Two Treatises on Mīqāt from the Maghrib (14th and 15th Centuries A.D.)

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1 Introduction

The aim of this paper is to analyse two treatises on *mīqāt* compiled in the Maghrib in the 14th and 15th Centuries. One of them is the *Kitāb fī ʿilm al-awqāt bi-l-ḥisāb* written by Abū-l-ʿAbbās Aḥmad b. Muḥammad b. ʿUzmān al-Azdī al-Marrākushī, known as Ibn al-Bannā' (d. 721 H/1321 A.D.), and preserved in the Ḥasaniyya Library of Rabat (n.10783). The second is the *Iqtiṭāf al-anwār min rawḍat al-azhār* written by Abū Zayd ʿAbd al-Raḥmān b. Abī Gālib al-Jādirī al-Muwaqqit (d. 839 H. /1435 AD) of which at least two copies survive, preserved in the Ḥasaniyya Library of Rabat (mss. 10410 and 8796). This treatise is an abridgement of an *urjūza* by the same author entitled *Rawḍat al-azhār fi ʿilm al-layl wa-l-nahār*, which is a poem on folk astronomy, compiled in 1391-92[1]. Muḥammad al-Jaṭṭābī edited both treatises in a book entitled *ʿIlm al-mawāqīt*[2], published in 1986, from the Ḥasaniyya manuscripts already mentioned. I have used this edition in the analysis of both texts.

2 *ʿIlm al-mīqāt* or Astronomical Timekeeping[3]

These two treatises mentioned are inscribed in the tradition called *tawqīt bi-l-ḥisāb* or "arithmetical timekeeping", which is related to the so-called

[1] See King ,1990a, pp. 224-225; King, 2004, p. 501.

[2] See Jaṭṭābī, 1986.

[3] See King,1990b; King, 1996a, pp.303-308 and King, 2004, pp..

ʿilm al-mīqāt. This expression describes the science of astronomical timekeeping by the sun and stars in general, and the determination of the times of the five prayers in particular. The limits of permitted intervals for the prayers are defined in terms of the apparent position of the sun in the sky relative to the local horizon. Therefore their times vary throughout the year and are dependent on the terrestrial latitude.

The definition of the times of prayer was standardized in the 8^{th} century and has been in use ever since. The day as well as the period for the Maghrib prayer begin at sunset, when the disk of the sun has set over the horizon. The interval for the *ʿishāʾ* prayer begins at nightfall and the interval for the *fajr* begins at daybreak. The permitted time for the *ẓuhr* begins according to Andalusī and Maghribī sources when the shadow of any object has increased over its midday minimum by one-quarter of its length. The interval for the *ʿaṣr* begins when the shadow increase equals the length of the object and ends when the shadow increase is twice its length.

These definitions of the times of the diurnal prayers in terms of shadow increases (as opposed to the shadow lengths, used in the *ḥadīth*) represent a practical means of regulating the prayers in terms of the seasonal hours. The aforementioned definitions of the *ẓuhr* and *ʿaṣr* correspond to the sixth and ninth seasonal hours of daylight. The links between them are provided by an approximate formula of Indian origin relating shadow increases to the seasonal hours:

$$\mathrm{T} = \frac{6n}{\Delta s + n}$$

where n is the length of the gnomon and Δs represents the increase of the shadow over its midday minimum at T seasonal hours after sunrise or before sunset. This formula has been known in the Islamic World since the 8^{th} century[4]. Here it appears in the last chapter of Ibn al-Bannā's *kitāb* and in chapter 19 of al-Jādirī's treatise.

[4] It appears for instance in the *zīj* of al-Fazārī, an 8^{th} century astronomer who was the author of one of the first Arab adaptations of the *Sindhind.* It was mentioned by al-Bīrūnī in his *Exhaustive Treatise on Shadows*. See Kennedy, 1976, vol I, 192-195, vol. II pp. 118-119 and King, 1990 p. 28.

2.1 *The institution of the muwaqqit*[5]

Before the 13th century the regulation of the prayer-times in Islam was the duty of the muezzins. They needed to know the rudiments of folk astronomy: the shadows at the *ẓuhr* and *ʿaṣr* for each month and the lunar mansion which was rising at daybreak and setting at nightfall.

But in the 13th century the figure of the *muwaqqit* appears, the professional astronomer associated with a religious institution, the mosque, whose primary responsibility was the regulation of the times of prayer. The first mention is found in Egypt[6] but the *muwaqqit* spread throughout the Islamic world and by the end of that century we find the first mention of an astronomer of this kind at the *Jāmiʿ* mosque of Granada[7]. A new kind of literature, intended to help the duties of the *muwaqqit,* began to be produced. The two treatises analysed here can be classified under this category.

3 The authors of the two treatises

The authors of these two texts were of Maghribī origin and their works were part of the Andalusī tradition. Most of the procedures described here can be found in earlier astronomical texts written in al-Andalus. As I say, these texts belong to the category of the *muwaqqit* manuals that were compiled by astronomers wishing to give instructions for performing the canonical duties as exactly as possible but in a very simple way, thus avoiding theoretical or technical explanations.

3.1 *Ibn al- Bannā'* [8]

The author of the first of the texts, Ibn al- Bannā', is one of the greatest mathematicians and astronomers in the Maghrib. His full name was Abū-l-ʿAbbās Aḥmad ibn Muḥammad ibn ʿUthmān al-Azdī. He was born

[5] On the role of the muezzin and the *muwaqqit* see King, 1996 a.

[6] See King, 1996 a., p.298 n. 35.

[7] See Calvo, 1993 pp. 24-25 and King, 1996 a. p. 299 n. 38.

[8] See Renaud,1938, pp 13-42; Suter, 1897,nº 399, p. 162; Vernet, 1970, pp.437-438; Calvo, 1989, pp. 21-50; Calvo, 1997, p. 404; Lamrabet, 1994, p. 83. His life and works have been the object of a recent study: see Djebbar-Aballagh, 2001.

in Marrakesh in 654 H. / 1256 A.D. and died, probably in the same city, in 721 H. / 1321A.D. He studied Arabic Language and Grammar, the Qur'ān, *ḥadīth* and also Mathematics, Astronomy, and Medicine. He is best known for his knowledge of Mathematics and is credited with writing more than 80 works. Among them we find:

- *Talkhīs aʿmāl al-ḥisāb,* probably his best known book: a very concise work on Arithmetic which is easy to memorise[9].
- An introduction to Euclid[10].
- An almanac entitled *Risāla fī'l-anwā'*, a book about asterisms and stars used in meteorology and navigation[11]. It also gives diverse information on agriculture, meteorology, and astronomical folklore for each day of the year. But, since Ibn al-Bannā' worked in Marrakesh and this almanac presents information based on a latitude of about 38°, that is, Cordova, his authorship is in doubt[12].
- Two abridgements of treatises by Ibn al-Zarqālluh on the use of the *ṣafīḥas*, the *zarqāliyya* and the *shakkāziyya*, and another on the *shabīha*[13].
- A *Zīj* (astronomical handbook with tables) entitled *Minhāj al-Ṭālib li-taʿdīl al-kawākib,* a highly practical book for calculating astronomical ephemerides. It is based on the work by Ibn Isḥāq and indeed both draw heavily on Ibn al-Zarqālluh's astronomical theories[14].
- Finally, the treatise analysed here, his *Kitāb fī ʿilm al-awqāt bi-l-ḥisāb,* which is one of his short works but which bears witness to the importance of this topic in 14th-century Maghrib[15].

[9] See Souissi, 1969; A.S. Saʿidān, 1984; Calvo, 1989, p. 21, n. 2 and Djebbar-Aballagh, 2001, pp. 89-99.

[10] See Djebbar, 1990 vol. II.

[11] See Rénaud, 1948 and Djebbar-Aballagh, 2001, p. 126.

[12] See King, 2004, p. 497.

[13] On the *zarqāliyya* see Puig, 1987; on the *shakkāziyya* see Calvo, 1989. See also Djebbar-Aballagh, 2001, pp. 122-124.

[14] See Vernet, 1952; Samsó-Millás,1994, X; Samsó-Millás,1998 and Djebbar-Aballag, 2001, p.112, n 39.

[15] See Djebbar-Aballag, 2001, p. 127. The copy of the incipit there lacks one line of text.

3.2 *Al-Jādirī*[16]

Abū Zayd ʿAbd al-Raḥman Muḥammad al-Jādirī was born in Meknes in 777 H./ 1375 A.D. and died in Fez, probably in 818 H./1416 A.D. He was *muwaqqit* at the Qarawiyyīn Mosque[17]. He was the author of several works, among them *Tanbīh al-anām ʿalà mā yaḥduthu fī ayyām al-ʿām,* a calendar adapted to the latitude of Fez[18], and the aforementioned *urjūza,* entitled *Rawḍat al-azhār fī ʿilm al-layl wa-l-nahār*, in 26 chapters and 335 verses, compiled in 1391 A.D. at an early time in his life; the treatise analysed here is an abridgement[19].

4 The contents of the treatises

As I have said, the two treatises give simple instructions, mainly arithmetical procedures, for performing the calculations needed for *mīqāt* purposes, avoiding for instance the use of astronomical instruments. It is evident that they were intended to produce straightforward solutions to the problems of calculation that the *muwaqqit* and *mu'adhdhin* might find in their everyday duties. Both treatises begin with an explanation of their aims for writing this kind of treatise:

Ibn al-Bannā' says that his treatise contains all that is needed in this science. He adds that it is not necessary to use instruments based on rays and shadows. Al-Jādirī says that he abridged his *urjūza* because he was asked to do so by some legal scholars and by "some of the best people".

