical sense.

Q 58: G 29:3 change *nihāyatāhu* to *nihāyātuhu* (RLH).

Q 60: G 29:5 change *mutasāwiya* (R) to *musāwiya* (H). In G 29:6 change *kāna* to *kam kāna* (H). In G 29:7 delete *makhraj*, which is G's reading of a second word *takhruj* in R. In G 29:7 change *wāḥid* to *kull wāḥida* (H) to make mathematical sense. In G 29:9 the text is incomplete; after *udīra* I I tentatively add < *khaṭṭ mustaqīm* >, and instead of *mā bayna jamīʿayhi* I read *māssa jamīʿahu* (H).

Q 61: G 29:10, 11 change *nihāyatāhu* to *nihāyātihi* (RLH, R omits the word in G 29:11). In G 29:11 change *aʿlā* to *akhfaḍ* (LH). Note that the end of Q 61 and the beginning of Q 62 are missing in R.

Q 62: G 30:1, 2 change *nihāyatāhu* to *nihāyātihi* (LH).

Q 63: G 30:3 after *al-musaṭṭah* add *al-uwwal* (L). In G 30:5 change *musaṭṭaḥan* (R) to *saṭḥan* (LH), which is the standard term for plane figure.

Q 64: G 30:7 after *ṣā'idan* add *abadan* (LH). G 30:8 add at the beginning of the line *alladhī yuḥīṭu* (RL).

Q 69: G 31:6 before *al-muthallath* add *al-mukhtalif al-aḍlāʿ al-qā'im al-zāwiya wa-l-mukhtalif al- aḍlāʿ al-munfarij al-zāwiya* as suggested by H 19:3-4. Q 70: G 31:10 change *yakūn* (R) to *yakūn kull ḍilʿayn* (LH), to make mathematical sense, cf. fn 133. G 31:12 change *al-mutawāṣilatayn*

(R) to *al-dākhilatayn* (H) (L is also corrupt here). G 31:13 change *zawāyā al-muthallath* (R) to *zawāyāhu al-thalāth* (LH). G 31:13-14 change *fīhi ayḍan* (R) to *fī muthallath wāḥid* (H), the passage is missing in L.

Q 71: G 32:3 Guergour's addition < *idhā jumiʿâ* > is confirmed by L and H. In G 32:7, change *nafsihi* to *'aḥad qismayhi* (LH). R has *nafsayhi*.

Q 72: In G 32:10 after *kull* add *wāḥid minhumā* (LH). G 32:10 before *al-thālith* add *al-ḍilʿ* (L).

Q 73: In G 32:16 after *nafsihi*, the passage in Guergour's fn 141 should be added (from L) to make mathematical sense. It seems that the passage was left out from R by a scribal error. A similar passage is contained in H 20:6-8.

Q 74: In G 33:2 after *al-murabbaʿāt* I have added *al-uwwal*, following H and L (not indicated in G's apparatus), because the classification into seven categories is further refined into 22 subcategories in Q 75-81. In G 33:3 change *thalāth aḍlaʿ* to *al-thalātha aḍlaʿ* following R, in which this passage is found (contrary to footnote 143). I have deleted from G 33:3 *mukhtalif al-bāqiyayn* (L), because these words are missing in R (fn 144) and H, and also because they are inconsistent with the previous words "having two pairs of equal opposite sides". In G 33:4 after *mutalāqiyayn* add a fifth category *mutasāwī ḍilʿayn mutaqābilayn mukhtalif al-bāqiyayn*. The addition, which is inspired by H, is not found in R and L, but is logically necessary, see Q 79. In G 33:4 read the sixth category as *mutasāwī ḍilʿayn mutalāqiyayn* (L) rather than *mutasāwī ḍilʿayn al-mutalāqiyayn*, compare Q 80.

Q 75: In G 33:6 add at the end of the line *mukhtalifuhā* (LH).

Q 78: G's emendation < *qā'ima* > in G 34:6 is confirmed by H.

Q 79: In G 34:8-9 delete G's addition < *dhū zāwiya qā'ima ʿalā l-akthar* >, which is not in any of the manuscripts, see my note to Q 79. In G 34:10 change *an yakūna* (RH) to *an lā yakūna* (L), see note to Q 79. In fn 162 delete *yumkinu an*.

