OPERATION GOALS IN MAINTENANCE SCHEDULING FOR POWER GENERATORS

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The Generator Maintenance and Operation Scheduling problem is presented as a large-scale mixed integer nonlinear programming case. Several relaxations of the conditions of variables and constraints are discussed. The optimal solution of the models based on these relaxations is viewed as the lower bound of the optimal solution in the original problem. A combined implicit enumeration and branch-and-bound algorithm is used. Typical dimension of the problems for which computational experience is reported are 25 generator in the system, 19 of these are to be maintained and a planning horizon of 52 weeks; the corresponding dimensions of the model are about 2300 constraints, 700 binary variables and 1300 bounded nonlinear separable variables.

1. INTRODUCTION

The increased cost of fossil fuels used in the production of electricity has prompted the utility industry to seek more efficient operating procedures. One of the most promising of these requires new methods for the automated scheduling of generator maintenance. These refined techniques will help minimize the cost of power generation.

It is expected that better generator maintenance schedule planning will result in two areas of savings. First, such planning would allow more efficient generators to be available more often during the yearly production cycle. Lessened fuel usage can amount to several million dollars a year in reduced cost of power generation. Second, better maintenance planning may postpone generation expansion. This results in postponed capital construction costs. In addition to reduced cost saving, the maintenance crews and operating plants can be utilized more efficiently.

The purpose of this work is to find fast quasi-optimal solutions to the generators maintenance and operation scheduling, so that (a) an ample variety of maintenance scheduling constraints are satisfied, (b) the estimated power demand level at different types of hours in each period (usually, a week) is satisfied, and (c) the nonlinear power generation cost function is minimized over the planning horizon (usually, one year) or, at least, the difference between the best feasible solution that is found and the optimal solution is not greater than a given value.

The paper is organized as follows. Sec. 2 describes the problem. Sec. 3 discusses the production cost function. Sec. 4 presents several relaxations. A general description of the algorithm is presented in Sec. 5. Sec. 6 describes the extensions of the model including the derating of the power generation capacity and the hourly distribution of the weekly power demand level. Some computational experience is reported in Sec. 7.

2. PROBLEM FORMULATION

See in Escudero et al./15/ a full discussion of the application area, maintenance scheduling constraints and types of the objective functions to be optimized. In this paper we present an extension to the model described in /15/ so that the generators maintenance scheduling must allow a power generation level at each period. Briefly, the problem is as follows. In power generation system the new goal consists in obtaining the power generators maintenance and operation scheduling to minimize the cost of satisfying an hourly distribution of the prescribed demand for power level in each period over the plan

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ning horizon, such that the maintenance scheduling constraints are also satisfied. In the next secs, we will only consider the peak hour; see sec. 6.2 for the extension to the general case.

Let T denote the set of periods over the planning horizon, J denote the set of generators in the system and I denote the ser of generators $j \in J$ to be maintained over the planning horizon. Let /T/, /J/, /I/ denote, respectively, the cardinals of T, J, and I.

Suppose that at periods (usually, weeks) $\ell = 1,2,\ldots,/T/$ in the planning horizon, it is known that the power demand levels on the system are $E_1, E_2, \ldots, E_{/T/}$.

The problem is to determine appropiate outputs from the power generators $j=1,2,\ldots,/J/$ at each of these weeks so as to minimize the cost of satisfying the demands. Recall that we only consider here the output, cost and demand of the peak load hour for each week over the planning horizon; see in Sec.6.2 a better representation of the problem since more than one hour is considered in each week. Note that it is not necessary that all generators j in the power system have to be maintained, but at least $I \subseteq J$.

At each week ℓ a generator may be available for the production system, in which case power generation level, say $Q_{j\ell}$ must be either $m_j \leq Q_{j\ell} \leq M_j$ (where m_j and M_j are the given lower and upper bounds for $j \in J$), or the generator may be unavailable for the production system (it is the case when it is in maintenance, and then $Q_{j\ell} = 0$). Let X_{jt} be a binary variable such as $X_{jt} = 1$ if generator $j \in I$ begins maintenance in week ℓ, and $X_{jt} = 0$ if the maintenance is not beginning in this week. Generator j will be unavailable for the production system in week ℓ if $X_{jt} = 1$ and $t \leq \ell \leq t + D_j - 1$, where D_j is the maintenance duration in integral and consecutive weeks. Let E_j and L_j denote the earliest and latest available weeks for beginning maintenance in generator j. Usually, generators are maintened once and only (if any) over the planning horizon (see other variant in /13/). Then for the generators to be maintained, $\sum_t X_{jt} = 1$ for $t = E_j, \ldots, L_j$ is the classical special ordered set of type 1 or S1 /3-6,10, 16/.