In fact, the two texts coincide on many points, although Ibn al-Bannā's *Kitāb fī ʿilm al-awqāt bi-l-ḥisāb* has only ten unnumbered chapters, whereas al-Jādirī's *Iqtiṭāf al-anwār min rawḍat al-azhār* is divided into 27 chapters, also unnumbered. The main topics dealt with in both are:

- **questions of the calendar**, in particular related to the conversion of solar and lunar calendars and the equivalences between them.

[16] See Brockelman, SII pp. 217-218 and King, 1990, pp. 224-225

[17] See Mannūnī, p. 156; Lamrabet, p. 114; Suter, nº 424, p. 172; Brockelmann S II, pp 218-219 and Alkuwaifi & Rius, 1998, pp.454-455.

[18] The latitude of Fez in modern sources is $\varphi = 34;05°$.

[19] The *Rawḍa* is preserved in several manuscripts and was the object of a number of commentaries. See Alkuwaifi & Rius, 1998, p. 455.

- **spherical astronomy**: determination of longitudes, latitudes, right and oblique ascensions, meridian altitudes, etc.
- **shadows,** calculated from the altitude of the sun or the stars.
- **time reckoning,** which can be applied to determine the times of the five prayers or any other time of day and night as well.
- **trigonometry:** al-Jādirī includes some trigonometrical elements, but Ibn al-Bannā' does not.
- **the azimuth of the *qibla*:** al-Jādirī devotes the last chapter to giving instructions on how to determine it.

4.1 *Questions of the calendar*

4.1.1 *Ibn al- Bannā'*

In the first chapter[20] the names of the 12 months of the Christian year are given as well as the number of days for each. The chapter includes the information that, every four years, there is a supplementary day in December, a practice in use in al-Andalus in the Middle Ages[21]. Ibn al-Bannā' gives a mnemotechnical rule to remember which months have 31 days, using the Arabic words:

فاز رجل ختم بحج[22]

The instructions are that for each month there is a letter. The letter with a dot corresponds to a month of 31 days. The letters without dots correspond to months of 30 days, with the exception of February, which has 28 days, and December in the leap years, which has 32.

The second chapter[23] gives the procedure for determining the year in which there is what he calls *izdilāf* (advance). He says that there is

[20] See Jaṭṭābī, 1986, pp. 86-87.

[21] This procedure is well documented in Andalusī sources. It appears for instance in the "Calendario de Córdoba". See Dozy & Pellat, 1961, pp. 16-17; Martínez Gázquez & Samsó, 1983, pp. 9-78; Samsó & Martínez Gázquez, 1981, pp. 319-344. It also appears in the adaptation made by Maslama al-Majrīṭī of the *Zīj al-Sindhind* by al-Khwārizmī. See Neugebauer, 1962, pp. 11-13 and 224 n. 25 and Samsó, 1992, p. 89.

[22] The translation would be: *A man obtained a seal by pilgrimage.*

[23] See Jaṭṭābī, 1986, pp. 87-88.

izdilāf when the Arabic year does not have a day corresponding to the first day of the Christian year. This occurs, according to him, every 32 years[24]. A formula is then given to calculate the *madkhal*[25] of the solar year corresponding to a given Islamic year:

$$\frac{(AH-670)}{4}+(AH-670)$$ [26]

The result has to be divided by 7 and the remainder has to be taken. Then, starting from Sunday (weekday 1) we can calculate the weekday to which this number corresponds. This will be the weekday of the beginning of the solar year.

The explanation for this formula is that, every year, the weekday changes by one, except after the leap year when it changes by two. Therefore we have to add as many days as years have passed since 670, and then add as many supplementary days as leap years have passed.

To know the weekday of the beginning of the other months of the solar year, the author gives another rule which consists of attributing to every month a value, beginning with January:

month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
letter	ا	د	د	ز	ب	ه	ز	ج	و	ا	د	و
value	1	4	4	7	2	5	7	3	6	1[27]	4	6
difference		+3	=	+3	+2	+3	+2	+3	+3	+2	+3	+2

This equivalence is derived from the structure of the months of the solar calendar[28].

[24] In fact there is *izdilāf* every 33 or 34 Islamic years. The year of the *izdilāf* is a year free of taxes to compensate the difference between the solar year of 365 1/4 days and the lunar year of 354 11/30 days. See Díaz Fajardo, 2002, p. 44; Forcada, 2000, p. 124 & Arabic text p. 161 and Van Dalen, 1998 p. 282.

[25] The *madkhal* is the day of the week with which a given year begins.

[26] This treatise was probably composed shortly after the year 670 H. / 1271-72 A.D. In the year 671 H. the year 1272 began, which was a leap year, and the four year cycle was completed. The first of January 1273 (9 Jumāda II) corresponds to Sunday (weekday 1). Therefore, it was the best moment to apply this formula.

[27] The edition gives : ح (8) .

[28] When the month has 31 days the difference is $\Delta = 3$, since $31 = 7\text{x}4 + 3$. For months of 30 days, $\Delta = 2$; and for February (28 days) $\Delta = 0$.

The third chapter[29] describes how to calculate the day of the month of the Christian year from the Arabic one.

The procedure, as explained in the text, is as follows:

$$354\ \frac{1}{5}\ \frac{1}{6}\ (\mathrm{AH} - 660) + \mathrm{n} = \mathrm{A}$$

where *AH* is the number of complete years passed of the Islamic calendar, the amount 354 1/5 1/6 corresponds to 354 + 11/30, the value of one mean lunar year[30], and *n* is the number of days corresponding to the months passed of the incomplete year and the days passed of the incomplete month.

The total result, *A*, is called *al-aṣl,* "the basis" and equals the number of days passed since the first Muḥarram 661 H. It is then divided by seven and the remainder beginning with Tuesday will give the weekday[31].

The next instruction is

$$\frac{A+317}{365.25} = \mathrm{R}$$

where A + 317 will be the total amount of days since the beginning of 1262[32] and R will be the number of solar years corresponding to this number of days. When there is no remainder, we have a number of complete solar years, and it is the last day of December. If there is a remainder, it will correspond to the number of days of the following year. First we have to divide the number of years obtained by 4 in order to know if we have to add one more day. This must be done if we obtain a remainder of 3[33]. Finally, we have to ascribe these days to the months of

[29] See Jaṭṭābī, 1986, p. 88.

[30] The reason is that the Islamic calendar is structured in cycles of 30 years, 19 of which have 354 days and 11 have 355.

[31] The first day of 661 is Wednesday.

[32] The 1st of Muḥarram of the year 660 H. corresponds to 26th November 1261 A.D. The 7th of Safar corresponds to the beginning of year 1262 A.D. and there are 317 days left until the end of the lunar year.

[33] The reason is that 1264 was bissextile and, therefore, after the third year passed of the cycle of four, beginning in 1262, we have to add a supplementary day.

the solar year beginning with January. When we finish with these days we obtain the day of the month of the corresponding solar year.

4.1.2 *al-Jādirī*

In the first chapter[34] he describes the Arabic year and defines it as lunar (*qamariyya*) with 354 1/5 1/6 days. Next, he gives the names of the months and then two verses, probably to help memorise them[35]:

محرم صفر ربيع إثنان   كذا جمادى رجب وبعده شعبان
ورمضان شوال وقعدة   وشهر ذي حجة أيضا له شأن

Then he says that odd months are *complete*, meaning that they have 30 days and that even months are *defective,* that is to say, they have 29 days except in the leap years when the last month, *dhū-l-ḥijja*, also has 30 days.

In the second chapter[36], the author gives different instructions for determining the *madkhal*. One of them is to convert the years into days by multiplying by the number of days in a year. Then, we add 5[37] and divide by seven. After that we take the remainder and begin with Sunday in order to determine the weekday which is the first day of *Muḥarram*.

Another way is to divide the years by 8. Then we take the remainder and apply this list of *abjad* letters to a cycle of eight years:

year	I	II	III	IV	V	VI	VII	VIII
letter	ز	د	ا	و	ج	ز	ه	ب
value	7	4	1	6	3	7	5	2
difference	+5	+4	+4	+5	+4	+4	+5	+4

[34] See Jaṭṭābī, 1986, pp. 100-101.

[35] The translation of these two verses is:

Muḥarram , ṣafar, rabīʿ there are two   and also *jumāda, rajab* and after it *shaʿbān*

Ramaḍān, shawāl and *qaʿda*   and the month of *dhū al-hijja* also has something to do.

[36] See Jaṭṭābī, 1986, pp. 101-102.

[37] This instruction has to do with the fact that the first day of the Islamic calendar was Friday (weekday 6).

The cycle of eight years described here to determine the *madkhal* for the beginning of the Islamic year is right only from 761 H. to 784 H. After this period the *madkhal* does not coincide for all the years of the cycle[38].

Finally the author gives another list of *abjad* letters to determine the first day of the other months of the Arabic year:

month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
letter	ا	ج	د	و	ز	ب	ج	ه	و	ا	ب	د
value	1	3	4	6	7	2	3	5	6	1	2	4
difference		+2	+1	+2	+1	+2	+1	+2	+1	+2	+1	+2

These values can be easily checked in any Islamic calendar[39].

