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On the Truncated-Newton Approach for the Hydropower Generation Management

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In the paper we present the motivation for using the Truncated Newton methodology when obtaining the superbasic stepdirection in an algorithm that maximise the hydropower generation in a multireservoir, multiperiod power system. The decision variables are the water to be released from and stored in each reservoir in each time period over a given time horizon. The function that relates the hydropower generation with the decision variables has a special structure -- that allows to use second-order information without requiring too-much computer storage and - time-consuming.

Keywords: NON-LINEAR PROGRAMMING. NUMERICAL ALGORITHMS.
ELECTRIC POWER GENERATION.

1. INTRODUCTION.

PROBLEM DESCRIPTION.

This paper reports the motivation for using second-order information about the objective function of a nonlinear network flow problem so that the computer storage and time-consuming are affordable even for very-large cases. The problem consists in the maximization of the hydropower generated along a time horizon (usually, one year) by a multireservoir power system. The decision variables are the amount of water to be released from and stored in each of the interconnected reservoirs in each time period (usually, a week). The - constraints are of two kinds: linear equations to ensure flow balance and bounds on - the variables.

The purpose of the bounds on the variables - is threefold. First, to ensure that the water released serves the flood control, irrigation and navegational purposes. Second, to ensure that the amount of water released --- from a given reservoir to any of its directly downstream reservoirs must not exceed its canal capacity, so that the overflow is not used for producing electricity. Third, to penalise the amount of water stored that exceeds a safety capacity in a given reservoir.

The constraints form a linear system that can be represented by a special direct graph whose nodes and arcs correspond to the state of the reservoirs in each time period and the - decision variables, respectively.

Assuming a multireservoir system of 40 reservoirs and a planning horizon of 52 time periods, the graph has over 2000 nodes and 4000 arcs. The dimensions of the problem are affordable for linear objective functions given -- the current state-of-the art of special data structures for storing and updating the network (see /5/, /13/, /14/, /15/ among others); a sounding nonlinear network algorithm must - require a computing time with the same order of magnitude of the linear ones /7/.

Recently, new algorithms have been designed - for specializing linear primal data structures to nonlinear network flow problems; see /1/, /8/, /17/, /18/, /20/ and related references. Apart from recent developments (see - /2/, /3/, /7/, /8/ among others), the methods use a linearized subproblem to generate stepdirections so that the reduced gradient methodology is used in the basic-nonbasic environ-

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- Article rebut el Maig del 1983.

ment /19/ or in the more efficient basic-superbasic-nonbasic environment /16/. The convergence is very poor since no second-order information about the objective function is used. The current methods that use the ----- Hassian matrix are designed for solving medium scale problems.

This work proposes to use the Truncated Newton method to solve the hydropower generation problem. It was introduced in /9/ for general unconstrained nonlinear problems, extended in /10/ to linearly constrained problems, and specialized in /8/ to network flow problems. The new algorithm (whose details and computational experience with real-life cases are reported elsewhere /11/) uses second-order information in a very large scale network flow problem, given the special structure of both the objective function and constraints.

The paper is organized as follows. Section 2 presents the network flow formulation of the reservoirs management problem. Section 3 describes the hydropower generation function. Section 4 summarises the variable-reduction environment of the algorithm. Section 5 presents the approach for obtaining the superbasic stepdirection in the hydropower generation problem and introduces the new concept of independent superbasic sets. Section 6 describes the de-activating process and introduces the new concept of independent sets of candidate nonbasic arcs to be de-activated. And finally, Section 7 outlines some topics for future research and experimentation.

2. PROBLEM FORMULATION.

Let J denote the set of reservoirs, T the time periods set, $W \subset J$ the water storage reservoirs (and, then, J/W is the set of run-of-river plants), $E \subset J$ the set of reservoirs that are not used for hydropower generation, P_j (Q_j) the set of reservoirs directly upstream (downstream) from reservoir j. The decision variables are denoted t_{tji}, the amount of water released from reservoir j to reservoir i in period t and s_{tj} the amount of water stored in reservoir k at the beginning of time period t. The flow balance equations are

$$-\sum_{i\in P_j} r_{tij} - s_{tj} + \sum_{i\in Q_j} r_{tji} + s_{t+1,j} = b_{tj} \qquad \forall t\in T, \quad j\in J \tag{2.1}$$

where b_{tj} is the next exogenous inflow to reservoir j in time period t, and s_{1j} and $s_{53,j}$ are fixed.

Letting X be a vector of all decision variables and b a vector of the exogenuous inflows, system (2.1) can be written

$$AX = b \tag{2.2}$$

where A is the node-arc incidence matrix. For $|Q_j|=1$ this network is refered to as a <u>temporally expanded arborescence</u> /18/. Since in our case, it is not excluded, although it is not frequent, that $|Q_j|>1$ let us term it, temporally expanded <u>quasi</u>-arborescence. Each column of A corresponds to an arc (whose flow is r_{tji} or s_{tj}) and each row to a node for each pair (period, reservoir). The nonzero elements in a column are +1 in the row corresponding to the node where the arc originates and -1 in the row where the arc terminates. In addition, there is a <u>root</u> node which represents the Exit in all time periods for the reservoirs $j\in J$ such that $\exists i\in Q_j$, $i = |NJ|+1$.

The bounds on water released and stored are given by

$$l_{ji} \le r_{tji} \le u_{ji} \qquad \forall i\in Q_j, \quad j\in J \tag{2.3a}$$

$$m_{tj} \le s_{tj} \le M_{tj} \qquad \forall t\in T, \quad j\in W \tag{2.3b}$$

such that usually, $m_{tj}=m_j$ and $M_{tj}=M_j$ $\forall t\in T$, where m_j and M_j are constant values, being $m_{1j}=M_{1j}$ $\forall j\in W$.

Following a traditional approach /16/, matrix A can be partitioned as follows.

$$A = (B \; S \; N)$$

where the columns of B form a basis and correspond to the basic arcs, and the columns of S and N correspond to the superbasic and nonbasic arcs, respectively; let $\bar{B}$, $\bar{S}$ and $\bar{N}$ denote the related <u>basic</u>, <u>superbasic</u> and <u>nonbasic</u> sets of arcs. Nonbasic arcs are temporarilly fixed to their bounds, and the flow in sets $\bar{B}$ and $\bar{S}$ vary between their bounds.

By construction of A it can be shown

(1) Any basis B may be ordered such that it is upper triangular /6/.

(2) The arcs corresponding to columns in any basis form a spanning tree of the net--- work /5/.

(3) A maximal basis spanning tree for a gi-- ven feasible solution avoids a basic-su- perbasic degenerate pivot /8/; otherwise, null steps are more frequent than in pro blems with a general structure.

(4) Let Z be the variable-reduction matrix - whose columns form a basis for the null space of A, given AZ=0, such that /16/

$$Z = \begin{pmatrix} -B^{-1}S \\ I \\ 0 \end{pmatrix} \begin{matrix} \}\ n \\ \}\ a-n \\ \end{matrix} \qquad (2.5)$$

where n denotes the number of nodes --- without including the root and a denotes the number of arcs. Let Basic-Equiva-- lent Path (BEP) define the unique path - in the basis spanning tree that leads -- from originating node, say i_k in superba sic or nonbasic arc k to determinating - node, say j_k; let β_k denote the set of - arcs in the BEP of arc k. Arc $k' \epsilon \beta_k$ -- has a forward orientation in the BEP of arc k if $p(i_{k'})_k = j_{k'}$, where $p(.)_k$ is - the predecessor of arc (.) in the BEP - of k; it has a reverse orientation if -- $p(j_{k'})_k = i_{k'}$. Let ρ_k denote the column - $(B^{-1}S)_k$; in any case, ρ_k is not expli-- citly stored. The nonzero elements of ρ_k for $k \epsilon \bar{S} \cup \bar{N}$ are +1 for a forward orienta-- tion and -1 for a reverse orientation. - The inexpensive implicit computation of matrix Z (2.5), via specialized network data structures as in /18/, and the ---- structure of the Hessian matrix G (3.5) (see below), makes affordable the using of second-order information in this very large-scale problem.