Usually, there are many exclusivity constraints along the periods in which the generators are to be maintained. The most typical constraints are (see in /15/ the details and mathematical formulation):

(1) For a particular week, the total rating of generators in maintenance cannot be greater than a given amount (termed gross reserve).

(2) Maintenance crews are assigned to power plants, or set of generators, and are not available to simultaneously work on different generators. No more than one generator belonging to the same physical plant set may be in maintenance in the same week.

(3) It is forbidden that more than a given number of generators belonging to the same special class may be out of the production system in the same week.

(4) It is frequent that there are constraints, such as the elapsed time between the beginning of the maintenance in generators, say i and j, must be greater than a given number of weeks (termed frozen time); other type of constraints, termed precedence relations, require that generator j cannot begin maintenance before a given number of weeks following the ending of maintenance in generator i; etc.

These types of restrictions may amount to several hundreds of mathematical constraints. The corresponding constraints matrix is very sparse; consider that in each constraint there are involved only a few generators per each week and that weeks produce different mathematical variables and constraints for the same restriction. Let $AX \leq b$ denote these constraints system, where A is the constraints matrix, X is the column vector of binary variables $\{X_{it}\}$, and b is the restriction vector (with many 1's in its nonzero elements). In a typical problem the number of rows varies from 52 (since /T/=52 and, then, number of gross reserve constraints) to several hundreds. In the case for which computational experience is reported in Sec. 7, there are 19 generators to be maintained with a total of 670 possible weeks to begin maintenance, and the number of rows in matrix A is 749 (so that its dimensions are 749x670 with a density of

1.02% of nonzero elements). The system $AX\leq b$ is linear with $X\in\{0;1\}$.

Since if generator j for $j\in I$ is in maintenance in week ℓ, it is unavailable for the production system (then, $Q_{j\ell}=0$) and otherwise, --- $m_j\leq Q_{j\ell}\leq M_j$, we may represent the model with the formulation shown in table 1.

Eq.(3) guarantees that the power demand level E_ℓ for $\ell\in T$ is satisfied. Note that in this paper we do not consider the transmission -- losses in the power network. See in /11/ a formulation that includes this restriction; in other context see also /22/ along others.

Eq. (4) guarantees that $Q_{j\ell}$ is a semi-continuous variable: either $Q_{j\ell}=0$ or it is bounded by m_j and M_j, depending on the 0-1 variable $Y_{j\ell}$ for $j\in I$.

Eq. (5) assures that $Y_{j\ell}=0$ if generator j -- for $j\in I$ is in maintenance in week ℓ; otherwise $Y_{j\ell}=1$. Eq. (6) takes the bounds on $Q_{j\ell}$ for the generators that are not to be maintained over the planning horizon (that is, - $j\notin I$); it takes also the bounds on $Q_{j\ell}$ for -- $j\in I$ when $\ell<E_j$ or $\ell>L_j+D_j-1$.

If the sign = in constraint (5) is substituted by the sign $\leq$, and if constraint (6) - is substituted by $m_jY_{j\ell}\leq Q_{j\ell}\leq M_jY_{j\ell}$ being ----- $Y_{j\ell}\in\{0;1\}$ for the same set $\{j\}$ and (ℓ) of -- constraint (6), it results that generator j- is allowed to be unavailable for the production system in week ℓ (then, $Q_{j\ell}=0$) without necessarily being in maintenance. This formulation is more general*, but it is not considered by the algorithms that support this -- work; in any case, the extension to treat -- this case does not present much difficulty.

The power generation cost function to be minimized can be written

$$(7) \qquad \min. C = \sum_{j\in J} \sum_{\ell\in T} C_{j\ell}(Q_{j\ell})$$

Function (7) represents the power generation cost of the /J/ generators while they are in production; it has separable components in - the sense that at week ℓ, the cost of generating the power $Q_{j\ell}$ by generator j is independent of the other generators output.