This chapter includes a section, called *faṣl*[40], in which the author proposes a way to determine the *kabs*: whether the year is intercalary, meaning that it has 355 days, or not. The instructions are to divide the year by 30 and to take the remainder; we then multiply this remainder by 11/30 and when the result has a fraction between 1/4 and 9/10, the year will be *kabīsa*. This has to do with the structure of the Islamic calendar in which there are cycles of 30 years in which eleven are intercalary. There are several distributions that are different from these eleven years throughout the 30-year cycle[41].

The third chapter[42] describes the solar year[43] as having 365 1/4 days. According to the text the supplementary day is added in al-Andalus at the end of December, so this month has 32 days and the year is called a leap

[38] This is because Islamic years have 50 complete weeks and 4 or 5 days more depending on whether the year has 354 or 355 days. For instance, in the period 761-768 years 763, 766 and 768 have 355 days. We find this structure of 8-year cycle fully developed in earlier astronomers, for instance in al-Battānī's *zīj*. See Nallino, 1899-1907, II p. 7.

[39] This is also the result of the number of days of the months of the Islamic years: odd months have 29 days, that is 4 weeks and 1 day, and even months have 30 days or 4 weeks and 2 days more.

[40] See Jaṭṭābī, 1986, p. 102.

[41] See Van Dalen, 1998, p. 277; Ocaña, 1981, p. 31. The most popular is the one established by al-Battānī. See Nallino, 1899-1907, II, p. 7. This procedure is also explained in al-Bīrūnī's *Tafhīm*. See Wright, 1934, section 271, p. 163.

[42] See Jaṭṭābī, 1986, p. 104.

[43] Literally *al-sana al-šamsiyya,* although in the edition it appears as *al-šamiyya*.

year (*kabīsa*)[44]. The author also gives the names of the months and a mnemotechnical sentence to remember the number of days in each month:

فاز رجل ظفر بحج

This sentence is slightly different from the one given by Ibn al-Bannā'[45]. The letters with dots correspond to months with 31 days and the letters without dots to months with 30 with the exception of February which has 28 days.

In the fourth chapter[46] he explains how to calculate the weekday of the beginning of the solar year corresponding to any Islamic year by an arithmetical formula[47]:

$$(AH-786)+\frac{(AH-786)}{4} \qquad \text{or} \qquad (AH-790)+\frac{(AH-790)}{4}-2$$

The instructions are to discard the decimal fractions and to divide the result by 7. The remainder obtained has to be applied to the weekdays, beginning with Sunday, and this gives the *madkhal* for January of the equivalent solar year. If the division by four has no remainder, the year will be *kabīsa*.

The author then explains that this operation leads to *izdilāf*. As in Ibn al- Bannā''s text, al-Jādirī says that there is *izdilāf* when in the Arabic year there is no day corresponding to the first day of January. Finally he gives some years in which there is *izdilāf*. This occurs, according to him, in years 824, 857, 891 H. and 924 H.[48], and every 33 years. This value is

[44] This coincides with the explanations in the text by Ibn al-Bannā'.

[45] The translation would be: *A man obtained a victory by pilgrimage.*

[46] See Jaṭṭābī, 1986, pp. 104-105.

[47] These formulae are very similar to those given by Ibn al-Bannā' but here al-Jādirī is using the years 786 H./1384-85 A.D. or 790 H./1388 A.D., only a few years before the composition of the *Rawḍa* (1391 A.D.). The year 1384 is a leap year and 1385 begins on Sunday, which are the two elements required to work. The year 1388 is also a leap year and is also the year of the *izdilāf*.

[48] Years 824, 857 and 891 are correct but 924 H is dubious. January 1st 1519 corresponds to 29 Dhū-l-Ḥijja 924, which was an intercalary year (*kabīsa*). Since this last month had 30 days, the year of the *izdilāf* should be 925. See Ubieto, 1984, p. 91

more accurate than the one given by Ibn al Bannā' (32 years)[49]. The *madkhal* of the other months can be found from a list of *abjad* letters from January to December that coincides with the one given by Ibn al Bannā':

month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
letter	ا	د	د	ز	ب	ه	ز	ج	و	ا	د	و
value	1	4	4	7	2	5	7	3	6	1[50]	4	6

4.2 *Spherical astronomy*

4.2.1 *Ibn al-Bannā'*

The fourth chapter[51] describes the ecliptic and gives the names of the twelve zodiacal signs, divided into 30 degrees each. Here, the solstitial and equinoctial points are described and the values of the declinations for the end of each sign and the increments in right ascension between the beginning and end of the signs placed north of the equator are given

Sign	$\Delta\delta$	$\Delta\alpha$
Aries, Virgo	12°	28°
Taurus, Leo	8°	30°
Gemini, Cancer	4°	32°

These are approximate values. The exact formula to determine the right ascension is:

$$Sin\alpha = R\frac{Tan\delta}{Tan\varepsilon}$$ [52]

[49] As has been said, there is *izdilāf* every 33 or 34 Islamic years.

[50] The edition gives: ح (8).

[51] See Jaṭṭābī, 1986, p. 89.

The author says that in the other two quadrants the values are symmetrical. These values are rounded or truncated and imply an obliquity of the ecliptic of 24°[53].

He then gives a table of fixed stars with the names of 18 of them, their degree and zodiacal sign of mediation, their declination and whether they are northern or southern stars.[54]

Name	**Mediation Degree**	**Mediation Sign**	**Declination**	**Direction of declination**
Ra's al-gūl (1)	7	Taurus	39	North
al-Dabarān (2)	1	Gemini	15	North
Rijl al-Jawzā' (3)	11	Gemini	10	South
Mankib al-Jawzā'(5)	18	Gemini	6	North
al-ᶜAbūr (6)	3	Cancer	16	South
al-Gumayṣā' (7)	14	Cancer	6	North
ᶜUnq al-Shujāᶜ (-)	11	Leo	6	South
Qalb al-Asad (9)	20	Leo	15	North
al-Aᶜzal (12)	13	Libra	6	South
al-Rāmiḥ (13)	30	Libra	24	North
al-Fakka (14)	18	Scorpio	29	North
al-Ḥayya (-)	19	Scorpio	18	North
Qalb al-ᶜAqrab (15)	29	Scorpio	23	South
al-Hawwā' (-)[55]	17	Sagittarius	33	North

[52] See Martí&Viladrich, 1983, p. 90.

[53] The exact values for an obliquity of 24° would be:

Sign	$\Delta\delta$	$\Delta\alpha$
Aries, Virgo	11;44°	27;48°
Taurus, Leo	8;53°	29;54°
Gemini, Cancer	3;23°	32;18°

[54] See Jaṭṭābī, 1986, p. 90. This table has many points in common with the one attributed to Maslama's observations in 367H/978 A.D. following al-Battānī's method. There, it is also said that these stars are the ones usually included in the astrolabe. See Kunitzsch, 1966, p. 17 Type I. Between brackets I give the corresponding number in Maslama's aforementioned table. The values corresponding to mediation and declination are also similar although in our text they usually appear rounded or truncated with respect to the values given in Maslama's table. See Samsó, 2000, pp. 506-522.

al-Naṣr al-wāqiʿ[56] (16)	3	Capricorn	39	North
al-Naṣr al-ṭā'ir[57] (17)	17	Capricorn	7	North
al-Ridf (19)	3	Aquarius	43	North
Mankib al-Faras (20)	5	Pisces	29	North

The fifth chapter[58] gives the degree of the sun from the day of the year. The instructions given are to add 17 to the days obtained from the first of April. Therefore, it seems clear that the vernal equinox corresponds to the 14th of March. So we have to take 31 days for every northern sign and 30 days for every southern sign beginning by Aries[59]. The degree of the sign, corresponding to the end of the days previously obtained, is the degree of the sun corresponding to this day.[60]

An alternative way given to determine the degree of the sign, corresponding to any day, is provided by this set of *abjad* letters, ascribing a letter for each month of the solar year beginning with January:

Month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Sign	♑	♒	♓	♈	♉	♊	♋	♌	♍	♎	♏	♐
Letter	ز	ح	و	ز	و	و	ه	ه	د	د	ه	و
Value	7	8	6	7	6	6	5	5	4	4	5	6

To calculate the degree of the sun one must take the days passed of the month, then add the value of the corresponding letter, and then add ten. The number obtained corresponds to the degree of the sun. If the result is

55 In the edition: *al-hawwā*

56 In the edition: *al-mawāqiʿ*.

57 In the edition: *al-kitāb*.

58 See Jaṭṭābī, 1986, pp. 92-93.

59 The solar apogee is in the northern half and the sun apparently moves more slowly than in the southern half.

60 The equivalence of the equinox with the 14th March can be located in the 12th century. By applying the formula:

$$f = 10.5 +- \frac{1582 - y}{128.5}$$

where *f* is the day corresponding to the equinox and *y* the year corresponding to this date, we have that for f = 14 then y = 1132. See García Franco, 1945, pp. 126-127 and Martí&Viladrich, 1983, p. 16.

greater than 31 in the northern signs (30 in the southern ones)[61], the degree will correspond to the next sign[62].

The sixth chapter[63] is devoted to determining the declination of any degree calculated by linear interpolation. By multiplying the degree of the sign by the value of increment of declination ascribed to the sign and dividing by 30, and adding the result to the declination of the preceding sign, the complete value is obtained.