(5) The temporally expanded arborescence net- works have the property that a-n=n; note that, usually, a-n>>n in a general net- work. As a result, $|\beta_k|$ is very small. - Any time segment of our network is a qua si-arborescence and, then, $|\beta_k|$ is still relatively small; this property will be exploited in the related algorithm.

3. THE HYDROPOWER GENERATION FUNCTION.

The important property of the hydropower ge- neration function is spatial and temporal qua- si-separability. See /2/, /7/, /8/ for separable functions; /8/ for temporal separable and spa tial nonseparable fuctions in very large-sca- le problems. The hydropower generated in re-- servoir j at time period t can be expressed as follows.

$$h_{tj} = K_{tj} \sum_{i \epsilon Qj} r_{tji} \qquad \forall t \epsilon T, \quad j \epsilon J/E \qquad (3.1)$$

where K_{tj} may be constant or a nonlinear func tion $f_{n\ell}$ of the s-variables for each reser--- voir in set J/E. Let j be termed linear reser- voir (and, then, (t,j,i) is a pure linear arc) if $K_{tj}=K_j$, where K_j is a constant; otherwise, j is termed nonlinear reservoir such that --- function K_{tj} can be written

$$K_{tj} = f_{n\ell}((c_{tj}+c_{t+1,j})/2) \qquad (3.2)$$

where c_{tj} is the water level in reservoir j at the beginning of time period t, such that

$$c_{tj} = f_{n\ell}(s_{tj}) \qquad (3.3)$$

Let PL denote the set of pure linear arcs --- (t,j,i), VL the set of linear arcs (t,j,i) -- with variable coefficients, and NL the set of nonlinear arcs (t,j). Arc $(t,j,i) \epsilon PL$ if ----- $K_{tj}=K_j$. Arc $(t,j,i) \epsilon VL$ if K_{tj} is a nonlinear function given by (3.2)-(3.3); note that h_{tj} is a linear function of $\sum_i r_{tji}$ if s_{tj} and --- $s_{t+1,j}$ are fixed. Arc $(t,j) \epsilon NL$ if K_{tj} is gi-- ven by (3.2)-(3.3), since s_{tj} and $s_{t+1,j}$ are nonlinear variables.

Without penalizing any amount of water stored or released, the objective can be written

$$\max \sum_{t \epsilon T} \sum_{j \epsilon J/E} h_{tj} \qquad (3.4)$$

Let g_{tji} for $(t,j,i) \epsilon PL \cup VL$ and g_{tj} for ------ $(t,j) \epsilon NL$ denote the gradient elements rela-- ted to the r-variables and s-variables, res-- pectively. Note that the gradient related to set PL is constant, $g_{tj}=0$ and $g_{tji}=0$ $\forall j \epsilon E$, - and g_{tji} for $(t,j,i) \epsilon VL$ does not change for - fixed values of s_{tj} and $s_{t+1,j}$. The Hessian - matrix G has the form

$$G = \begin{array}{c|c|c|c}
 & PL & VL & NL \\ \hline
PL & G_1=0 & 0 & 0 \\
VL & 0 & G_2=0 & G_3 \\
NL & 0 & G_3^t & G_4
\end{array} \qquad (3.5)$$

such that G_3 is a two-diagonal mateix for -- $|Q_j|=1 \ \forall j\epsilon J$ and G_4 is a symmetric tri-diagonal matrix; see an example in form (3.6).

$$G_3 = \begin{array}{c|ccccc}
 & s_{31} & s_{41} & s_{51} & s_{61} & s_{71} \\ \hline
r_{312} & X & X & & & \\
r_{412} & & X & X & & \\
r_{512} & & & X & X & \\
r_{612} & & & & X & X \\
r_{712} & & & & & X
\end{array} \qquad (3.6a)$$

Note that the off diagonal elements in a given column of matrix G_3 have the same value for $|Q_j|>1$, $j\epsilon W/E$

$$G_4 = \begin{array}{c|ccccc}
 & s_{31} & s_{41} & s_{51} & s_{61} & s_{71} \\ \hline
s_{31} & X & X & & & \\
s_{41} & X & X & X & & \\
s_{51} & & X & X & X & \\
s_{61} & & & X & X & X \\
s_{71} & & & & X & X
\end{array} \qquad (3.6b)$$

Let R_{ji} denote the power generation capacity of canal (j,i) for $j\epsilon J/E$, and T_{tj} a safety - upper bound on the amount of water to be stored in reservoir j for $j\epsilon W$ at the time period t; note that upper bound u_{ji} may be regarded as the maximum physical capacity of the segment (j,i) in the river. Now,since water --- overflow $r_{tji}-R_{ji}$ (where $\ell_{ji}\leq R_{ji}\leq u_{ji}$) cannot be used for hydropower generation, and it -- could be interesting to penalise the excess of water stored $s_{tj}-T_{tj}$, the objective can - be expressed by (3.7) instead of using (3.1) and (3.4).

$$\max \{\sum_{t\epsilon T}\sum_{j\epsilon J/E} h_{tj} - \sum_{t\epsilon T}\sum_{j\epsilon W} P_{tj}\max\{0,s_{tj}-T_{tj}\}\} \qquad (3.7a)$$

where

$$h_{tj} = K_{tj}\sum_{i\epsilon Q_j}\min\{r_{tji},R_{ji}\} \qquad (3.7b)$$

and P_{tj} is the unit penalty for excess on water stored. The <u>nondifferentiability</u> introduced by (3.7) can be treated without great -- difficulty (see Section 6).

4. SKELETAL ALGORITHM FOR OBTAINING FEASIBLE-ASCENT SOLUTIONS.

Let d define the stepdirection from feasible solution, say $\bar{X}$ such that the new iterate -- can be expressed

$$X:=\bar{X}+\alpha d \qquad (4.1)$$

where scalar α is the steplength. Given eqs. (2.2) and matrix partition (2.5), by linearity it results

$$(B\ S\ N)\begin{pmatrix} d_B \\ d_S \\ d_N \end{pmatrix} = 0 \qquad (4.2)$$

being $d=(d_B^t,d_S^t,d_N^t)^t$. The basic stepdirection d_B is used to satisfy the constraints system (2.2), the elements of nonbasic stepdirection d_N, say d_k for $k\epsilon\bar{N}$ are temporarilly fixed to - zero, and the superbasic stepdirection d_S is used to maximise the objective function ---- (3.7).

At each iteration, the problem then becomes determining vector d and scalar α, such that αd is feasible and <u>ascent enough</u>, and the algorithm is globally and, if possible, Q-su-- perlinearly convergent. Direction d is feasible if system (4.2) is satisfied. Since $d_N=0$ and d_S is allowed to be free, it results

$$d_B = -B^{-1}Sd_S \qquad (4.3)$$

such that

$$d = Zd_S \qquad (4.4)$$

The ascent enough stepdirection d_S can be obtained by 'solving' the problem

$$\max\{h^td_S + 1/2d_S^tHd_S\} \qquad (4.5)$$

where the reduced gradient h and the reduced Hessian H can be written

$$h\equiv Z^tg = g_S - S^t\mu_B \qquad (4.6)$$

$$H\equiv Z^tGZ \qquad (4.7)$$

such that the <u>basic estimation</u> μ_B of the --- constraints Lagrange multipliers solves the system

$$g_B = B^t\mu_B \qquad (4.8)$$

and $g = (g_B^t, g_S^t, g_N^t)^t$. Note that the solution of problem (4.5) and, then, the solution d_S of system

$$Hd_S = -h \tag{4.9}$$

is feasible-ascent for a positive definite - matrix -H and a maximal basis spanning tree.