Q 80: In G 35:1-2 delete G's addition < *dhā zāwiyatān qā'imatān ʿalā al-akthar* >, see my note to Q 80. In G 35:2 *al-mutaqābilatān* (RH) should be changed to *al-mutalāqiyatān* for mathematical sense. In G 35:4 change the last word *watar* to *tawāz* ("parallelism") as in R; H and L have *mutawāzin*.

Q 81: In G 35:7 change *fīhi qā'ima* to *ghayr qā'ima fīhi* (H); R has *qā'ima fīhi*. G 35:7 change the last word to *mutawāzī* (RH).

Q 82: In G 35:11 before *faqaṭ* add *wa-kull zāwiyatayn mutaqābilatayn* (LH).

Q 84: In G 36:2 after *al-zawāyā* add *hiya* (LH). In G 36:5 change *ḍilʿ qā'im* (R) to *ḍilʿ ḍilʿ* (H).

Q 87: Guergour's emendations (footnotes 171,172) are confirmed by H.

Q 88: In G 37:1 the *bi-khaṭṭ* (R) should be deleted following H; instead, I have added *ghayr mutaqāṭiʿa* following L and in analogy with Q 89.

Q 89: In G 37:2 after *muthallathāt* add the following passsage: *idhā ṣuyyira li-kull ithnayn minhā ḍilʿ mushtarak wa-ijtamaʿat zawāyāhu kulluhā ilā nuqṭa wāḥida yanqasimu ilā arbaʿ muthallathāt*. This passage in R 6a:10-11 was omitted from G 37:2 by an oversight. Similar passages are found in L 4b:4 and H 22:12-13. In G 37:3 change *ilā l-zāwiya* (R) to the plural *ilā l-zawāyā* (H).

Q 90: In G 37:5 after *min muthallath* add *yatarakkaba* (H); L has a similar word which is not very legible, cf. fn 176. In G 37:8, change *fa-dhālika* (R) to *bi-dhālika* (LH). In G 37:11 after *taqūlu* add: *inna l-muʿashshar yatarakkabu min thamāniya muthallathāt* (H), compare fn 178 for a similar passage in L.

Q 91: In in G 37:14 change the emendation *majmūʿ zawāyā* to *kam fī* (LH); R has the nonsensical *ḥadī*. In G 37:15, change *nisba li-zāwiya* to *min zāwiya* (RLH). In G 37:15 after *fa-l-nanẓur* add *kam* (LH). In G 38:1 change *li-dhālika* to *fi dhālika* (LH). In G 38:3 change *min al-shakl* to *fī al-shakl* (L). In G 38:5-6, the passage which is missing in R (fn 183) should be inserted between *wa l-musabbaʿ* and *kulluhā*. Thus the following changes should be made to G's text: in G 38:5 delete *kulluhā*; in G 38:6 change *aʿnī* to *zawāyā aʿnī an*; in G 38:6 before *idhā* add *kulluhā*.

Q 92: In G 38:9 change *zāwiya* to *zāwiyatihi* (RL). The sentence in G 38:10 continues on G 38:11, so the boldface in G 38:11 is unnecessary. In G 38:12 after *dhālika* add *al-shakl* (LH). In G 38:13 after *nisba* add *zāwiya shakl* (LH). In G 38:13, instead of *min zāwiya* (R), H has *wa l-zawāyā ilā l-zāwiya al-qā'ima,* and L has *al-qā'im al-zawāyā ilā l-zāwiya al-qā'ima*, which reading shows that the scribe of L understood nothing of the subject. I believe that *min zāwiya* in G 38:13 should be changed to *min zāwiya qā'ima*.

Q 96: In G 39:11 and 39:13, change *al-zāwiyatayn* to *zāwiyatayhi* (RLH). In G 39:12, change *yuwattiruhā* to *yuwattiruhumā* (RLH). In G 39:14, change *al-qā'ima* (R) to *al-bāqiya* (LH, see fn 194).

Q 97: In G 39:15 before *muthallathan* add *ʿalayhi* (RLH). In G 39:16 change *al-zāwiyatayn* to *zāwiyatayhi* (RLH).

Q 99: In G 40:9 after *al-bāqiya* add the following passage in R: *fa-kullamā zāda al-shakl ḍilʿan*. The scribe of R marked these words as corrupted, but the passage makes sense and is probably genuine. For *al-mutasāwiyayn* in R and G 40:10 I prefer *al-mutasāwiyatayn*. The text in R is incomplete. L and H omit the passage corresponding to G 40:7-10.