The right ascension is measured from the beginning of Capricorn. It is also calculated by linear interpolation beginning from Capricorn, adding the right ascension corresponding to every sign and the fraction corresponding to the degrees of the last one.

The seventh chapter[64] determines the meridian altitude from the latitude of the locality and the solar declination corresponding to this day according to the well known formula:

$$h_m = (90 - \varphi) + \delta$$

If the result is more than 90 degrees it must be subtracted from 180 and the meridian altitude will be the difference.

4.2.2 Al-Jādirī

The fifth chapter[65] is devoted to the lunar mansions and the zodiacal signs. The author begins by giving the names of the twelve zodiacal signs, which are divided into 30 degrees: six northern signs and six southern signs. He describes the equinoxes and the solstices and their equivalence to the months. Finally, he gives a list with the names of the lunar mansions.

[61] There is a gap in the Arabic edition of the text where, instead of: "31 in the northern ones and 30 in the southern ones", we only read: " 31 in the southern ones".

[62] For instance March 14th will correspond to 14 + 6 + 10 = 30 degrees of Pisces => Aries 0º. Therefore it would correspond to the vernal equinox. The summer solstice will correspond to June 14th : 14 + 6+ 10 = 30 => Cancer 0º. As for the autumnal equinox, it will correspond to September 16th: 16 + 4 + 10 = 30 => Libra 0º. Finally, the winter solstice will correspond to December 14th: 14 + 6 + 10 = 30 => Capricorn 0º.

[63] See Jaṭṭābī, 1986, pp. 92-93.

[64] See Jaṭṭābī, 1986, p. 102.

[65] See Jaṭṭābī, 1986, p. 105-106.

The sixth chapter[66] gives the equivalence between day of the month and degree of the corresponding sign. He adds to the day of the month 10 and the characteristic of the sign according to a list which is exactly the same as the one given by Ibn al-Bannā'[67]:

Month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Sign	♑	♒	♓	♈	♉	♊	♋	♌	♍	♎	♏	♐
Letter	ز	ح	و	ز	و	و	ه	ه	د	د	ه	و
Value	7	8	6	7	6	6	5	5	4	4	5	6

When the result is more than 30, the excess corresponds to the next sign. The inverse procedure gives the day from the degree of the sign.

In the seventh chapter[68] the declination of the sun is defined as the distance from the equator (in fact from *nuqṭat al-iʿtidāl,* the equinoctial point). The complete value is, according to the text, 24° in the beginning of Cancer to the north and the beginning of Capricorn to the south. Al-Jādirī adds that this value is only approximate because it does not have a constant value, since it varies as time passes, and that in his time was 23;30°. He then gives the increase in declination, $\Delta\delta$, for each sign:

Sign	Δδ
Aries	12°
Taurus	9°
Gemini	3°

He adds that this increase is the opposite in the next quadrant and it is symmetrical in the next two quadrants.

These values are slightly different from the ones given by Ibn al-Bannā'. Al-Jādirī also calculates the declination of a given degree of a sign by linear interpolation. Then he explains the inverse procedure.

[66] See Jaṭṭābī, 1986, p. 106-107.

[67] There is a variant in ms. 8796: زحو زوو د ههد هو (7-8-6-7-6-6-4-5-5-4-5-6).

[68] See Jaṭṭābī, 1986, p. 107-108.

The eighth chapter[69] gives the values of the differences in right ascensions between the beginning and the end of each sign, $\Delta\alpha$,:

Sign	$\Delta\alpha$
Pisces, Aries, Virgo, Libra	28°
Aquarius, Taurus, Leo, Scorpio	30°
Sagittarius, Capricorn, Cancer , Gemini	32°

These values are identical to the ones given by Ibn al-Bannā'. The author adds that when the declination increases the right ascension diminishes.

To calculate the right ascension of a particular degree he operates by interpolation. He also considers the first degree of Capricorn as the origin of ascensions, and then explains the inverse procedure.

He adds a section *(faṣl)* [70] in which he explains how to calculate the oblique ascensions. The origin is, according to him, the first degree of Aries. To obtain the value of the oblique ascension of a degree for a given latitude, we must calculate the difference between the right ascension of that degree and the length of half daylight, *AD/2,* corresponding to the same degree. In fact, the difference between the right and the oblique ascension equals the difference between 90 and half the daylight (this is called the equation of the daylight, e):

$$\alpha - \alpha_{\varphi} = \frac{AD}{2} - 90 = e$$

Since the author measures the right ascensions from the beginning of Capricorn instead of the beginning of Aries he will obtain α':

$$\alpha' = \alpha + 90$$

Therefore, the oblique ascension will be the difference between the right ascension of that degree and the length of half daylight corresponding to the same degree, as al-Jādirī explains:

[69] See Jaṭṭābī, 1986, p. 110.

[70] See Jaṭṭābī, 1986, p. 110-111.

$$\alpha_{\varphi} = \alpha' - \frac{AD}{2}$$

Then, the text indicates how to obtain the oblique ascension for each sign separately. Here the author introduces the concept of *faḍla*, *F*, which is the difference between the diurnal arc and 180º:

$$F = AD - 180^{\circ}$$

The instructions are to obtain the difference between half the *faḍla* (F/2= e) of each sign separately and the right ascension, for the ascending signs from Capricorn to Cancer, or the addition of these two values for the descending ones, from Cancer to Capricorn. The result will be the value of the ascension for every sign.[71] He also says that in his country the values are approximately[72]:

Sign	α_{φ}
Aries & Pisces	19;12°
Taurus & Aquarius	24;23°
Gemini & Capricorn	29;48°
Cancer & Sagittarius	34;12°
Leo & Scorpio	36;36°
Virgo & Libra	36;48°

Finally, he obtains the ascension of a given degree by linear interpolation.

[71] This instruction seems to be wrong because the value of the equation of daylight (half the *faḍla*) is a function of the declination and the latitude and, therefore, it is symmetrical on the northern and southern halves of the ecliptic. This difference (adding or subtracting) is the reason why instead of being symmetrical, the values given next for his latitude differ from one side to the other of the ecliptic. The exact formula to calculate the equation of daylight is:

$$\text{Sin } e = R \cdot \tan \varphi \cdot \tan \delta$$

[72] These values apply to a latitude of approximately 34º which may correspond to the city of Fez where the author was probably working. This city is mentioned elsewhere in the text.

The ninth chapter[73] describes how to calculate the latitude of a locality by applying the formula:

$$h_m - \delta = 90 - \varphi$$

which means that

$$\varphi = 90 - (h_m - \delta)$$

From this formula it is also clear, as he points out, that:

for $\delta = 0$	$\varphi = 90 - h_m$
for $h_m = 90$	$\varphi = \delta$
for $h_m > 90$	$\varphi = h_m + \delta - 90$ [74]

He explains in a section *(faṣl)* [75] how to determine the latitude from the stars that have neither rising nor setting in this latitude from the maximum and minimum altitudes of the star by applying the formula:

$$\varphi = \frac{h_{max} + h_{min}}{2}$$

In the tenth chapter[76] he determines the meridian altitude from the latitude and the declination applying the same formula as in the precedent chapter:

$$h_m = 90 - \varphi + \delta$$

4.3 *Shadows*

4.3.1 *Ibn al-Bannā'*

[73] See Jaṭṭābī, 1986, p. 111-112.

[74] In this case we have to calculate $h'_m = 180 - h_m = 90 - \varphi + \delta$.

[75] See Jaṭṭābī, 1986, p. 112.

[76] See Jaṭṭābī, 1986, p. 113.

The eighth chapter[77] begins stating that the value of the gnomon (*g*, here called *qāma*) is 12 fingers or 6 2/3 feet. These values were very common in al-Andalus and the Maghrib, and also appear in other works by Ibn al-Bannā'[78].

He then gives the values of the shadows in fingers for different values of the altitude of the sun: the first for 27 degrees, the second for 45 and the third for 63, and for the intermediate values. He also gives some arithmetical calculations to determine the Fingers of Extended Shadow (FES =12 cotan α) and the Fingers of Reversed Shadow (FRS=12 tan α)[79].

For h = 27 => FRS = 6[80]

For h = 45 => FES = FRS =12

For h= 63 => FES = 6[81]

$$\text{For } h < 27 \Rightarrow \text{FRS} = \frac{h}{4{,}5}$$

$$\text{For } 27 < h < 45 \Rightarrow \text{FES} = \frac{h-27}{3} + 6$$

$$\text{For } 45 < h < 63 \Rightarrow \text{FES} = \frac{63-h}{3} + 6$$

It is possible to find the equivalence between one shadow and the other. The factor of conversion is 144:

[77] See Jaṭṭābī, 1986, p. 94.

[78] The value 6 2/3 is found among others in al-Bīrūnī who ascribes it to Abū Maᶜshar. See Kennedy, 1976 vol. I p. 78 vol. II p.35; Calvo, 1989, pp. 30, 48 and Calvo, 1993 pp. 104 and 25 of the Arabic text.

[79] He is applying the formulae:

FES = g cotan h and FRS = g tan h

for a gnomon, g of 12 fingers.

[80] 12 tan 27 = 6.1.

[81] 12 cotan 63 =12 tan 27 = 6.1.

$$\frac{144}{FES} = FRS \quad \text{and} \quad \frac{144}{FRS} = FES$$

To obtain shadows in feet he uses the factor of conversion 5/9[82]. This value also appears in certain Andalusian and Maghribi treatises [83].