Solving the n-system (4.8) when the arcs --- corresponding to the columns of B form a --- spanning tree does not need a great computational effort /5/, but the LP simple rules - for updating μ_B do not apply when the objective function is nonlinear (even if basic -- set $\bar{B}$ does not change). From other point of view, using (4.8) in (4.6) is computationally advantageous, since (1) a-n=n for any --- arborescence tree and, then $|\beta_k|$ is small -- for $k \in \bar{S} \cup \bar{N}$, (2) the cardinality of the set to be used while iteratively solving problem -- (4.5) is much smaller than a-n, and (3) β_k - must be used, in any case, for obtaining the upper bound α_m of the steplength α. Then, it can be written /18/

$$h_k = g_k - \sum_{k' \in \beta_k} \rho_{k'k} g_{k'} \qquad \forall k \in \bar{S} \tag{4.10}$$

Matrix H is likely very dense even for sparse matrices Z and G. Since our problem is very large, we cannot afford to use matrix H,- nor any of its approximations suggested in - the literature. We suggest to use the Truncated Newton method at independent series of - iterations (see Section 5), such that matrix H does not need to be stored and the computer time and storage are within affordable - limits. Note that system (4.9) is not needed to be completely solved at every iteration - for getting, under mild conditions, a Q-superlinear rate of convergence /9/.

The steplength α must be feasible and ascent enough. Being d ascent, a feasible α must be such that $0 < \alpha \leq \alpha_m$, where α_m is the upper bound for keeping feasibility; α_m is obtained by - analysing the sign of each element d_k and -- the related active bound, say ab_k in the direction of the sign for $\forall k \in \bar{B} \cup \bar{S}$. Let $\underline{a}_k$, $\bar{a}_k$ - and $\bar{\underline{a}}_k$ denote the lower bound (l_{ji} or m_{tj}), 'intermediate' bound (R_{ji} or T_{tj}, see Section 3) and upper bound (u_{ji} or M_{tj}) of the feasible flow in arc k, respectively. The active - bound ab_k is obtained as follows.

(1) If $d_k < 0 \wedge \bar{X}_k \leq \bar{a}_k$, $ab_k := \underline{a}_k$

(2) If $d_k < 0 \wedge \bar{X}_k > a_k$, $ab_k := \bar{a}_k$

(3) If $d_k > 0 \wedge \bar{X}_k < \bar{a}_k$, $ab_k := \bar{a}_k$

(4) If $d_k > 0 \wedge \bar{X}_k \geq \bar{a}_k$, $ab_k := \bar{\underline{a}}_k$

After obtaining an ascent enough steplength α at the current iteration (here, termed major iteration), say ℓ, a new iterate ----- $X := \bar{X} + \alpha d$ is obtained and, theoretically, the algorithm continues till $\|h\| = 0$ or the superbasic set is empty and, then, the de-activating process is executed; the Lagrange multipliers if the solution is optimal or their estimates if the solution is quasi-optimal - are used for selecting the nonbasic arc to - be deactivated (see Section 6).

While maximizing in a given manifold, it is possible that either a basic or a superbasic arc strikes a bound during the search. If a superbasic arc strikes a bound then it becomes nonbasic, the cardinality of the basic-- superbasic set on is reduced by one, and the search continues. If a basic arc strikes a - bound then it is exchanged with an appropriete superbasic arc and the resulting new superbasic arc is made nonbasic. Note that the related pivoting and, then, the new (maximal) basis spanning tree may be easily obtained by using LP special data structures without any matrix manipulation; we use the data structures described in /18/.

5. OBTAINING THE SUPERBASIC STEPDIRECTION.

5.1. BRIEF REVIEW OF THE TRUNCATED NEWTON-METHOD.

See in /9/, /10/ the motivation for using -- the Truncated Newton methodology when 'solving' sistem (4.9); it is a natural extension of the conjugate gradient method for solving system

$$-Hd_S - h = 0 \tag{5.1}$$

At each iteration, say i (here, termed minor iteration) of the conjugate gradient method, a stepdirection $\delta_S^{(i)}$ is obtained as a linear combination of the residual error --------- $e^{(i-1)} = -Hd_S^{(i-1)} - h$ and the stepdirections ----

$\left\{\delta_S^{(j)}\right\}$ of previous minor iterations such --- that they are conjuaate.

Let $d_S^{(i)}=d_S^{(i-1)}+\alpha^{(i)}\delta_S^{(i)}$ be the solution -- (probably, inexact) of system (5.1), where -- $\alpha^{(i)}$ is the(exact) steplength that solves -- the quadratic problem

$$\min\{e^{(i-1)^t}\alpha\delta_S^{(i)}-1/2\;\alpha^2\delta_S^{(i)t}H\delta_S^{(i)}\} \quad (5.2)$$

Note that $e^{(i)}$ can also be written

$$e^{(i)}=e^{(i-1)}-\alpha^{(i)}H\delta_S^{(i)} \quad (5.3)$$

If $\|e^{(i)}\|$ satiafies the accuracy tolerance given at major iteration ℓ, then $d_S\equiv d_S^{(i)}$ is the truncated solution of system (5.1). The procedure is as follows:

SKELETAL ALGORITHM A1

(0) Assign $e^{(0)}:=-h;\ \delta_S^{(1)}:=-e^{(0)};\ q^{(1)}:=H\delta_S^{(1)}$

If $\delta_S^{(1)^t}q^{(1)}\geq-\varepsilon_1\|\delta_S^{(1)}\|_2^2 \rightarrow d_S:=\delta_S^{(1)}$, stop

(1) $d_S^{(1)}:=\delta_S^{(1)};\ e^{(1)}:=e^{(0)}-q^{(1)}$

Stopping rule:

If $\|e^{(1)}\|_{1+t}/\|h\|_{1+t}\leq\eta_\ell\rightarrow d_S:=d_S^{(1)}$, stop

$i:=2$

(2) $\beta^{(i)}:=\|e^{(i-1)}\|_2^2/\|e^{(i-2)}\|_2^2$

$\delta_S^{(i)}:=-e^{(i-1)}+\beta^{(i)}\delta_S^{(i-1)};\ q^{(i)}:=H\delta_S^{(i)}$

If $\delta_S^{(i)^t}q^{(i)}\geq-\varepsilon_1\|\delta_S^{(i)}\|_2^2\rightarrow d_S:=d_S^{(i-1)}$, stop

(3) $\alpha^{(i)}:=-\|e^{(i-1)}\|_2^2/\delta_S^{(i)^t}q^{(i)}$

$d_S^{(i)}:=d_S^{(i-1)}+\alpha^{(i)}\delta_S^{(i)}$

If $i=\tau_1\rightarrow d_S:=d_S^{(i)}$, stop

$e^{(i)}:=e^{(i-1)}-\alpha^{(i)}q^{(i)}$

(4) Stopping rule:

If $\|e^{(i)}\|_{1+t}/\|h\|_{1+t}\leq\eta_\ell\rightarrow d_S:=d_S^{(i)}$, stop

(5) $i:=i+1$, go to (2)