* It has been suggested by a referee.

Table 1

(1) $\sum_{t=E_j}^{L_j} X_{jt} = 1$ (special ordered set of type 1) for $j\in I$.

(2) $AX\leq b$ (maintenance conflict constraints matrix) See /15/.

(3) $\sum_{j\in J} Q_{j\ell} \geq E_\ell$ for $\ell\in T$

(4) $m_jY_{j\ell}\leq Q_{j\ell}\leq M_jY_{j\ell}$ and $Y_{j\ell}\in\{0;1\}$ for $j\in I$ and $E_j\leq\ell\leq L_j+D_j-1$.

(5) $Y_{j\ell}+\sum_{t=t_5}^{t_6} X_{jt}=1$ for $j\in I$ and $E_j\leq\ell\leq L_j+D_j-1$, with $t_5=\max\{E_j,\ell-D_j+1\}$ and $t_6=\min\{\ell,L_j\}$

(6) $m_j\leq Q_{\ell j}\leq M_j$ for $j\notin I$ and $\ell\in T$, and for -- $j\in I$ and $(\ell<E_j$ or $\ell>L_j+D_j-1)$

Let P1 denote problem (1)-(7); it is a mixed integer nonlinear programming problem. There are some approaches /7/,/11/ in which the -- problem is relaxed by using non-linear constraints instead of the binary Y-variables -- and constraints (4)-(5). The new problem is only an approximation and its solution still requires much CPU time. We choose the alternative outlined in Sec.5 that does not re--- quire nonlinear constraints, nor increasing the problem's dimensions.

3. PRODUCTION COST FUNCTION

For the calculation of function $C_{j\ell}$ we use - the procedure described in /13/,/26/. In any case, let the following notation.

NK(j). Number of capacity states in the -- power generation level of generator j; being k=1,2,...,NK(j) a given capacity state.

RK(j,k). Maximum power generation level of - generator j while operating at capacity state k. The minimum level in k is the maximum level in k-1, that is RK(j,k-1). If k=1, RK(j,1) is - maximum and minimum generation level.

PC(j,ℓ,k) Power generation cost in generator j while operating at the maximum power generation level RK(j,k) of capacity state k in period ℓ. Note - that some periods (usually, the weeks -

that belong to the same month) have the same power generator cost function; see /26/

There are basically four types of production cost functions: continuous nonlinear function, bloks function, convex piecewise linear ---- function and nonconvex piecewice linear ---- function. The first function is not usual, but in anycase it may be approximated by the third function.

The second function considers that the pro-- duction variable $Q_{j\ell}$ is discrete in the since that the only possible 'states' are 0, RK(j,1) RK(j,2),..., RK(j,NK(j)) for $j \in J$. See /13/.

The third type of cost function is very usual in the utility industry; the forth function is a special nonconvex piecewise function -- (see /13/). In these two cases, the power - generation level $Q_{j\ell}$ can be expressed as fo- llows.

Case a

Consider that the situation $j\ell$ is not to be in maintenance. Then variable $Q_{j\ell}$ is bounded by $RK(j,1) \le Q_{j\ell} \le RK(j,NK(j))$ for $j \notin I$, and $\ell < E_j$ or $\ell > L_j + D_j - 1$ for $j \in I$.

$$Q_{j\ell} \equiv \sum_{k=1}^{NK(j)} RK\ (j,k) \times Y_{j\ell}^{(k)} \tag{8a}$$

such as

$$\sum_{k=1}^{NK(j)} Y_{j\ell}^{(k)} = 1 \tag{8b}$$

where the set $j\ell: \{Y_{j\ell}^{(k)}$ for $k=1,2,\dots,NK(j)\}$ is a Special Ordered Set of type 2 or S2 --- /3/,/5/,/10/. In a S2 the sum of the 0-1 -- continuous variables must be one and, at most, two variables may be different from zero and consecutive.

In this case, constraint (6) is substituted by constraint (8b) and expresion (8a) substi tutes variables $Q_{j\ell}$ in constraint (3).