This chapter contains a section (*faṣl*) [84] in which the inverse procedures are described, that is to say, how to determine the altitude from the shadow. The factors of conversion are as follows:

$FRS < 6 \Rightarrow h = 4.5\ FRS$[85]

$FRS = 6 \Rightarrow h = 27$

$12 > FRS > 6 \Rightarrow h = 3\ (FRS - 6) + 27$[86]

4.3.2 *al-Jādirī*

In chapter 12[87] the author gives the value of the gnomon as $S_d = 12$ fingers (or digits) or $S_s = 8$ spans or $S_f = 6\ 2/3$ feet. The shadow can be extended or reversed. He gives some verses of an unknown author with indications how to convert from reversed shadow into an extended one giving approximate values, according to the factor of conversion of 144[88]:

إذا ظلك المنكوس عشرا وواحد   فمبسوطك الجيم المزاد على العشر
وإن كان عشرا فهو (يـد) ونصفه   وإن كان تسعا فهو ست مع العشر

[82] It comes from the relationship between 6 2/3 (= 20/3) and 12 $\Rightarrow \frac{20/3}{12} = \frac{20}{36} = \frac{5}{9}$.

[83] See Calvo, 1989, p. 30 and Calvo, 1993 p. 104. It can also be found in Latin texts. See Millás Vallicrosa, 1943-50, pp. 26, 129 and 148.

[84] See Jaṭṭābī, 1986, p. 95.

[85] It means that, between 1 and 6, *h* increases four degrees and a half for every finger of FRS.

[86] Which means that , between 6 and 12, *h* increases three degrees for every finger of FRS.

[87] See Jaṭṭābī, 1986, p. 114.

[88] It means that he is operating with fingers.

وثـامنه (يـح[89]) وللسبـع كافـهـا ونصف كمـا للست (كـد) بلا نكر
و(كـط) لـه (ها ولـو) لأربع و(مح[90]) لـه جيم و(عب) على الإثر

The indications are for different values of reversed shadows as follows:

FRS	FES
11	13
10	14.5
9	16
8	18
7	20.5
6	24
5	29
4	36
3	48
2	72

The author adds one more verse of his own:

(قمد) لــواحـــد و(رفح) لنصفـــه و(ثعو) إلى ربع فكن حاذقا وادر

With the following equivalences:

FRS	FES
1	144
1/2	288
1/4	576

He then gives some more rules to obtain the altitude from the shadow. For the reversed shadow:

[89] In the edition: يـج

[90] In the edition: يـج

If $S = \frac{1}{2}g$ then $h = 27$

If $S < \frac{1}{2}g$ then $h < 27 \Rightarrow h = S_d \cdot 4.5 = S_s \cdot 6.75 = S_f \cdot 8.1$

If $\frac{1}{2}g < S < g$ then $27 < h < 45 \Rightarrow h - 27 = 3(S_d - \frac{1}{2}g)$

$= 4.5(S_s - \frac{1}{2}g) = 5.4\,(S_f - \frac{1}{2}g)$

If $S = g$ then $h = 45$

For the extended shadow:

If $g - S < \frac{1}{2}g$ then $45 < h < 63 \Rightarrow h - 45 = (g - S_d) \cdot 3 =$

$= (g - S_s) \cdot 4.5 = (g - S_f) \cdot 5.4$

If $g - S = \frac{1}{2}g$ then $h = 63$

If $g - S > \frac{1}{2}g$ then $h > 63 \Rightarrow h - 63 = 4.5\,(\frac{1}{2}g - S_d) =$

$= 6.75\,(\frac{1}{2}g - S_s) = 8.1\,(\frac{1}{2}g - S_f)$

The author is describing a method that is an alternative to the trigonometric calculation of the altitude of the sun from the shadows by means of arithmetical formulae. But at the end of the chapter he also gives the trigonometric rule (see fig. 1):

$$\text{Sin}\,(h) = 60\,\sin\,(h) = 60\,\frac{g}{\sqrt{g^2 + s^2}}$$

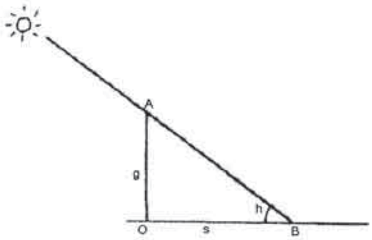

Fig. 1

In chapter 13[91] he determines the altitude of the sun or a star on a cloudy day by observing its reflection in a recipient holding water. The relations between the height of the eyes of the observer (n) and the distance of the observer from the recipient (d) equals the relation between the gnomon (g) and the shadow corresponding to the altitude of the star or the sun (s, see fig. 2):

$$\frac{g}{s} = \frac{n}{d}$$

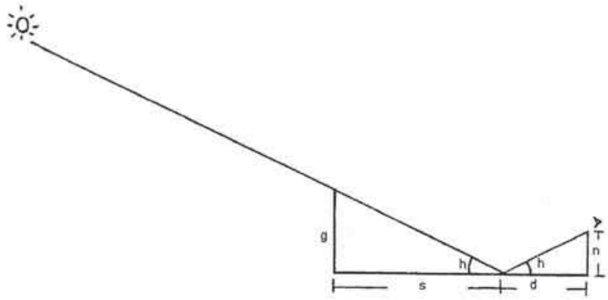

Fig. 2

Another possibility is to erect a column (g) higher than the observer (n) and to observe the star or the sun from the distance necessary (d) to see it as if it were at the top of the column. The shadow (s) equals the gnomon multiplied by the distance between the column and the observer (d) and divided by the difference in height between the column and the observer

[91] See Jaṭṭābī, 1986, pp. 117-118.

(g-n). From the value of the shadow (s) one can obtain the value of the altitude (h, see fig. 3):

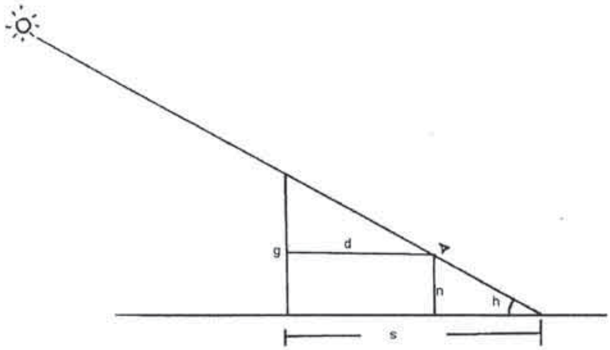

Fig. 3

$$\frac{s}{g} = \frac{d}{g-n} \qquad => \qquad s = \frac{g \cdot d}{g-n}$$

Chapter 14[92] determines the shadow from the altitude. The procedure is the reverse of the second part of chapter 12. At the end, as an alternative, the author gives the equivalent to the trigonometric formula:

$$ES = g \text{ cotan } (h)$$

He calculates sin (h), then cos (h), and, finally,

$$\frac{g \cos(h)}{\sin(h)}$$

which, as he indicates, equals the extended shadow.

In chapter 15 al-Jādirī explains how to change from one kind of shadow to another using these formulae:

$$S_f = \frac{5}{9} S_d = \frac{5}{6} S_s \qquad \text{and} \qquad S_s = \frac{2}{3} S_d$$

[92] See Jaṭṭābī, 1986, pp. 118-119.

Where S_f is the shadow in feet, S_d is the shadow in fingers and S_s is the shadow in spans.

4.3 Time reckoning

4.3.1 *Ibn al-Bannā'*

The ninth chapter[93] describes how to determine the diurnal and nocturnal arc of the sun or a star, and also the equinoctial hours in these two periods. Here the author gives an approximate formula equivalent to

$$AD = 180 + F$$

Where

$$F = \frac{11 \cdot 12 \tan(\varphi) \cdot \delta}{60}$$

The results obtained with this formula are very similar to those obtained with the exact one[94]. The exact formula for AD/2 = 90 + e, and e = F/2, is:

$$\text{Sin e}(\lambda) = 60 \sin e = 60 \cdot \tan \delta(\lambda) \cdot \tan \varphi$$ [95]

The last chapter[96] determines the time elapsed in hours since sunrise. The first step is to determine the shadow for this time in fingers. This can be done directly or by converting the shadow in feet to shadow in fingers. Then, his indication is to calculate

$$T = \frac{72}{S_h + 12 - S_{h_m}}$$ [97]

[93] See Jaṭṭābī, 1986, p. 96.

[94] The differences are between 4 and 5 minutes of time in the determination of the arc of daylight for a latitude of 34°.

[95] See King, 1973 Variorum IX p. 357 and Kennedy, 1976, vol. I pp. 251-253, vol. II pp. 157-158.

[96] See Jaṭṭābī, 1986, pp. 96-98.

[97] This formula can also be found in Qāsim b. Muṭarrif's *Kitāb al-Hay'a* although with some errors. See Samsó, 1992, pp. 68-69.

where S_h is the shadow corresponding to the solar altitude at any given time and S_{hm} is the shadow corresponding to the meridian altitude. This is exactly the formula given by al-Fazārī in his *zīj* and is the application to a gnomon of 12 divisions of the one mentioned above:

$$T = \frac{6n}{\Delta s + n}$$

where n is the length of the object and Δs represents the increase of the shadow over its midday minimum at T seasonal hours after sunrise or before sunset[98].