It can be shown /9/ that for $\varepsilon_1>0$ small ----- enough, d_S is an ascent stepdirection, the -- steplength $\alpha=1$ is ascent enough in the vecinity of the local optimal point $\overset{*}{X}$ of the ----- current manifold (if $-H$ is positive definite (pd) and $\eta_\ell\to0$ for $\ell\to\infty$), and the above algo--- rithm is globally convergent; in addition, if $-\overset{*}{H}$ is pd the rate of convergence on $\{X\}\to\overset{*}{X}$ is superlinear iff

$$\lim\|e\|/\|h\|\to0 \quad (\text{i.e., } \eta_\ell\to0) \quad (5.4)$$

such that its order is $t+1$, where $0<t\leq1$ iff

$$\lim\|e\|_{1+t}/\|h\|_{1+t}^{1+t}<\infty \quad (5.5)$$

Thus, tolerance η_ℓ can be written

$$\eta_\ell=\min\{\eta_0,\gamma\|h\|_{1+t}^{t}\} \quad (5.6)$$

for $0<\eta_0<1$ and $\gamma>0$; for $t=1$ the rate of convergence is quadratic as the Newton method.- If $\|h\|$ is large (X is away from $\overset{*}{X}$), only - few minor iterations are required for obtaining d_S; when X is getting close to $\overset{*}{X}$ then -- $\|h\|\to0$ which implies $\eta_\ell\to0$ and, then, d_S is getting close to a Newton stepdirection. Tolerance τ_1 is a safeguard against unstabilities on calculating $q^{(i)}$. Although ε_1 only - needs to be positive, it is other safeguard; it avoids that d_S is not ascent (e.g., if $-H$ is not pd). Typical values, $\varepsilon_1=\varepsilon_M^{1/2}$, where ε_M is the machine precision in floating ---- point calculations (in our case, 10E-15), -- $\tau_1=3|\bar{S}|$, $\gamma=1$ and $\eta_0=1/\ell'$, where ℓ' is the major iteration of the subproblem defined by the current manifold; ℓ' is reset to 1 when set $\bar{S}$ is changed.

5.2 INDEPENDENT SETS OF SUPERBASIC ARCS.

Note that algorithm A1 does not require the calculation of any Hessian matrix, but the - product

$$q^{(i)}\equiv H\delta_S^{(i)}=Z^tGZ\delta_S^{(i)} \quad (5.7)$$

For obtaining $q^{(i)}$, superbasic set $\bar{S}$ is partitioned into, say $|P|$ disjoint so-termed -- independent superbasic sets, such that

$$\bar{S}\triangleq\bigcup_{p\in P}\bar{S}^{(p)} \quad (5.8)$$

$$\bar{S}^{(p)}\cap\bar{S}^{(q)}=\{\emptyset\} \qquad \forall p,q\in P \quad (5.9)$$

For stating the necessary and sufficient conditions for the unique valid partition (5.9), let $\bar{B}^{(p)}\subset\bar{B}$ define the set of basic arcs covered by the superbasic arcs included in set $\bar{S}^{(p)}$; that is,

$$\bar{B}^{(p)}\triangleq\bigcup_{k\in\bar{S}^{(p)}}\beta_k \quad (5.10)$$

(i) Superbasic arc k will be included in set $\bar{S}^{(p)}$ if the following condition is satisfied

$$\bar{B}^{(p)} \cap \beta_k \neq \{\emptyset\} \qquad (5.11)$$

That is, two superbasic arcs will belong to the same independent superbasic set (and, then, they will be simultaneously used for obtaining the independent superbasic stepdirection $d_S^{(p)}$) if, at least, any flow change in one of them effects the other's solution feasibility. Note that condition

$$\beta_k \cap \beta_\ell = \{\emptyset\} \qquad (5.12)$$

is not sufficient, since it could be possible that β_k and β_ℓ are disjoint sets and the following condition is satisfied

$$\bar{B}^{(p)} \cap \beta_k \neq \{0\} \quad \wedge \quad \bar{B}^{(p)} \cap \beta_\ell \neq \{\emptyset\} \qquad (5.13)$$

(ii) Two superbasic arcs, say k and ℓ will belong to the same independent set if any flow change in any of them effects the other's objective function coefficient.

Let $\bar{C}^{(p)} \triangleq \bar{B}^{(p)} \cup \bar{S}^{(p)}$ define the set of basic and superbasic arcs to be used for obtaining $d_S^{(p)}$.

Superbasic arc k will be included in set $\bar{S}^{(p)}$ if the following condition is satisfied

$\exists G_{gg'} \neq 0$ such that

$$(g \in \{k\} \cup \beta_k) \wedge (g' \in \bar{C}^{(p)}) \qquad (5.14)$$

Then, arc k will be included in set $\bar{S}^{(p)}$ if conditions (5.11) or (5.14) are satisfied. Now, assume that there is other independent set, say $\bar{S}^{(q)}$ for which, at least, one of the two following conditions is satisfied

$$\bar{B}^{(q)} \cap \beta_k \neq \{\emptyset\} \qquad (5.15a)$$

$G_{gg'} \neq 0$ such that

$$(g \in \{k\} \cup \beta_k) \wedge (g' \in \bar{C}^{(q)}) \qquad (5.15b)$$

In this case, the new independent set will be

$$\bar{S}^{(p)} \cup \bar{S}^{(q)} \cup \{k\} \cup \beta_k \qquad (5.16)$$

Since $|\beta_k|$ is small, G very sparse and, likely, $|\bar{C}^{(p)}|$ and $|\bar{C}^{(q)}|$ are also small (note that the de-activating process is only used for optimal and quasi-optimal solutions in the current manifold) then the number of situations for which conditions (5.15) are satisfied, could be reduced without too-much time-consuming. In any case, analysing the two above conditions is not time-consuming.

Let $\bar{VL} \subset VL \cap (\bar{B} \cup \bar{S})$ define the set of variable-coefficient linear variables whose coefficients are only related to nonbasic set $\bar{N}$. Let $\bar{C} \triangleq \bigcup_{p \in P} C^{(p)}$.

Note that at major iteration ℓ, $|P|$ independent iterations are consecutively executed; note also that there are {i} minor iterations to be executeu at each iteration p for $p \in P$.

The advantages of using independent sets at sucessive major iterations are as follows.

(1) The computational time for obtaining vector $q^{(i)} := Z^t(G(Z\delta_s^{(i)}))$ is drastically reduced.

(2) Faster minor iterations at the price of more (but much cheaper) major iterations. Note that the elements of matrix G out of set $\bar{C}^{(p)}$ are not modified after obtaining $d_s^{(p)}$. Note also that only the elements of matrices G_3 and G_4 related to set $\bar{C}$ and the gradient related to set $\bar{C}/$.$PL \cup \bar{VL}$ are to be evaluated at the beginning of a given major iteration.

(3) Strong reduction on the number of arcs (i.e), cardinality of $\bar{C}^{(p)}$) to be used for obtaining the steplength related to set $\bar{C}^{(p)}$. Note also that only the terms of set $\bar{C}^{(p)}$ in the objective function (3.7) are to be recomputed for obtaining the objective function value $F(X_{BS}^{(p)})$ related to each trial step.

5.3. REDUCED GRADIENT USED AS A SUPERBASIC STEPDIRECTION.

Assume that the p-th stepdirection is being obtained for $p \in P$. Assume that $\bar{C}^{(p)}/(PL \cup \overline{VL}) = \{\emptyset\}$; in this case, a LP-network subproblem must be maximised, such that the related stepdirection $d^{(p)}$ can be rxpressed

$$d_k^{(p)} := h_k = g_k - \sum_{k' \in \beta_k} \rho_{k'k} g_{k'} \qquad \forall k \in \bar{S}^{(p)} \tag{5.17}$$

$$d_{k'}^{(p)} := - \sum_{k \in \bar{S}^{(p)}} \rho_{k'k} d_k \qquad \forall k' \in \bar{B}^{(p)} \tag{5.18}$$

Note that, in this case, $\alpha^{(p)} = \alpha_m^{(p)}$. See also Section 6.6.