Q 102: In G 41:4 change *ilā* (R) to *yalī* (LH).

Q 103: In G 41:6 change the unusual word *al-qawsa* (R) to *qawsuhu* (LH).

Q 104: In G 41:9 after fn 210 add *aṣghar min al-qaws al-ākhar* (RLH) (compare fn 210). In G 41:9 and 10, change *fa-hiya* (R) to *fahiya min* (LH). In G 41:11 change *al-dā'ira* (R) to *min dā'ira* (LH). In G 41:12 change *wa-dā'ira* (R) to *wa-min dā'ira* (LH).

Q 107: In G 42:4 I emend *tuqsam* to *tusammā*. R does not have any dot on top of the letter *qāf*. L has *laysa*.

Q 108: *fīhā* in G 42:8 and R should probably be read as *minhā* (H) or *faminhā* (L). Following L and H, change *qaṭʿ watar* in G 42:10 to *qaṭṭāʿ* to make mathematical sense; R has *qiṭʿa*.

Q 109: In G 42:10 the beginning of a new question should be inserted: after *al-kathra* insert *kam anwāʿ* (LH), and change *wa-qiṭaʿ* (R) to *qiṭaʿ* (H).

Q 111: In G 43:3 change *qawsiya* (R) to *qawsihi* (L). H omits the word.

Q 113: In G 43:9 delete *alladhī yuḥīṭu bihi khaṭṭ muqawwas*, which is a corrupt passage in R only, and add *mutashābih al-ajzā'* (L); H has *mutasāwī al- ajzā'*.

Q 114: G 43:11 after *wataruhā* add *thābit* (L). In G 43:12 *ibtidā* to *ibtada'a al-ḥaraka* (L).

Q 115: In G 44:1 and G 44:2 change *yaʿruḍu* (R) to *yufraḍu* (LH).

Q 116: In G 44:3 change *mutasāwī* (R) to *mutashābih* (LH).

Q 117: In G 44:7 change *bada'at minhū al-ḥaraka* (R) to *ibtada'a bi-l-ḥaraka* (L), compare Q 114.

Q 118: In G 44:9, 10, 11 change *yaʿruḍu* (R) to *yufraḍu* (LH). In G 44:10 change *al-dā'ira* (R) to *al-dawā'ir* (LH). Q 120: In G 44:16 change *al-ḥaraka* (R) to *bi l-ḥaraka* (LH). In G 45:1 change *bi-zāwiya* (R) to *fī dawrānihi* (L); H omits this passage.

Q 121: Change G 45:4 *dā'iratān* (R) to *ithnatān* (LH).

Q 122: G 45:6 change *taqdīr* to *anwāʿuhā bi-qadr* (LH) (R has *anwāʿuhā taqdīr*). G 45:7 for *kāna* I prefer *mā fa-in* (LH). G 45:8 change *baṭīn* (R) to *naẓīr* (LH), compare fn 248. In G 45:9 *al-muqabbabat* (fn 250) should be read as al-*muqaʿʿara* (R), L is unclear. In G 45:9 after *yanbanī* add *ayḍan* (RL). In G 45:9 I emend the last word *al-muqaʿʿara* (RL) to *al-muqabbaba* to make mathematical sense.

Q 124: In G 46:8 change *mustaqīma* (L) to *muqabbaba wa-musaṭṭaḥa* (H). The word is missing in R, cf. fn 255.

Q 127: In G 47:1 change *al-muthallathāt* (R) to *muthallathāt* (LH). In G 47:1 delete *mutasāwī l-aḍlaʿ yusammā shakl nārī* (these words are in L but not in R); in G 47:2 change *qā'ima al-zawāyā wa-yusammā kaʿban shaklan* to *yusammā shakan nāriyan* (H).

Q 128: In G 47:3 change *al-murabbaʿāt* (R) to *murabbaʿāt* (LH).

Q 129: In G 47:5 change *al-muthallathāt* (R) to *muthallathāt* (LH).

Q 130: In G 47:7 change *al-mukhammasāt* to *mukhammasāt* (LH); R has *al-mujassamāt*.

Q 131: In G 47:9 change *al-muthallathāt* (R) to *muthallathāt* (LH).