There is also a section *(faṣl)* [99] with the reverse procedure: how to obtain the shadow for the corresponding altitude when the hours are known. In this case the formula implied in the instructions is:

$$S_h = \frac{72}{T} + S_{h_m} - 12$$

The author then gives the different values of the shadow increase:

$$\Delta s = S_h - S_{h_m}$$

for every one of the seasonal hours of the day, T:

T	Δs
1 & 11	60 (= 5n)
2 & 10	24 (= 2n)
3 & 9	12 (= n)
4 & 8	6 (= n/2)
5 & 7	2 (≅ n/5)

[98] The exact formula in modern notation is:

$$T = \arcsin[\tan(\delta)\tan(\varphi)] + \arcsin\frac{\sin(h) - \sin(\delta)\sin(\varphi)}{\cos(\delta)\cos(\varphi)}$$

as a function of the altitude of the sun at that moment, h, the declination of the sun in that day, δ, and the latitude of the locality, φ.

[99] See Jaṭṭābī, 1986, p. 98.

There is a *fā'ida* (notice)[100] explaining how to obtain the meridian line in a place by using a compass and a *balāṭa*. The author indicates that one has to make the instrument oscillate until the meridian shadow, previously determined with the compass, will fit with the meridian line in the *balāṭa*. In this way it will become a *mizwala*, another name for a sundial[101].

Finally, there is another *fā'ida*[102] to determine the leap year without tables. The indications are to take the number of years exceeding 700[103] years since the *Hijra*, to add 3 and to divide by 4. If the remainder is zero it corresponds to a leap year; if the remainder is 1, 2, or 3, then we have the first, the second or the third year of the 4-year cycle corresponding to the solar calendar. He adds that the year 47 and 48 are not leap years and that the year 49 is a leap year[104].

4.4.2 *Al-Jādirī*

In chapter 16[105] the author explains how to determine the diurnal and nocturnal arc of the sun or a star by different methods from the equality:

$$AD = 180 + F$$

Where

$$F = 2e = AD - 180$$ [106]

[100] See Jaṭṭābī, 1986, pp. 98-99.

[101] See King, 1991, pp. 210-211.

[102] See Jaṭṭābī, 1986, p. 99.

[103] The edition gives the reading "900" but the author was probably working with the years of his century. The year 700 H. corresponds to 1300-1301 A.D. The year 1300 was a leap year and, therefore, by adding 3 to 1301 and dividing by four we obtain a remainder of zero.

[104] The editor adds a note that this treatise may have been written in the year 646 H/ 1248/49 A.D. when, according to him, the author was still very young. This does not seem possible since Ibn al-Bannā' was born in the year 654 H./ 1256 A.D. In any case, as stated in the text, while year 1252 A.D. (649 H.) was effectively a leap year, 1250 A.D. (647 H.) and 1251 A.D. (648 H.) were not.

[105] See Jaṭṭābī, 1986, pp. 120-121.

[106] As in the case of Ibn al-Bannā's text, he defines the *faḍla* as the difference between the diurnal arc and 180.

The first method is the same formula as the one given by Ibn al-Bannā':

$$F = \frac{S_f(90-\varphi)\cdot 1\,1\cdot\delta}{60} = 12\tan(\varphi)\cdot 0;1\,1\cdot\delta$$

Where S_f is the shadow in feet. Since it is the same formula, the results are again very near to the exact values for a latitude of 34 degrees.
The author gives another approximate formula:

$$F = \frac{\delta\cdot\varphi}{\varepsilon}$$

with results that are also very accurate, though not exact[107].

Finally, the author gives an approximate formula for the latitude of Fez:

$$F = \delta + \frac{\delta}{2} - 1$$

with results that are not bad but not as good as the preceding ones[108]. The author recognises that the preceding formulae are better.

Two other possibilities given by the author of the treatise to determine the diurnal arc are:

$$AD = \alpha_\varphi(\lambda+180) - \alpha_\varphi(\lambda)$$ [109]

$$AD = 2[\alpha'(\lambda) - \alpha_\varphi(\lambda)]$$ [110]

[107] For an obliquity of the ecliptic ε=23;30°, this formula works quite well for a latitude between 33;30° and 34;30°, especially at the solstices. For instance, in the solstices we have that F = φ, which almost perfectly matches a φ = 34°.

[108] In this case, for instance in the solstices, for a latitude φ = 34°, we have that F= 34;15° which is a good approximation.

[109] The oblique ascension determines the rising point of this degree. The oblique ascension of the opposite degree determines its rising point (corresponding to the setting point of the first one). Therefore, we have the arc of daylight for this degree.

[110] As has been said, the difference between the right and the oblique ascension corresponds to half the *faḍla* but, since he is measuring the right ascension from Capricorn, it corresponds in fact to half the arc of daylight:

Chapter 17[111] determines the number of equinoctial hours of daylight and the number of degrees in one seasonal hour. Since equinoctial hours always have 15 degrees, once we know the number of degrees of the arc of daylight we divide the figure by 15 and we obtain the number of equinoctial hours. Also, by dividing the *faḍla* by 15 and adding it to or subtracting it from 12 we can obtain the number of these hours.

As for the seasonal hours, they are the result of dividing the diurnal or nocturnal arc by 12. And we can also divide the *faḍla* by 12 and then we add to or subtract from 15 the result, to obtain the degrees of one seasonal hour. We then subtract this result from 15 and we obtain the difference between equinoctial and seasonal hours at that night.

Chapter 18[112] explains how to change from one kind of hour to the other. If we multiply the number of hours by the degrees of one hour we obtain the degrees corresponding to these hours. Then, we divide by the degrees of the other type of hour to obtain their number

$$q = \frac{s \cdot d}{15} \quad \text{or} \quad s = \frac{15 \cdot q}{d}$$

where s is the number of seasonal hours, d is the number of degrees of one seasonal hour and q is the number of the corresponding equinoctial hours[113].

Chapter 19[114] determines the time elapsed since sunrise from the shadow and the altitude. Al-Jādirī uses the aforementioned approximate formula of Indian origin:

$$T = \frac{6n}{\Delta s + n} \quad [1]$$

$$\alpha'(\lambda) = 90 + \alpha(\lambda) \Rightarrow \alpha'(\lambda) - \alpha_\varphi(\lambda) = 90 + [\alpha(\lambda) - \alpha_\varphi(\lambda)] = 90 + e$$

Doubling this result we have the complete arc of daylight.

[111] See Jaṭṭābī, 1986, pp. 122-123.

[112] See Jaṭṭābī, 1986, p. 123.

[113] These hours always have 15 degrees.

[114] See Jaṭṭābī, 1986, p. 123-124.

Where n is the length of the object and Δs represents the increase of the shadow over its midday minimum at T seasonal hours after sunrise or before sunset.

As an alternative way to determine equinoctial hours, he offers the approximate formula:

$$T = \frac{1}{15}\arcsin\left[\frac{\sin(h)}{\sin(h_m)}\right] \qquad [2]$$

There is a section *(faṣl)* [115] in which he explains the reverse procedure, that is to say, how to determine the shadow from the hour using formula [1] or how to find the altitude from it using formula [2].

He also gives the increase to the meridian shadow, Δs, to obtain the shadow corresponding to the hours of the day, which corresponds to the first formula with the exception of the 5th hour:

T	Δs
1	5n
2	2n
3	n
4	n/2
5	n/4[115]

Therefore, for a gnomon divided in fingers (n = 12), in spans or in feet he gives the values:

Shadows	T=1	T=2	T=3	T=4	T=5
Fingers (n = 12)	صـ (60)	كد (24)	يب (12)	و (6)	ج (3)
Spans (n = 8)	م (40)	يو (16)	ح (8)	د (4)	ب (2)

[115] See Jaṭṭābī, 1986, p. 124-125.

[116] According to the formula it should be n/5. This is used for the gnomon divided in fingers, usually in 12 divisions. Since 12 is divisible by 4 but not by 5, this is probably the reason for the change.

Feet (n = 6 2/3)	لج (33)	يج (13)	و (6)	ج (3)	ا (1)

In the last case (gnomon in feet) 1/3 has to be added to each value in order to obtain the shadow corresponding to these hours.

In chapter 20[117] the author describes the method to determine the moments of the *ẓuhr* and *ʿaṣr* prayers, by finding the corresponding shadows from the meridian shadow and the reverse procedure. The indications are the standard ones:

$$S_z = S_m + \frac{1}{4} n$$

$$S_a = S_m + n$$

Where S_z is the shadow corresponding to the moment of the *ẓuhr* prayer, S_a is the shadow of the *ʿaṣr* prayer, S_m is the meridian shadow and *n* is the length of the gnomon. The author says that these formulas follow the Imām Mālik's method.[118]

He then gives some approximate arithmetical formulae to determine the altitude of the sun at the moment of the prayers from the meridian altitude. But some lines are missing in the text at this point, and there is some confusion between the formulae for obtaining the altitude of the sun at the moment of the *ẓuhr* prayer and for obtaining its altitude for the beginning of the *ʿaṣr* prayer.

For the reverse procedure he also gives some formulae. To determine the meridian altitude, h_m, from the altitude at the moment of the *ẓuhr* prayer, h_z, he gives this formula:

$$h_m = h_z + \frac{1}{5} h_z$$

Which implies that:

$$h_z = \frac{5}{6} h_m$$

[117] See Jaṭṭābī, 1986, pp. 125-126.