5.4. OBTAINING VECTOR $q^{(i)}$ IN THE TRUNCATED NEWTON METHOD.

Assume that $q^{(i)}$ is related to the superbasic stepdirection $d_S^{(p)}$; see Section 5.1. Let $\bar{C}_n^{(p)} \triangleq (\bar{C}/\bar{C}^{(p)}) \cup \bar{N}$ define the complement of set $\bar{C}^{(p)}$.

(1) Obtain intermediate vector $\underline{\delta}^{(i)} := Z^{(p)} \delta_S^{(i)}$, such that

$$\underline{\delta}^{(i)} := \begin{cases} - \sum_{k \in \bar{S}^{(p)}} \rho_{\ell k} \delta_k^{(i)} & \forall \ell \in \bar{B}^{(p)} \\ \delta_\ell^{(i)} & \forall \ell \in \bar{S}^{(p)} \\ 0 & \forall \ell \in \bar{C}_n^{(p)} \end{cases} \tag{5.19}$$

Note that the $(0, \pm 1)$ matrix ρ is not required to be explicitly stored; we use data structures very similar to those described in /18/.

(2) Obtain intermediate vector $\bar{\delta}^{(i)} := G \underline{\delta}^{(i)}$, such that

$$\bar{\delta}_\ell^{(i)} := \begin{cases} 0 & \forall \ell \in \bar{C}_n^{(p)} \cup PL \cup \overline{VL} \\ \sum_{g' \in \bar{C}^{(p)} \cap NL} G_{3\ell g'} \underline{\delta}_{g'}^{(i)} & \forall \ell \in VL/\overline{VL} \\ \sum_{g' \in VL/\overline{VL}} G_{3g'\ell} \underline{\delta}_{g'}^{(i)} + \sum_{g' \in \bar{C}^{(p)} \cap NL} G_{4\ell g'} \underline{\delta}_{g'}^{(i)} & \forall \ell \in \bar{C}^{(p)} \cap NL \end{cases} \tag{5.20}$$

Computation of vector $\bar{\delta}^{(i)}$ is very fast since each row of G_3 and G_4 has only two and three nonzero elements, respectively, the elements of $\underline{\delta}^{(i)}$ related to set $\bar{C}_n^{(p)}$ are zero (and, then, the related columns of G_3 and G_4 are not used), and only the rows of the Hessian matrix related to set $\bar{C}^{(p)}/(PL\ \overline{VL})$ are used.

(3) Obtain vector

(3) $q^{(i)} := Z^{(p)} \bar{\delta}^{(i)} = \bar{\delta}_S^{(i)} - (B^{-1}S)^{(p)t} \bar{\delta}_B^{(i)}$, such that

$$\bar{\delta}^{(i)} = (\bar{\delta}_B^{(i)t}, \bar{\delta}_S^{(i)t}, \bar{\delta}_N^{(i)t})^t$$

$$q_k^{(i)} = \bar{\delta}_k^{(i)} - \sum_{k' \in \beta_k} \rho_{k'k} \bar{\delta}_{k'}^{(i)} \qquad k \in \bar{S} \tag{5.21}$$

6. DE-ACTIVATING STRATEGY.

6.1. DEFINITIONS.

Let the following stopping tests (with values true and false) for the optimization on the manifold, provided that the solution $X = \bar{X} + \alpha d$ is feasible and ascent.

$t1: \|h\|_2 \le \varepsilon_2 \vee \|d\|_2 / (1 + \|X\|_2) \le \varepsilon_3 \vee \bar{S} = \{\emptyset\}$

$t2: |F(X) - F(\bar{X})| / |1 + F(X)| \le \varepsilon_4$ in the last τ_2 major iterations

$t3: \|h\|_2 \le \varepsilon_5$

An optimal solution is assumed to be found in the current manifold if t1; the current solution is quasi-optimal if $\ulcorner t1 \wedge (t2 \vee t3)$. Typical values for the (positive) tolerances are $\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 10E{-}04$, $\tau_2 = 3$, initial $\varepsilon_5 = \varepsilon_6 \|h^I\|_2$, where h^I is the reduced gradient related to the initial feasible solution, ε_5 is dynamically adjusted, and $\varepsilon_6 = 0.2$.

Let $\bar{U}$ define the set of unsafe arcs; a unsafe arcs is a nonbasic arc that we made basic-superbasic and, again, become nonbasic at any major iteration executed after obtaining the optimal solution of the previous manifold.

Let us define indicator γ_k $\forall k\epsilon\bar{N}$ as follows. $\gamma_k=0$ means that nonbasic arc k is not a <u>candidate</u> to be de-activated; otherwise, it takes the sign of its de-activating direction (+ for up-direction and - for down-direction) A nonbasic arc will not be a candidate to be de-activated if it is an unsafe arc, the pricing is not favorable or it is a blocked arc; see Sections 6.2 and 6.3

Let $\bar{D}$ define the set of nonbasic arcs to be <u>de-activated</u>; that is, the arcs that will be moved from the nonbasic set to the superbasic set. Let $\bar{D}\underline{\Delta}\bigcup_{p\epsilon P}\bar{D}^{(p)}$ and $\bar{D}^{(p)}\cap\bar{D}^{(q)}=\{\emptyset\}$, where $\bar{D}^{(p)}$ is the independent nonbasic set to be de-activated and, then, joined with independent superbasic set $\bar{S}^{(p)}$. A candidate nonbasic arc will not be de-activated if $|\bar{D}|$ is at its (upper) bound and there is, at least, other candidate arc with higher (first-order) guarantee for a stronger increase in the objective function; see Section 6.4.

6.2. PRICING NONBASIC ARCS.

When a solution on the current manifold is quasi-optimal, $\gamma_k=0$ $\forall k\epsilon\bar{U}$; when the solution is optimal, set $\bar{U}$ is declared empty.

The nonbasic Lagrange multipliers estimation λ can be written

$$\lambda_k = g_k - \sum_{k'\epsilon\beta_k} \rho_{k'k}g_{k'} \equiv g_k - \bar{g}_k \qquad \forall k\epsilon\bar{N}/\bar{U} \tag{6.1}$$

For $\bar{X}_k=\underline{a}_k$ (lower bound), $\gamma_k=+$ if $\gamma_k>\varepsilon_7$; otherwise, $\gamma_k=0$, where ε_7 is a positive tolerance, (typically, 10E-04). For $\bar{X}_k=\bar{\underline{a}}_k$ (upper bound), $\gamma_k=-$ if $\lambda_k<-\varepsilon_7$; otherwise, $\gamma_k=0$. Note that $g_{k'}$ is the usual k'-th gradient element for $\bar{X}_{k'}\neq\bar{a}_{k'}$. But for $\bar{X}_{k'}=\bar{a}_{k'}$ ('intermediate' bound), $g_{k'}$ for $k'\epsilon\beta_k$ is assigned as follows: $g_{k'}:=g^+_{k'}$ for (g^+_k and $\rho_{k'k}=-1$) or (g^-_k and $\rho_{k'k}=+1$), and $g_{k'}:=g^-_{k'}$ otherwise (see below).