Q 132: In G 47:11 after *hādhihi* add *al-khamsa* (L). In G 47:12 change the emendation *bi-l-khamsa* to *bi-l-arbaʿa* (RL). H omits this passage. In G 48 at the end of line 1 add *bi-l-hawā' wa-shabbahū dhā l-ʿishrīn qāʿida muthallathāt* (LH). This passage was probably left out in R by scribal error. In G 47:12 change Guergour's emendation *al-khamsa* to *al-arbaʿa* (RL).

Q 133: I have deleted the word *mas'ala* at the beginning of G 48:4, because it does not occur in R and H. In G 48:5 change *yadhkur* (R) to *yadhkur dhālika* (L). Q 134: Guergour's emendation of *wa-* (R) to < *min* > in G 48:9 is confirmed by L and H.

Q 134: In G 48:12 delete *thalāth*; the word is not found in L and H and it is inconsistent with the definition of a pyramid in Q 135. In G 48:14 after *wa-murabbaʿāt* add *maʿan* (LH).

Q 136: In G 49:3 change *kull* (R) to *ʿalā* (LH). In G 49:4 after *ʿalā* add *kull* (LH). In G 49:4 change *al-qawā'id* (R) to *al-qā'ida* (LH). In G 49:6 before *al-arbaʿ* delete *al-muthallathāt*; the word does not occur in R and H. In G 49:7 after *al-nārī* add *alladhī* (H). Alternatively, one can change *al- nārī* to *nāriyan* (L).

Q 137: In G 49:11 change *min jism yuḥīṭu* (R) to *murabbaʿ yuḥīṭu bihā* (LH). In G 49:12 change the second word *sitta* (R) to *arbaʿa* (LH). In G 49:13 change *al-mukhammas* (R) to *al-khamsa* (L). The answer in Q 137 begins with the definition of (a) a cube and then presents three definitions which concern rectangular parallelepipeds with (b) two equal sides and a third side which is shorter, (c) two equal sides and a third side which is longer, and (d) three different sides. L and H give the definitions in the order (a), (b), (c), (d), whereas R only presents (a), (b), (c). Guergour first presents (a) and in G 49:14-17 he continues with (c) from R, then (b), then (c) from L, finally (d). To restore Qusṭā's original, delete *wa-minhā...*

randan in G 49:14-15; put footnote 282 on the word *libna* in G 49:16, change footnote 282 to *wa minhā mā... libna: m nāqiṣa*. In G 49:17 the word *tsr*, indicated by footnote 283, should probably be read as *tīr*, meaning (supporting) beam. The reading *rndan* in R is probably corrupted.

Q 140: In G 50:6 change *taqdīr li-'anwāʿ* (R) to *bi-qadr anwāʿ* (LH).

Q 141: In G 50:7 delete *jism* (R), the word is not found in L and H. Q 144: Instead of *yamurru* in G 50:13 I read *yajūzu* (RLH).

Q 147: In G 51:2 change *min qiṭʿa dā'ira* (R) to *qaṭʿuhu dā'iratan* (LH). In G 51:2 after *bi-niṣfayni* add *wa-inna kull al-dawā'ir allatī yajūzu ʿalā aqṭārihā yaqṭaʿuhā bi-niṣfayni* (LH) (H reads *aqṭābihā*, "its poles" but this reading must be incorrect because one would need the dual *quṭbāyhā*); in G 51:3 delete *wa-'in kānat al-dawā'ir* and in G 51:4-5 delete all the text. G did not realize that *wa-lā... dā'iratayn* in G 51:2-3 and *wa-annahu lā ...mutasāwiyatayn* G 51:4-5 are versions of the same passage in the two different manuscripts R and L.

Q 149: In G 51:10 change *ibtada'at al-ḥaraka* (R has *ibtada'at al-kura*) to *ibtada'a bi l-ḥaraka* (LH).

Q 150: Change *yamurru* in G 51:13 twice to *yajūzu* (RLH). In G 51:13 change *tablaghu* (R) to *tantahī* (LH), cf. the same terminology in Q 151 and Q 154.

Q 154: Change *tamurru* in G 52:1 to *tajūzu* (RLH).

Q 156: In G 52:1 change *watarā* (R) to *wataruhā* (LH). In G 52:6 change *udīra* to *udīra al-qaws* (R), and change *al-ḥaraka* (R) to *bi l-ḥaraka* (LH).

Q 157: In G 52:8 change *fa-ka-anna* to *makān* (RLH). Q 158: In G 25:10, Guergour's emendation < *yaḥduthu* > is confirmed by H. In G 52:10 delete *muḥīṭ* which is not in L and R. In G 52:11 change *ilā l-muḥīṭ min* (R) to *min al-muḥīṭ ilā* (LH).