[118] It seems that this statement is not true. See Kennedy, 1983, p. 302-303.

To determine the meridian altitude from the altitude at the moment of the beginning of the ʿ*aṣr* prayer, h_a, the instructions are to calculate:

$$h_m = 2h_a - \frac{1}{4}(90 - 2h_a)$$ [119]

To obtain the shadow for the end of the ʿ*aṣr* prayer, S_f, from the meridian shadow, S_m, he gives the equivalence:

$$S_f = S_m + 2n \qquad [1]$$

He adds that this formula corresponds to the one given by Abū Ḥanīfa to calculate the beginning of the ʿ*aṣr* prayer.[120] He also adds the opinions of other scholars such as Ašhab b. ʿAbd al-ʿAzīz al-Qaysī (204 H./ 819 A.D.), Ibn al-Mawāz (d. 269 H. / 882 A.D.) and Ibn Abī Zayd (d. 386 H. / 996 A.D.) as to whether there is continuity between the end of the *ẓuhr* prayer and the beginning of the ʿ*aṣr* prayer.

Finally he gives an arithmetical approximate formula to determine the altitude of the sun at the end of the ʿ*aṣr* prayer, h_f:

$$h_f = \frac{h_m}{4} + 5 \qquad \text{or} \qquad h_f = h_a - \frac{1}{6}h_a - \frac{1}{12}h_a \qquad [2]$$

The author says that it is better to operate with shadows[121].

[119] This means that the altitude of the sun for the beginning of the ᶜ*aṣr* prayer will be:

$$h_a = \frac{4h_m + 90}{10}$$

For instance, for a latitude φ= 34° and an obliquity ε=23;30°, in the summer solstice the meridian altitude will be h_m = 79;30° and the altitude for the beginning of the ᶜ*aṣr* prayer will be h_a = 40;48°.

[120] See Kennedy, 1983, p. 302.

[121] The first of these two formulae gives more similar results to the ones obtained by formula [1]. For instance, for a meridian altitude h_m = 79°, the altitude for the end of the ᶜ*aṣr* prayer according to the first of these two formulae [2] will be h_f = 24;45° and according to formula [1] it will be h_f = 24;30°.

Chapter 21[122] determines the times of nightfall and daybreak and the equivalence of both in degrees. The instructions are to calculate

$$T = \frac{6n}{48 - DES\{h_m(\lambda + 180)\}}$$

$$T = \frac{6n}{32 - SES\{h_m(\lambda + 180)\}}$$ [123]

$$T = \frac{6n}{26 - FES\{h_m(\lambda + 180)\}}$$ [124]

where DES is the shadow in fingers, SES is the shadow in spans and FES is the shadow in feet. From the instructions given it seems that he is applying the formula

$$\mathrm{T} = \frac{6n}{\Delta s + n}$$

to the diurnal arc of the nadir of the degree of the sun for an altitude of 18°[125]. The reason is that:

$$48 \cong 12 \text{ cotan } (18) + 12 = \text{DES } (18) + \text{n}$$

$$32 \cong 8 \text{ cotan } (18) + 8 = \text{SES } (18) + \text{n}$$

$$26 \cong 20/3 \text{ cotan } (18) + 20/3 = \text{FES } (18) + \text{n}$$

[122] See Jaṭṭābī, 1986, p. 126-128.

[123] In the edition $T = \frac{6n}{36 - SES\{h_m(\lambda + 180)\}}$

[124] In the edition $T = \frac{6n}{32 - SES\{h_m(\lambda + 180)\}}$

[125] This value can be found in other astronomical texts from al-Andalus and the Maghrib. See King, 1986, pp. 366-367; Goldstein,1977, pp. 97-100 and Calvo, 1993 p. 74.

The result is the time (measured in seasonal hours) of the end of nightfall[126]. The author also says that, subtracting this value from 12, we will obtain the time of the beginning of dawn.

As another possibility he gives the formula:

$$T = \frac{1}{15} arcSin \left[\frac{1112}{Sin(h_m)} \right]$$

Therefore the formula is:

$$T = \frac{1}{15} arcSin \left[60 \ \frac{Sin(h)}{Sin(h_m)} \right]$$

Where

$$60 \cdot \text{Sin}\,(h) = 1112$$

and, therefore, h =18°.[127]

He also gives an approximate method for the latitude of Fez:

For $\delta > 0$ (northern declination) $\quad T = \frac{1}{4}\delta + 22^\circ$

For $\delta < 0$ (southern declination) $\quad T = 22^\circ$

with the exception of the winter solstice, which is T = 23°[128].

In chapter 22[129] the author measures what he calls *darajat al-tawassuṭ*[130], that is to say, the degree of the ecliptic which crosses the

[126] This period is called *mudda*. The value obtained for an altitude of 18° for the opposite of the degree of the sun equals the time corresponding to the moment of the depression of the sun below the horizon for this degree of the sun.

[127] 60 Sin 18 = 1112.

[128] This is a simplification with very poor results. The exact formula for calculating the *mudda* in modern notation is:

$$m = \arccos \frac{\sin(h) - \sin(\delta)\sin(\varphi)}{\cos(\delta)\cos(\varphi)} - \arccos[-\tan(\delta)\tan(\varphi)]$$

[129] See Jaṭṭābī, 1986, p. 128-129.

meridian at sunset, at the end of twilight, at daybreak and at any other moment in the night. The procedure for sunset is to obtain

$$\lambda_s (\alpha'_s + D/2)$$

where α'_s is the right ascension of the solar degree of the day measured from Capricorn 0°, D/2 is the value of half daylight and λ_s is the ecliptic longitude corresponding to this right ascension. The degree of the ecliptic corresponding to it will be the *darajat al-tawassuṭ,* the degree crossing the meridian at sunset. For nightfall he calculates

$$\lambda_s (\alpha'_s + D/2 + m)$$

where *m* is the *mudda*[131].

For daybreak he calculates the degree of mediation as:

$$\lambda_s (\alpha'_{s'} + N/2 - m)$$

where N/2 is half the nocturnal arc and $\alpha'_{s'}$ is the right ascension of the *naẓīr* (opposite) of the degree of the sun.

In chapter 23[132] the author determines the lunar mansion corresponding to the degrees crossing the meridian line by using tables and his *Rawḍa.*

Chapter 24[133] determines the time elapsed of the night in seasonal hours from the mansion on the meridian line. To do this, the indication is to determine the degree of the ecliptic crossing the meridian line together with the aforementioned lunar mansion. We then take the difference between it and the degree of the ecliptic which crosses the meridian line at sunset, and then calculate the difference in right ascension between the two points of the ecliptic which cross the meridian line at the two

[130] Literally "degree of mediation".

[131] The *mudda* is a concept that appears in other works by Ibn al-Bannā'. See for instance Calvo, 1989 p. 25. There, it is defined as the period of time elapsed between sunset and nightfall and between daybreak and sunrise. Therefore, it can be measured in degrees of the equator, as a right ascension.

[132] See Jaṭṭābī, 1986, p. 129.

[133] See Jaṭṭābī, 1986, p. 129-130.

aforementioned moments and divide the result by the times of a nocturnal hour to obtain the time elapsed from sunset[134].

In chapter 25[135] al-Jādirī determines the altitude of a star at different times of the night. The instructions are to operate with the degree which crosses the meridian at the given times. It can also be determined from the right ascension of the arc corresponding to the hours elapsed.

Another procedure is to obtain the hour angle. There are different ways:

$$\text{For } t > 6 \quad d(6 - t) = H$$
$$\text{For } t < 6 \quad d(t - 6) = H$$

Where *t* are the seasonal hours corresponding to the time of the night, *d* are the degrees in each seasonal hour and H is the hour angle.

$$d = \frac{AD}{6}$$

Where AD is half the arc of night.

He then obtains the distance to the *maghrib* point and establishes whether the star is visible over the horizon.

Another procedure is to determine the shadow from the seasonal hours applying the formula

$$S(T) = \frac{6n}{T} + S_m - n$$

and then the altitude corresponding to this shadow.

In chapter 26[136] the author explains how to calculate the ascendant of a given moment, I', on a given day, from the time elapsed since sunrise or sunset, I.

The instructions are to obtain the oblique ascensions of the ascendant of that day from the degree of the sun, $\alpha_\varphi(I)$, and to add the arc of time elapsed since sunrise or sunset, d

$$\alpha_\varphi(I') = \alpha_\varphi(I) + d$$

[134] This procedure is described in Andalusī earlier texts. See Forcada, 1990 and Kunitzsch, 1994, p. 193.

[135] See Jaṭṭābī, 1986, pp. 130-131.

[136] See Jaṭṭābī, 1986, p. 131.

The result, α_φ *(I')*, is the oblique ascension for the given moment. Finally, the longitude corresponding to this value can be obtained.

4.5 *Trigonometry*

In chapter 11[137] Al-Jādirī includes several trigonometric functions, namely sines, cosines, versed sines and chords. The formulae involved are:

$$\text{Cos}\ \alpha = \text{Sin}\ (90-\alpha)$$

$$\text{Vers}\ \alpha = 60-\text{Cos}\ \alpha$$

$$\text{Chrd}\ \alpha = 2\text{Sin}\ (\alpha/2)$$

To calculate sines he uses linear interpolation. He assumes that

$$\text{Sin}\ \alpha = 60\ \sin\alpha$$

which means that since

$$\text{for}\ \lambda = 90° \Rightarrow \delta = 24°$$

therefore

$$\text{for}\ \lambda = 90° \Rightarrow \sin\lambda = 60^p = 2.5 \cdot \delta$$

He deduces that to obtain the sine of any longitude we have to take the corresponding declination and multiply it by 2.5 or 60/24. The results are only approximate but very near the exact values.