In general, g^+_ℓ and g^-_ℓ are the gradient elements of arc ℓ ($g^+_\ell=g^-_\ell$ for $\bar{X}_\ell\neq\bar{a}_\ell$) related to its up-direction and down-direction, respectively. Note that $g^+_\ell=0$ and $g^-_\ell=K_{tj}$ for $\ell\equiv(t,j,i)$, see (3.7); it is assigned $\max\{0,s_{tj}-T_{tj}\}:=s_{tj}-T_{tj}$ for obtaining g^+_ℓ for $\ell\equiv(t,j)$ such that $g^-_\ell=g^+_\ell+P_{tj}$. Note that the computation of $g_{k'}$ (and, then, $\bar{g}_k$) for $k'\epsilon\beta_\ell$ and $\bar{X}_{k'}=\bar{a}_{k'}$ is based on $\rho_{k'\ell}$ and γ_ℓ; then, the ambiguity of $g_{k'}$ for $k'\epsilon\beta_k\cap\beta_\ell$ is solved by blocking the nonbasic arc that satisfies test t5 (see below); note that there is not a null step for $\ulcorner$t5 since λ is used as the stepdirection of set $\bar{D}$ (see section 6.6) and a maximal basis spanning tree is assumed (i.e., $\nexists k'\epsilon\bar{B}^{(p)}$ $\forall p\epsilon P$ such that $\bar{X}_{k'}$ is at any of its bounds).

For $\bar{X}_k=\bar{a}_k$ ('intermediate' bound), γ_k is assigned as follows:

$$\gamma_k = \begin{cases} 0 & \text{if } (g^+_k-\bar{g}_k\le\varepsilon_7)\wedge(g^-_k-\bar{g}_k\ge-\varepsilon_7) \\ + & \text{if } (g^+_k-\bar{g}_k>\varepsilon_7)\wedge(g^-_k-\bar{g}_k\ge-\varepsilon_7) \\ - & \text{if } (g^+_k-\bar{g}_k\le\varepsilon_7)\wedge(g^-_k-\bar{g}_k<-\varepsilon_7) \\ j \text{ such that} & \\ \quad |g^j_k-\bar{g}_k|=\max\{|g^+_k-\bar{g}_k|,|g^-_k-\bar{g}_k|\} & \end{cases}$$

Finally, assign $g_k:=g^i_k$ for $i=\gamma_k$ and, then, expression (6.1) may be used for

λ_k $\forall k\epsilon\bar{N}/\bar{U}|\bar{X}_k=\bar{a}_k\wedge\gamma_k\neq0$.

Let the following anti-zigzagging test for any nonbasic arc being priced out.

$t_4: \|h\|_2\le\varepsilon_8|\lambda_k|$

where ε_8 is a positive tolerance (typically, 0.9). When the solution on the current manifold is quasi-optimal, arc k will not be considered as a candidate to be de-activated (and, then, $\gamma_k=0$) if $\ulcorner$t4.

6.3. BLOCKING NONBASIC ARCS.

A maximal basis spanning tree avoids degenerate basic-superbasic pivots, but it does not prevent null steps when a nonbasic arc is de-activated /8/. Therefore, a mecanism is needed for testing whether a nonbasic arc, say k must be considered as a candidate to be de-activated. It may be carried out at the same time the nonbasic arc is priced (i.e., its Lagrange multiplier estimate is calculated) and, then γ_k is set to 0 if otherwise a null step could not be prevented. Thus, $\gamma_k=0$ if t5$\vee$t6$\vee$t7, where t5, t6 and t7 are the result of the following <u>blocking tests</u>:

t5: $\lambda_k=\min\{|\lambda_k|,|\lambda_\ell|\}\bar{X}_{k'}=\bar{a}_{k'}$ for $k'\epsilon\beta_k\cap\beta_\ell$ such that $\ell\epsilon\bar{N}|\gamma_\ell\neq0$ and $\lambda_k\lambda_\ell\rho_{k'k}\rho_{k'\ell}>0\}$

Note that $\lceil$t5 if the flow in basic arc k' -- changes in the same direction for any flow - change in the appropriate direction of arcs k and ℓ (given by λ_k and λ_ℓ, respectively).

t6 (case γ_k=+): $k'\in\beta_k$ such that

$\rho_{k'k}=-1$ (reverse) $\wedge(\bar{X}_{k'}=\underline{\bar{a}}_{,})$ or

$\rho_{k'k}=+1$ (forward) $\wedge(\bar{X}_{k'}=\underline{a}_{k'})$

t7 (case γ_k=-): $k'\in\beta_k$ such that

$\rho_{k'k}=-1\wedge(\bar{X}_{k'}=\underline{a}_{k'})$ or

$\rho_{k'k}=+1\wedge(\bar{X}_{k'}=\bar{a}_{k'})$

If t5vt6vt7 we refer to arc k as a blocked - arc and, then, it will not be a candidate - to be de-activated.

6.4. OBTAINING SET $\bar{D}$ TO BE DE-ACTIVATED.

A multiple de-activating strategy is allowed such that as many as possible candidate nonbasic arcs are to be de-activated up a given -- bound, say $\min\{\tau_3,\varepsilon_9|\bar{N}/\bar{U}|\}$, where τ_3 and ε_9 are positive tolerances (typically, τ_3=30 and ε_9=0.5). For reducing the computational time and storage required by the Truncated-Newton method, it is interesting that the cardinality of any independent set $\bar{S}^{(p)}\cup\bar{D}^{(p)}$ does not exceed a given bound, say τ_4 (typically, 60).

Given the dimensions of our problem, we suggest to use for partial pricing if --------- $|\bar{N}/\bar{U}|\geq\varepsilon_{10}(a-n)$, where ε_{10} is a positive tolerance (tipically, 0.1), such that only a subset of $\bar{N}/\bar{U}$ is priced at each de-activating process. Basically, the procedure consists - in pricing sequentially the arcs in set $\bar{N}/\bar{U}$ so that they will be candidate to be de-activated if the pricing result is favorable. -- Once $\ddot{D}$ reaches the allowed bound, the next candidate scanned arc will replace the arc - from set $\bar{D}$ with the worst pricing result --- till $\underline{r}$ is not greater than a given bound, -- say ε_{11} (tipically, 0.1), $\underline{r}$ takes the ratio of the number of replacements to the number of candidate scanned arcs. When $\underline{r}\leq\varepsilon_{11}$ the -- scanning is interrupted; it will be restarted, at the next de-activating process, by -- pricing the arc where it was left out.

Let the following de-activating tests:

t8 : $|\bar{D}|<\min\{\tau_3,\varepsilon_9|\bar{N}/\bar{U}|\}$

t9 : $(|\lambda_k|>\min\{|\lambda_\ell|\forall\ell\in\bar{D}\})\wedge\lceil t8$

t10: $|\bar{N}/\bar{U}|\geq\varepsilon_{10}(a-n)$

t11: $\underline{r}\leq\varepsilon_{11}$

Formally, $\bar{D}\underline{\Delta}\bar{D}\cup\{k\}$ for $\gamma_k\neq0$ if t8vt9.

After the de-activating process, if the solution on the current manifold is quasi-optimal, tolerance ε_5 is reset to ------------ $\varepsilon_5:=\min\{\varepsilon_2,\varepsilon_5(1-\varepsilon_{12})\}$ (even if $\bar{D}=\{\emptyset\}$), ---- where ε_{12} is a positive tolerance (typically, 0.3). If the solution is optimal, $\bar{U}$ is declared empty and $\varepsilon_5:=\|\lambda_k\forall k\in\bar{D}\|_2$ for $\bar{D}\neq\{\emptyset\}$ if $D=\{\emptyset\}$, stop since it is assumed that the optimal solution of the problem has been found; note that (t5vt6vt7) for t1$\wedge|\bar{D}|=0$; that is,- there are not blocked arcs when the optimal solution of the current manifold is found -- and there is not any nonbasic arc with a favorable pricing result.