Q 159: In G 52:14 after *kam* add *anwāʿ* (LH). In G 52:15 change *al-qiṭaʿ* (not in R) to add *qiṭaʿ ukar wa-minhā qiṭaʿ asāṭīn wa-minhā qiṭaʿ makhrūṭāt* (L). H has *arbaʿa* instead of *khamsa* (in G 52:14) and H omits the *qiṭaʿ ukar*.

Q 160: In G 52:17 change *saṭḥ* to *saṭḥā* (LH).

Q 161: In G 53:3 change *bi-dawrānihi* in G 53:3 to *fī dawrānihi* (RL), compare Q 117.

Q 163: The passage: "and it contains... a right angle," which I have translated from G 53:8-9, is found in L, but not in R and H. The passage may be inauthentic.

Q 164: Compare Q 118. To make mathematical sense, I have emended *al-dā'ira* in R and G 53:10 to *al-dawā'ir* following H; L has *kull al-dawā'ir*. In G 53:11 change *al-khārija min* (R) to *al-mukhrija fī* (L); H has *al-mukhrija ilā*.

Q 167: In G 54:2 change *al-makān* to *aʿlā* (LHR) (cf fn 300); in R the word ends with normal alif. In G 54:2 change *bi-idāra* (R) to *yudār* (H); L has *mudār*. In G 54:3 after the second *al-khaṭṭ* add *ilā l-nuqṭa* (LH).

Q 170: In G 54:10 change *basīṭ* (R) to *basīṭihi* (LH), and change *dā'iratayn mutasāwiyatayn* to *dā'iratān mutasāwiyatān* (RLH). In G 54:11 change *mutasāwiyayn mutawāziyayn* (similar to R) to *mustaqīmān mutawāziyān* (LH), cf fn 303.

Q 172: In G 54:15 after *uqlīdis* add *alladhī* (LH). In G 54:16 after *alladhī* add *ʿanhu* (LH). Compare Q 173-174, In G 55:2, 5 change *muthallatha* (R) to *muthallathihi* (LH).

Q 176: In G 55:8, change *bi l-khaṭṭ* (R) to *al-khaṭṭ* (LH), and change *quṭrihi* (R) to *quṭr* (LH).

Q 177: In G 55:10 after *maʿa* add *dā'ira* (LH).

Q 179: In G 55:13, change *bi-saṭḥ* (R) to *bihi saṭḥ* (LH). In G 55:14 change *al-dā'ira* (R) to *al-kura* (LH).

Q 180: In G 55:16-56:1 change *miḥwaran li-kura* (R) to *miḥwaru l-kura* (LH). In G 56:1 change *markaz ākhar* (R) to *markazuhā* (LH).

Q 181: In G 56:3-4 change *al-dā'ira* (R) to *al-dawā'ir* (H). L has *al-zawāyā* which is probably a corruption of *al-dawā'ir* as well, cf. fn 309.

Q 183: In G 56:6 before *sahmihā* add *ʿalā* (LH). In G 56:7 change the first *wa-minhā mā yakūn qaṭʿuhā* (R) to *aw* (LH). In G 56:8 change *wa-khārijan* (R) to *aw khārijan* (H); L has *aw khārij*.

Q 184-185: R and G first present what I have called question 185 (G 56:9-12) and then question 184 (G 56:13-14). I have restored the logical order of the text, following L and H (cf. fn 312). In G 56:14 change *zāwiyatayn* (R) to *dā'iratayn* (LH).

Q 185: In G 56:10 before *yuḥīṭu* add *aw aʿẓam min niṣf usṭuwāna* (H), or greater than half a cylinder. L has the same passage with *wa-* instead of *aw*. This passage must be authentic because the rest of the answer specifically refers to a section of a cylindrical surface greater than half a cylinder.

Q 186: In G 56:15 delete *fī*, which is not in RLH. In G 56:15-16 delete *wa-qiṭʿa al-usṭuwāna allatī qaṭʿuhā muqāṭiʿ sahmahā fī khārijihā*, which passage is not in RLH. In R, the manuscript has only the words *wa-qiṭʿa* *al-usṭuwāna fī dākhilihā*, which are a partial repetition of the preceding passage. The scribe marked the two first words as erroneous.

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