4.6 *The* Qibla

Al-Jādirī devotes the last chapter[138] of his treatise to determine the azimuth of the *qibla* for a given locality.

The instructions are given in a series of steps:

[137] See Jaṭṭābī, 1986, pp. 113-114.

[138] See Jaṭṭābī, 1986, pp. 132-134.

$$Sin\ \theta_1 = \frac{Cos\varphi_M \cdot Sin\Delta\lambda}{60} \qquad [1]$$

Where φ_M is the latitude of Mecca and $\Delta\lambda$ is the difference of longitudes between Mecca and the locality. Al-Jādirī calls θ_1 *al-ʿamūd,* "the perpendicular".

$$\text{Cos } \theta_1 = \text{Sin } (90 - \theta_1)$$

Cos θ_1 is called *al-imām*, "the guide".

$$\text{Sin } \theta_2 = 60 \cdot \text{Sin } \varphi_M / \text{Cos } \theta_1 \qquad [2]$$

θ_2 is called *buʿd ʿan da'irat muʿaddil an-nahār* "the distance from the equator".

$$(90 - \theta_3) = (90 - \varphi_L) + \theta_2 \qquad [3]$$

where φ_L is the latitude of the locality.

$$[\theta_3 = \varphi + \theta_2]$$

$$\text{Cos } \theta_3 = \text{Sin } (90 - \theta_3) \qquad [4]$$

$$\text{Sin } \theta'_4 = (\text{Cos } \theta_3 \cdot \text{Cos } \theta_1)/60\ [= \text{Sin } (90 - \theta_4) = \text{Cos } \theta_4]$$

$$\Rightarrow \theta_4 = (90 - \theta'_4) \qquad [5]$$

θ_4 is called *al-buʿd bayna samt ru'ūs baladi-ka wa samt ru'ūs ahl Makka* "the difference between the zenith of your locality and the zenith of Mecca".

$$\text{Sin q} = \text{Sin}\theta_1 \cdot 60/\text{Sin}\theta_4 \qquad [6]$$

Where q, measured from the South, would be the *inḥirāf al-qibla*[139]. Another way of expressing this value would be

[139] See King, 1993-95, p. 1090.

$$q' = 90 - q$$

Therefore, q' measured from the East, would be the *samt al-qibla* [140].

The text adds that when the latitude of the locality is greater than the latitude of Mecca and the longitude of Mecca is greater, then the azimuth is south-eastern. The azimuth is north-eastern when the latitude of the locality is lower than the latitude of Mecca and the longitude of Mecca is greater. Finally, when the longitude of Mecca is lower, then the azimuth is north-western.

Al-Jādirī adds that the distance between the zenith of Mecca and the horizon of the locality is

$$\theta'_4 = (90 - \theta_4)$$

He finally indicates that

$$D = \theta_4 \cdot 66\ 2/3$$

Where *D* is the distance in miles between the two localities[141].

Although the author does not give further explanations, the method described to determine the azimuth of the *qibla* corresponds to an exact method, which al-Bīrūnī called *method of the zījes (al-ṭarīq al-mustaʿmal fī'l-zījāt)*[142], and which appeared for the first time in al-Andalus in the 11th century A.D. in the *zīj* of Ibn Muʿādh al-Jayyānī.[143] It is ascribed to Ibn Muʿādh in the anonymous edition of Ibn Isḥāq's *zīj* extant in the

[140] See King, 1993-95, p. 1091

[141] This factor of conversion is very common in Islamic astronomy, Khwārizmī (fl. C. 800), Ibn al-Haytham (c.965-1039) and Abū-l-Fidā' (d. 1331), among others, used this Ptolemaic value. See King, 2000 pp. 230-231. It is also found very often in Andalusian treatises on these topics. See Calvo, 1993 p. 82.

[142] Some other astronomers who mentioned this method were Ḥabash (fl. 850), the first known author to have proposed it explicitly, Abū-l-Wafā' (994-997), al-Kūhī (fl. 988), al-Bīrūnī (973-1048), Kushyār b. Labbān (c. 971-1029), Ibn Yūnus (1009) and Ibn al-Haytham (c. 965-1039). See Bulgakov, 1962, pp. 206-215; Berggren, 1981, pp. 237-245; Berggren 1985, pp. 1-16; Kennedy,1973, pp. 128-130, Debarnot, 1985, pp. 50, 102, 152-257; King, 1986a, pp. 82-149; King, 2004, p. 762 and Samsó, 1992, pp. 164-166.

[143] See Samsó, 1992 p. 164; Samsó, 1994 pp. 9-17; King, 1986b, pp. 88-89 and Samsó & Mielgo, 1994, VI p. 7.

Hyderabad manuscript[144]. The terminology used by Al-Jādirī also seems to be derived from Ibn Muʿādh.[145]

5 Concluding remarks

As we have seen, the aim of these two treatises is to give a wide range of arithmetical procedures for the most frequent calculations related to *mīqāt* matters. The authors are not particularly interested in giving precise values. For instance, both give the value of the obliquity of the ecliptic $\varepsilon=24°$ although al-Jādirī adds that this value is approximate because it does not have a constant value.

But we find the determination of the *qibla* using an exact method instead of another approximate method that was very popular throughout the Islamic world, the one in al-Battānī's *zīj*. The explanations are simple, but the underlying theory is occasionally more complex.

6 Appendix

6.1 *Titles of chapters in Ibn al-Bannā's Kitāb fī ʿilm al-awqāt bi-l-ḥisāb*

1: Determination of the solar year
2: Determination of the weekday at the beginning of the solar year
3: Determination of the month of the solar year and the present day
4: Determination of the zodiacal signs, declination, ascensions, mediation
5: Determination of the solar longitude
6: Determination of the solar declination
7: Determination of the meridian altitude of the sun
8: Determination of the shadows in fingers and in feet from the altitude
Faṣl: Determination of the altitude from the shadows
9: Determination of the diurnal and nocturnal arc of the sun and the stars and how to calculate the equinoctial hours of the day and night.

[144] See Mestres, 1996, p. 408.

[145] The terminology in Ibn Muʿādh is as follows:
θ_1: *al-faḍla al-ṭūliyya /al- ʿamūd*
θ_2: *al-buʿd min muʿaddil al-nahār*
θ_3: *buʿd al-balad*
θ_4: *masāfa mā bayna baladi-ka wa Makka*

10: Determination of the time elapsed (in hours) since sunrise (from the shadow).

Faṣl: how to calculate the shadow in fingers from the hour.

Fā'ida 1: how to determine the meridian line

Fā'ida 2: how to calculate the leap year (*kabīsa)* without tables.

6.2 *Titles of chapters in al-Jādirī's Iqtiṭāf al-anwār min rawḍat al-azhār*

1: Determination of the days and months of the Arabic year
2: Determination of the beginning of the Arabic year and its months
3: Determination of the days and months of the solar year
4: Determination of the months of the solar year
5: Determination of mansions and zodiacal signs
6: Determination of the place of the sun in the signs
7: Determination of the declination of the sun or of a degree of the ecliptic
8: Determination of right and oblique ascensions of a degree
9: Determination of the latitude of any locality
10 Determination of the meridian altitude of the sun or a star
11: Determination of the sine of the altitude, the cosine and the altitude from these two values.
12: Determination of the altitude from the shadow
13: Determination of the altitude of a star in a cloudy day
14: Determination of the shadow from the altitude
15: Determination of the equivalence between shadows measured in fingers and shadows measured in feet
16: Determination of the diurnal and nocturnal arc of the sun and the stars.
17: Determination of the equinoctial hours in a day and night and the number of degrees in a seasonal hour in a given day.
18: Determination of how to change from one type of hour to another
19: Determination of the hours elapsed in a day from the shadow and the altitude
20: Determination of the time of the *ẓuhr* and *'aṣr* prayers from the meridian shadow and the altitude
21: Determination of the time of the nightfall and the daybreak and the degrees of both
22: Determination of the degree of mediation for sunset, nightfall and daybreak and the other parts of the night
23: Determination of the mansion of mediation for these parts of the night
24: Determination of the time elapsed of the night from the seasonal hour
25: Determination of the altitude of a star for these moments of the night

26: Determination of the ascendant in the day and night
27: Determination of the azimuth of the *qibla*

6.3 *Abridged reference*

Topic	Ibn al-Bannā'	Al-Jādirī
Questions of the calendar	Chapters 1-3	Chapters 1-4
Spherical astronomy	Chapters 4-7	Chapters 5-10
Trigonometry		Chapter 11
Shadows	Chapter 8	Chapters 12-15
Times of day and night	Chapters 9-10	Chapters 16-26
Qibla		Chapter 27

7 Acknowledgements

This paper was written with the support of the research project "Ciencia y Sociedad en el Mediterraneo Occidental en la Baja Edad Media" sponsored by the Spanish Ministerio de Educación y Ciencia (HUM2004-02511) and FEDER.

I would like to express my gratitude to Julio Samsó for his helpful suggestions.

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