Finding the most suitable values for the tolerances is a subject for experimentation, - mainly for the multiple pricing tolerance τ_3.

6.5. OBTAINING THE INDEPENDENT SET $\bar{D}^{(p)}$ TO BE DE-ACTIVATED.

Recall $\bar{D}\underline{\Delta}\bigcup_{p\in P}\bar{D}^{(p)}$ and $\bar{D}^{(p)}\cap\bar{D}^{(q)}=\{0\}$. Let $\bar{C}_d^{(p)}\underline{\Delta}\bar{B}_d^{(p)}\cup\bar{D}^{(p)}$ and $\bar{B}_d^{(p)}\underline{\Delta}\bigcup_{k\in\bar{D}^{(p)}}\beta_k$. An arc, - say k to be de-activated could be included - in set $\bar{D}^{(p)}$ if any move $d_k\neq0$ effects the solution feasibility or the objective function coefficient of any arc from set $\bar{C}^{(p)}\cup\bar{C}_d^{(p)}$; - formally, $\bar{D}^{(p)}\underline{\Delta}\bar{D}^{(p)}\cup\{k\}$ for $k\in\bar{D}$ if ------- t12$\wedge$(t13vt14), where t12 for σ=1, t13 and t14 are the result of the following including tests:

$t_{12}: |\bar{S}^{(p)}|+|\bar{D}^{(p)}|+\sigma\leq\tau_4$

$t_{13}: (\bar{B}^{(p)}\cup\bar{B}_d^{(p)})\cap\beta_k\neq\{\emptyset\}$

$t_{14}: \exists G_{gg'}\neq0|(g\in\{k\}\cup\beta_k)\wedge(g'\in\bar{C}^{(p)}\cup\bar{C}_d^{(p)})$

It is suggested to perform the testing in -- the following sequence:

t12, t13 and t14.

How, to assure that sets $\bar{D}^{(j)}$ $\forall j\in P$ are independent, it is required to analyse if any mo ve $d_k \neq 0 \quad k \in \bar{D}^{(p)}$ will effect the solution feasibility or the objective function coefficient of any arc from set $\bar{D}^{(q)} \quad \forall q \in P/\{p\}$; - in that case, both sets must be joined. Formally, $\bar{D}^{(p)} \underline{\Delta} \bar{D}^{(p)} \cup \bar{D}^{(q)}$ and $\bar{D}^{(q)} \underline{\Delta} \{\emptyset\}$ if --- $t12 \wedge (t15 \vee t16)$ such that $\sigma = |\bar{S}^{(q)}| + |\bar{D}^{(q)}|$ -- for t12 and where t15 and t16 are the result of the following <u>joining tests</u>:

$$t_{15} : \ (\bar{B}^{(p)} \cup \bar{B}_d^{(p)}) \cap (\bar{B}^{(q)} \cup \bar{B}_d^{(q)}) \neq \{\emptyset\}$$

$$t_{16} : \ \exists G_{gg'} \neq 0 \mid (g \in \bar{C}^{(p)} \cup \bar{C}_d^{(p)}) \wedge (g' \in \bar{C}^{(q)} \cup \bar{C}_d^{(q)})$$

It is suggested to perform the testing in -- the following sequence:

t12, t15 and t16.

If $k \in \bar{D}$ is not included in $D^{(p)}$ (and, then, $j := |P|+1$, $\bar{D}^{(j)} \underline{\Delta} \{k\}$ and $P \underline{\Delta} P \cup \{j\}$) or $\bar{D}^{(q)}$ is - not joined to $\bar{D}^{(p)}$ due to t12 then $\bar{D}^{(j)}$ in - the first case and $\bar{D}^{(q)}$ in the second case - will be used for obtaining $\alpha^{(p)}$ and $\alpha_m^{(p)}$.

Selecting nonbasic arcs to be de-activating may be performed in several ways; one of the main criteria could be reducing $|\bar{D}^{(p)}| \quad \forall p \in P$ and, then, increasing the number $|P|$ of independent sets of arcs to be used for obtaining the stepdirections of the next manifold. Finding the best procedure is left open at this point.

<u>6.6. 'SOLVING' THE NEWTON EQUATION AFTER DE-ACTIVATING</u>.

The new ascent independent stepdirection --- $d_S^{(p)} = (\underline{d}_S^{(p)\,t}, \underline{\underline{d}}_S^{(p)\,t})^t$, where $\underline{d}_S^{(p)}$ takes the - direction related to the old superbasic set $\bar{S}^{(p)}$ and $\underline{\underline{d}}_S^{(p)}$ is related to set $\bar{D}^{(p)}$ is obtained as follows.

$$\underline{d}_S^{(p)} = \begin{cases} 0 \text{ if } \|h\|_2 = \varepsilon_2 \vee \bar{S}^{(p)} = \{\emptyset\} \\ \text{Truncated-Newton direction in} \\ H\underline{d}_S^{(p)} = -h, \text{ otherwise} \end{cases}$$

$$\underline{\underline{d}}_S^{(p)} = \{h_k \qquad k \in \bar{D}^{(p)}\}$$

where h and H are related to the current solution $X = \bar{X} + \alpha d$ and $h_k \equiv \lambda_k$. Note that the new - direction is a mixture of the scaled stepeest ascent direction and an accurate Truncated- - Newton direction; since the flow change in -- set $\bar{B}_d^{(p)}$ has the appropriate direction (see Sections 6.2 and 6.3) then a null step is --- avoided provided that $\bar{B}^{(p)}$ is a maximal basis spanning tree.

<u>7. FUTURE WORK</u>.

The work covered by this paper is in progress and we cannot say too-much about computational experience. We have experimented with a - shorter version (6 reservoirs and 24 time periods) of the real-life problem. It seems --- that the ideas described in this paper (mainly, the concept of independent sets of superbasic and de-activated nonbasic arcs) are --- worthy of extensive experimentation; a careful implementation of the algorithm is planned. In the sequel of this paper /11/ we are planning to report the results of the current algorithm as well as the results of the modifications described below.

Computational time is important because the model is to be run frequently for planning - purposes under several assumed inflow patterns. It is important to solve the 6-reservoirs problem efficiently if the computing - time for the full 40-reservoirs system is to be within affordable limits; in some cases, aggregating the last, say 14 weeks of the time horizon into 3 time periods (months) does not strongly deteriorate the planning objective.

Topics that are worthy of future experimentations are the following.

1) <u>Selecting candidate nonbasic arcs</u>.

The procedure could promote small independent sets of arcs to be de-activated, while de-activating as many as possible arcs up a given bound and the first-order estimation - of the objective function gains is not strongly deteriorated.

2) <u>Maximizing the hydropower benefit function</u>.

The important property of this function is -- temporal separability and spatial non-separability. Since there is an economic interaction between the decision variables of all reservoirs in the same time period, the whole - system must be treated in the same run, even if the reservoirs are not physically interconnected. By using the strategies described in /18/ for selecting candidate nonbasic arcs, -

it seems that the Truncated Newton methodology could be used to obtain independent superbasic stepdirections at each major iteration.

3) Activating as many as possible superbasic arcs at each major iteration.