Case b

Variable $Q_{j\ell}$ is semi-continuous. That is -- $Q_{j\ell}$ is either zero (there is not power gene- ration or $RK(j,1) \le Q_{j\ell} \le RK(j,NK(j))$). It is the case for $E_j \le \ell \le L_j + D_j - 1$ and $j \in I$; so that if -- $t \le \ell \le t + D_j - 1$ such as $X_{jt}=1$, generator j is in maintenance in period ℓ and then $Q_{j\ell}=0$.

$$Q_{j\ell} \equiv \sum_{k=1}^{NK(j)} RK(j,k) + Y_{j\ell}^{(k)} \tag{9a}$$

such as

$$\sum_{k=1}^{NK(j)} Y_{j\ell}^{(k)} + \sum_{t=\ell-D_j+1}^{\ell} X_{jt} = 1 \tag{9b}$$

where the set $j\ell$: $\{Y_{j\ell}^{(k)}$ for $k=1,2,\dots,NK(j)\}$ is a S2, but with the additional condition -- that the term $\sum Y_{j\ell}^{(k)}$ may be zero (it is the - case for which $X_{jt}=1$ and $t \le \ell \le t+D_j-1$). Then $\sum Y_{j\ell}^{(k)}$ is zero or one; let this set be called modified S2. In this case, constraints (4) - and (5) are substituted by constraint (9b) - and expression (9a) substitutes variable $Q_{j\ell}$ in constraint (3).

Based on Eqs.(8) and (9), objective function (7) can be written

$$\min.C = \sum_{j\in J} \sum_{\ell \in T} \sum_{k=1}^{NK(j)} PC(j,\ell,k) \times Y_{j\ell}^{(k)} \tag{10}$$

subject to constraints (1)-(3),(8) and (9). Let P2 denote this problem.

4. LOWER BOUND ON THE OPTIMAL SOLUTION

The goodness of a feasible solution (X,Y)may be measured by the difference between its -- objective function value and a lower bound - on the optimal solution value. This lower - bound is the optimal solution value of a re- laxation on problem P2. From a practical -- point of view, four conditions are required - in any relaxation: (a) any solution that is feasible in the original problem it must be also feasible in the relaxed problem, (b) -- the objective function value of the optimal solution of the relaxed problem is better -- (less in our case) or equal than the optimal solution value of the original problem, (c) the relaxed problem is simpler than the ori- ginal problem in terms of the CPU time re--- quired to obtain the optimal solution, and - (d) both problems are very similar so that more similarity, stronger lower bound is ob- tained on the optimal solution of the origi- nal problem. If the optimal solution of the relaxed problem is feasible in the original problem, it is also optimal in this problem. We have obtained good results (see Sec. 7 -- and /14/) with the two following relaxations of problem P2.

Relaxation R1

If the maintenance scheduling constraints -- (1) and (2) are relaxed in problem P2 then - it results in /T/ knapsack problems. These problems may be continuous or semicontinuous, and convex or nonconvex; for each period ℓ -- the formulation can be written

$$\min . C_{1\ell} = \sum_{j \in J_1} \sum_k c_j^{(k)} \delta_j^{(k)}$$

$$\text{subject to} \quad \sum_{j \in J_1} \sum_k w_j^{(k)} \delta_j^{(k)} \geq E_\ell \qquad (11)$$

where each set $j:\{\delta_j^{(k)}$ for $k=1,2,\ldots\}$ for $j \in J_1$ forms an Special Ordered Set (see Sec.3) and J_1 denotes the set of generators inclu-- ded in the problem corresponding to period ℓ for $\ell \in T$. For obtaining the lower bound $C_1 = \sum_\ell C_{1\ell}$ for $\ell \in T$ to the optimal solution of -- problem P2, we will consider that $J_1=J$; but for obtaining the lower bound C_1 to the opti mal solution of problem P2 in a given node - of the combined implicit enumeration and --- branch-and-bound algorithm outlined in Sec. 5, $J_1=J-\{i\}$, where $\{i\}$ are the generators - that must be in maintenance in period $\ell \in T$.