Note that, barring exceptional circumstances, at most one basic-superbasic arc per each set $\bar{C}^{(p)}$ $p \epsilon P$ can be added to nonbasic set $\bar{N}$ in the algorithms based on manifold suboptimization and active set strategies as above; so if, say 1000 arcs are active at the optimal solution $\overset{*}{X}$ and the initial feasible solution is interior, then the method will require at least 1000 major iterations to converge. Let

$$\alpha_B^{(p)} = \min\{ |(\bar{X}_k - ab_k)/d_k| \quad \forall k \epsilon \bar{B}^{(p)} \} \tag{7.1}$$

$$\alpha_S^{(p)} = \min\{ |(\bar{X}_k - ab_k)/d_k| \quad \forall k \epsilon \bar{S}^{(p)} \} \tag{7.2}$$

denote the upper bounds on steplength $\alpha^{(p)}$ for keeping feasibility on sets $\bar{B}^{(p)}$ and $\bar{S}^{(p)}$ respectively such that $\alpha_m^{(p)} = \min \{ \alpha_B^{(p)}, {}_S^{(p)} \}$. If $\alpha_m^{(p)} = \alpha_B^{(p)}$, $\alpha^{(p)}$ is obtained with a traditional linesearch /12/ that preserves Q-superlinear convergence. If $\alpha_m^{(p)} = \alpha_S^{(p)}$, fewer major iterations could be required by allowing more that one superbasic arc from set $\bar{S}^{(p)}$ to be activated at each major iteration; given the special structure of matrix ρ (recall that it is not explicitly stored), the time consuming is likely to be within affordable limits.

See in /4/ a nonlinear algorithm for a general system of linear constraints; the objective function F(X) is expressed as a function of Y such that $Y = \bar{A}X$, where $\bar{A}$ takes the submatrix related to 'active' constraints and bounds and, then Y gives the solution vector of 'active' slack and structural variables. A key point in the new algorithm is the transformation X=T(Y) such that the problem, at each major iteration, becomes: $\alpha \Delta \arg\max\{f(\bar{Y}+\alpha d_y) : 1_y \leq \bar{Y}+\alpha d_y \leq u_y, \quad 0<\alpha\leq\alpha_m\}$ where $\bar{Y}$ is the feasible solution at the previous iteration. The feasible set is referred to as the active rectangle at the current iteration. The algorithm has, under mild conditions, Q-superlinear convergence.

In out context, a version of this algorithm is as follows. Assume that the independent superbasic stepdirection $d_S^{(p)}$ has been obtained as follows: $d_k = h_k \quad \forall k \epsilon I^{(p)}$, where $I^{(p)}$ denotes the set of quasi-active arcs in the p-th superbasic set such that

$$I^{(p)} = \{ k \epsilon \bar{S}^{(p)} \mid |\bar{X}_k - ab_k| \leq \varepsilon^{(p)} \} \tag{7.3}$$

where scalar $\varepsilon^{(p)}$ is given by

$$\varepsilon^{(p)} = \min\{ \varepsilon_{13}, \| \bar{X}_S - [\bar{X}_S + h]^{\#} \| \} \tag{7.4}$$

for $\varepsilon_{13} > 0$ (typically, 0.01), $[.]^{\#}$ will be described below and, by slight abuse of the active bound notation used un Section 4, ab_k denotes the active bound in the direction of the sign of h_k. Let $I_n^{(p)} \Delta \bar{S}^{(p)} / I^{(p)}$; d_k for $k \epsilon I_n^{(p)}$ is a Truncated Newton direction and $d_k = 0$ for $k \epsilon \bar{N}$. Any new feasible solution $X_{BS} = \{X_{BS}^{(p)} \quad \forall p \epsilon P\}$, $X_{BS}^{(p)} = \{X_k \ \forall k \epsilon \bar{C}^{(p)}\}$, $X_B^{(p)} = \{X_k \ \forall k \epsilon \bar{B}^{(p)}\}$, and $X_S^{(p)} = \{X_k \ \forall k \epsilon \bar{S}^{(p)}\}$, takes the form

$$X_S^{(p)} = [\bar{X}_S^{(p)} + \alpha^{(p)} d_S^{(p)}]^{\#} \equiv [x]^{\#} \tag{7.5a}$$

$$X_B^{(p)} = \bar{X}_B + P^{(p)} (X_S^{(p)} - \bar{X}_S^{(p)}) \tag{7.5b}$$

where $\rho^{(p)}$ is the submatrix of ρ related to set $\bar{C}^{(p)}$, $0<\alpha\leq\alpha_B^{(p)}$, and $[.]^{\#}$ denotes the projection on the feasible superbasic rectangle for $X_S^{(p)}$ such that, it results

(i) Case $\bar{X}_k < \bar{a}_k$ $k \epsilon \bar{S}^{(p)}$

$$[x_k]^{\#} = \begin{cases} \underline{a}_k & \text{if} \quad x_k \leq \underline{a}_k \\ x_k & \text{if} \quad \underline{a}_k < x_k < \bar{a}_k \\ \bar{a}_k & \text{if} \quad x_k \geq \bar{a}_k \end{cases}$$

(ii) Case $\bar{X}_k > \bar{a}_k$ $k \epsilon \bar{S}^{(p)}$

$$[x_k]^{\#} = \begin{cases} \bar{a}_k & \text{if} \quad x_k \leq \bar{a}_k \\ x_k & \text{if} \quad \bar{a}_k < x_k < \underline{\bar{a}}_k \\ \underline{\bar{a}}_k & \text{if} \quad x_k \geq \underline{\bar{a}}_k \end{cases}$$

(iii) Case $\bar{X}_k = \bar{a}_k$ $k \epsilon \bar{D}^{(p)}$; $d_S^{(p)}$ is obtained as described in Section 6.6.

$$[x_k]^{\#} = \begin{cases} \underline{a}_k & \text{if} \quad x_k \leq \underline{a}_k \quad (\text{case } \gamma_k = -1) \\ x_k & \text{if} \quad \underline{a}_k < x_k < \underline{\bar{a}}_k \\ \underline{\bar{a}}_k & \text{if} \quad x_k \geq \underline{\bar{a}}_k \quad (\text{case } \gamma_k = +1) \end{cases}$$

Note that the strategy used for de-activating nonbasic arcs (see Section 6) prevents degenerate pivots, provided that a maximal basis spanning tree is used.

Following the same approach described in /4/, it can be shown that, under mild conditions, the solution $X_{BS}^{(p)}$ (7.5) is feasible and ascent enough, such that

$$\alpha^{(p)}=\beta^{m(p)} \tag{7.6}$$

where $m^{(p)}$ is the first nonnegative integer m (0,1,2,...) that satisfies the two following conditions

$$\beta^{m}\leq\alpha_B^{(p)} \tag{7.7}$$

$$F(X_{BS}^{(p)})-F(\bar{X}_{BS}^{(p)}) \geq \mu(\beta^{m}\gamma^{(p)} \sum_{k\in I_n^{(p)}} h_k d_k + \sum_{k\in I^{(p)}} h_k(X_k-\bar{X}_k)) \tag{7.8}$$

where $\beta\in\{0;1\}$, $\mu\in\{0;0.5\}$ (typically, β=0.5 and μ=10E-04) and the scalar $\gamma^{(p)}$ is given by

$$\gamma^{(p)}= \min\{1,|(\bar{X}_k-ab_k)/d_k| \quad \forall k\in I_n^{(p)}\} \tag{7.9}$$

Note that for $m\geqslant 0$, the right-hand side of (7.8) is nonnegative and it is positive iff $\bar{X}_{BS}^{(p)}$ is not an optimal solution in the current manifold.

It is suggested to start the linesearch with m_I such that $\beta^{mI}=\min\{\alpha_B^{(p)},\beta^0\}$; if at any major iteration a fixed number τ_5 (typically, 2 or 3) of trial steplengths fail to satisfy the Armijo-like condition (7.8), then, $\gamma^{(p)}$ is used as the next trial value.

The main advantage that the new linesearch offers over the manifold suboptimization-based algorithm as described in the above sections is that as many as $|\bar{S}|$ new arcs may become active in a single major iteration. However, the computational performance of the new approach and the comparison with the manifold suboptimization alternative is a subject for future experimentation.

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