Let J_2 denote the set of generators that must be in production in period ℓ; then $J_2 \subseteq J_1$. -- For obtaining the lower bound C_1 to the opti mal solution of problem P2, we will consider that $J_2=\{j \notin I\} \cup \{j \in I$ if $\ell < E$ or $\ell > L_j + D_j - 1\}$. See in Sec. 5 the characterization of the -- set J_2 at a given node.

For $j \in J_2$ coefficient $c_j^{(k)}$ represents the pro duction cost PC (j,ℓ,k) and coefficient $W_j^{(k)}$ re presents RK(j,k) for k=1,2,...,NK(j). But - for $j \notin J_2$ (that is, generators that may be in maintenance in period ℓ) $c_j^{(1)}=w_j^{(1)}=0$, $c_j^{(k)}=$ PC$(j,\ell,k-1)$ and $w_j^{(k)}=$RK$(j,k-1)$ for $k=2,3,\ldots$, NK(j)+1, and if $\delta_j^{(1)}>0$ it must be $\delta_j^{(1)}=1$ -- and the others $\delta_j^{(k)}=0$.

In function (11) the set j in case (8) (then, $j \in J_2$) is an S2 and the set j in case (9) --- (then, $j \notin J_2$) is a modified S2. In /12/ we - describe the algorithms dealing with relaxa- tion R1.

Relaxation R2

This relaxation is simply the linear program ming problem associated with the original -- problem P2; that is, the integrality condi-- tion of X-variables and the special charac-- ter of Y-variables are relaxed. Then problem R2 is the same problem P2 with $0 \leq X, Y \leq 1$ and - continuous. Let C_2 denote the optimal solu- tion of relaxation R2.

5. GENERATORS MAINTENANCE ALGORITHMS

Three methods are basically used in the sys- tem: an implicit enumeration, a branch-and-- bound and a combination of both methods.

Previously to executing any algorithm, two - task are performed: a prefiltering phase --- that eliminates all redundant constraints and infeasible periods to begin maintenance /13 Sec.8/ and the optimization of the two re--- laxations of problem P2 (Sec. 4). These --- tasks are also performed at each node of the tree (see e.g. in /17/ the framework on this methodology) obtained while executing the al gorithms.

Let G denote the set of generators $j \in I$ that have already been branched at given node or its predecessors by explicitly or implicitly fixing their period t to begin maintenance; then T_j is the set of periods in which gene rator $j \in G$ is in the production system. Let us now be more precise in the definition of the sets J_1 and J_2 corresponding to the knap sack problem (11) associated with period $\ell \in T$ in relaxation R1 at a given node; J_1 repre-- sents the set of generators that may or must be in production in period ℓ, and J_2 repre-- sents the set of generators $j \in J_1$ that in this period must be in production. Then $J_2=\{j \notin I\}$ $\cup \{j \in G$ if $\ell \in T_j\} \cup \{j \notin G$ if relation $t \leq \ell < t + D_j - 1$ - does not hold for any $t \in F_j\}$ (where F_j is the set of feasible periods to begin maintenance for $j \in I$; see /13/), and $J_1 = J_2 \cup \{j \notin G$ if rela- tion $t \leq \ell \leq t + D_j - 1$ holds for some $t \in F_j\}$. Any generator $j \notin J_1$ must be in maintenance, any - generator $j \in J_2$ must be in production, and -- any generator $j \in J_1 - J_2$ may be in maintenance in period ℓ at the given node.

Quasi-optimal solutions

The system may find a feasible schedule (if any), the optimal solution and the desired - number of quasi-optimal solutions. A feasible solution is quasi-optimal if its power generation cost $C^{(\delta)}$ is such that

$$(12) \quad C^{(\delta)} - \underline{C}^{(o)} \leq qC^{(o)}$$

where $\underline{C}^{(o)} = \max \{C_1, C_2\}$ where C_i is the optimum value of relaxation Ri; then $\underline{C}^{(o)}$ is the strongest lower bound on the optimal solution $C^{(o)}$ of problem P2. The quasi-optimality tolerance is represented by q.

Let Z denote the incumbent solution, that is $Z=\min\{C^{(\delta)}\}$. If the optimal solution of a relaxed problem is also feasible in the original one, the solution $C^{(\delta)}$ is the new incumbent if $C^{(\delta)}<Z$. A node is fathomed if the best solution of any of its relaxations, say L is such that $L>F$, where $F=(1-q)Z$ is the fathoming bound. Note that it is not always necessary for a node to solve the /T/ independent problems of relaxation R1; e.g. if the total power generation cost corresponding to periods, say 1 to k in the given node plus the total power generation cost in periods k+1 to /T/ in the predecessor node is greater than the bound F, the node is fathomed. The reason is that the problem associated with a node is tighter than the problem associated with its predecessor; see other cases in /14/.

See in /13/ the description of the algorithms

Diagram 1 : General organization

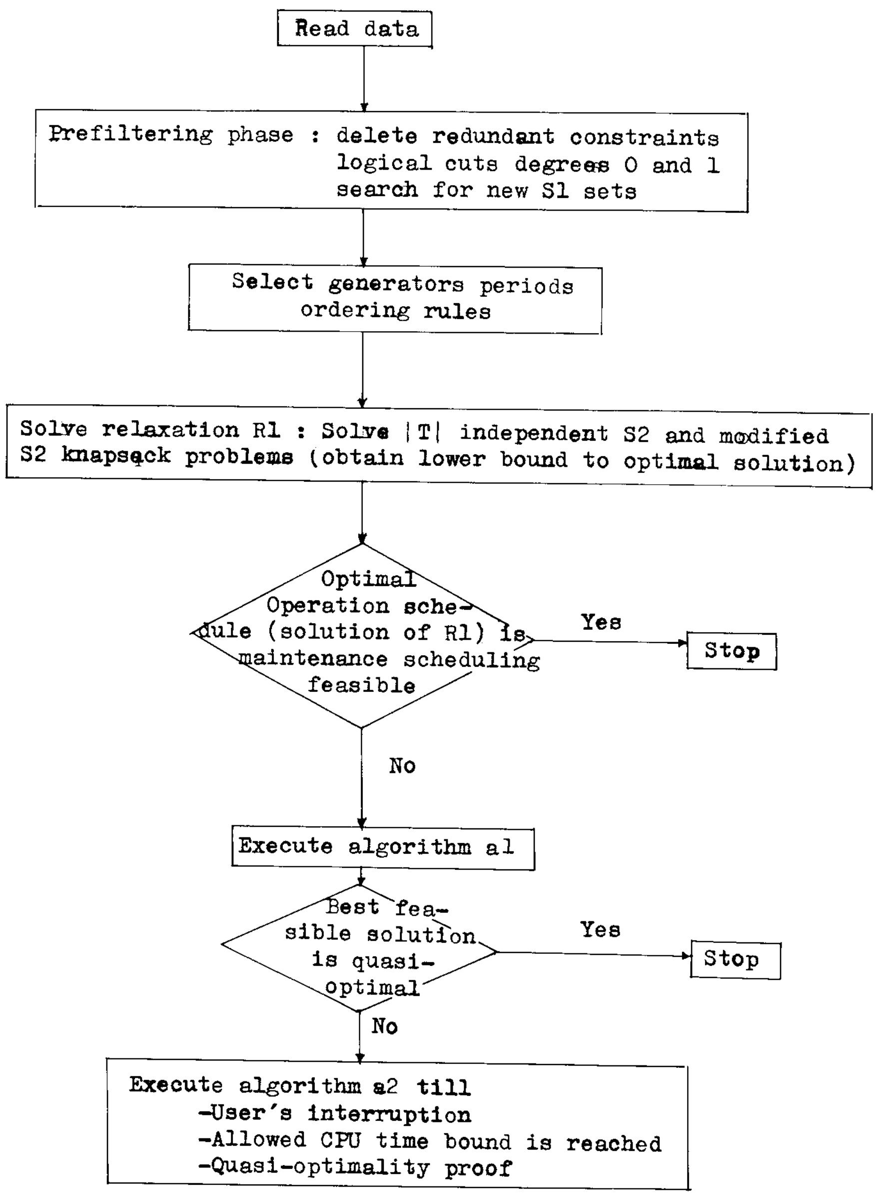
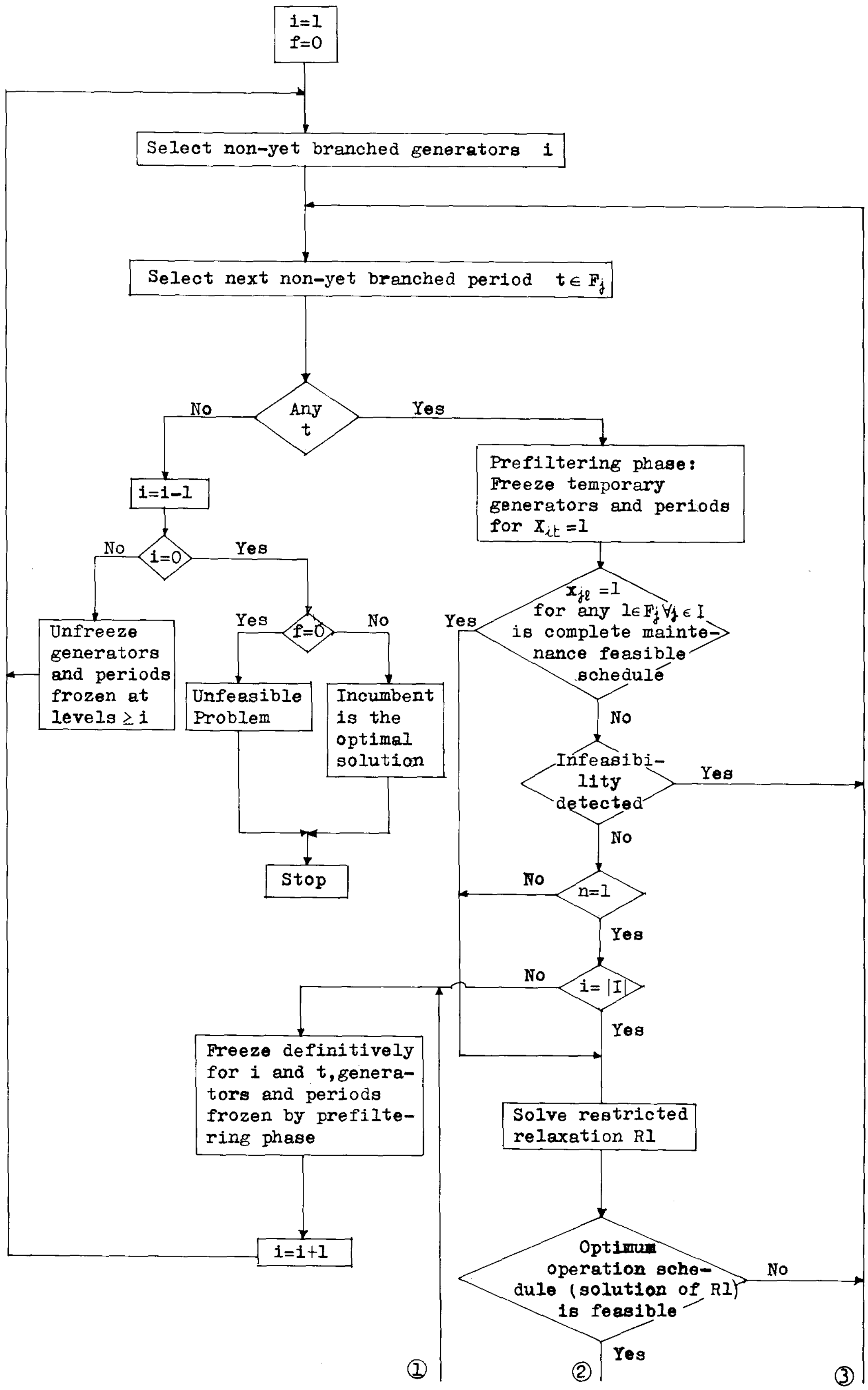
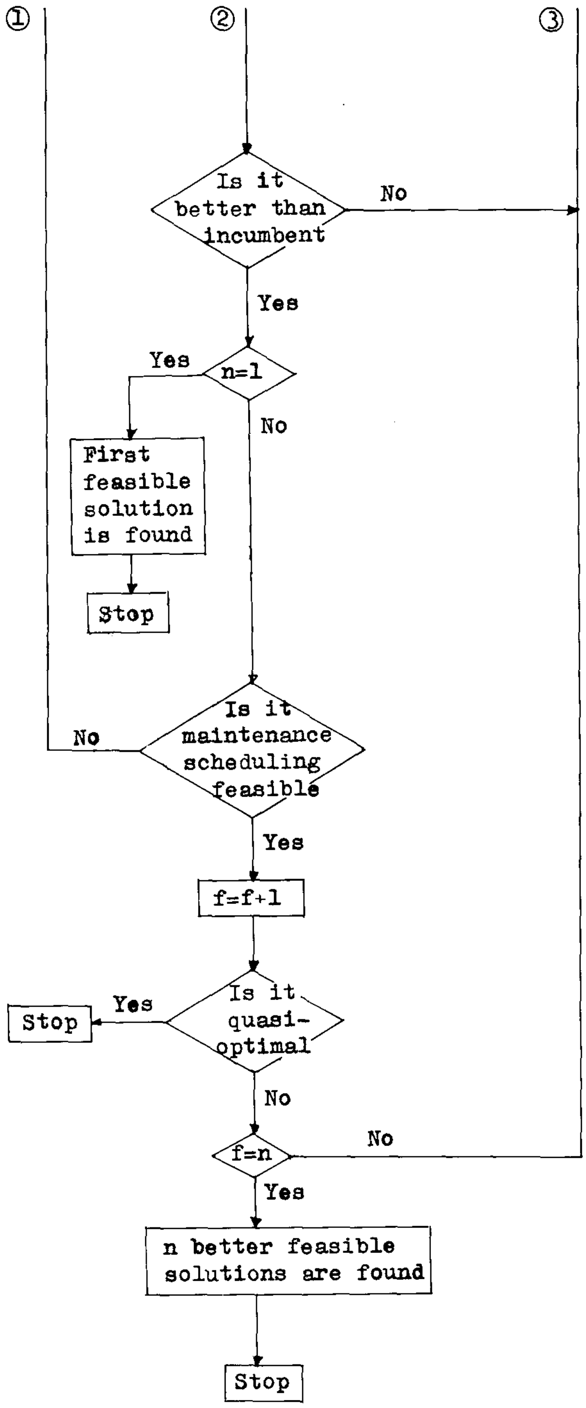


Diagram 2 : algorithm a1

(Obtain n feasible solutions)





and strategies used in this problem; in /14/ we report the computational experience obtained in their comparison. But we obtain our best results when the initial feasible solution - is obtained with algorithm a1 and the algo-- rithm a2 is used to improve it. See diagram 1.

Algorithm a1 (diagram 2)

It is an implicit enumeration algorithm of - the type described in /2/,/23/. See in /8/, /24/ another algorithm of this type designed for dealing specifically with the maintenance scheduling constraints. Algorithm a1 obtains the maintenance schedule by chosing gene--- rator $j \in I$ and maintenance beginning period - $t \in F_j$ to be branched first, according to the selected ordering rules; this selection de-- pends on the problem's condition; see in /13/ the available strategies and the criteria in which they are based. The basic principle - is: select the next generator to be scheduled and its period so that (1) the estimation of its optimum operation schedule produces the minimum cost, and (2) it allows the succesor branches to produce a complete feasible maintenance schedule.

The partial schedule obtained at each node - is tested to analyze if the complete schedule to be obtained in its successor nodes - could be feasible and better than the incumbent solution. The future feasibility is -- analized by using the prefiltering phase. -- If the optimum value of relaxation R1 is not better than the functional value of the incumbent solution, the node is fathomed.

Once the maintenance beginning period on a - generator is fixed, an entire string of ---- periods for this and for the not-yet-scheduled generators in this node are prohibited. The implicit enumeration algorithm produces very fast solutions, although sometimes it - does not guarantee the optimality of the incumbent solution; typical case: 40 generators, 52 periods, small maintenance duration, large L-E ranges and very unconstrained problem. In this case, the number of quasi-optimal solutions is very high.

Algorithm a2 (diagram 3)

It is in principle a branch-and-bound algorithm that (a) exploits the structure of the Special Ordered Sets S1 in eq.(1) /6/, (b) - uses the implicit enumeration strategy for - finding feasible solutions, and (c) uses the branch-and-bound method for obtaining the -- branching and fathomed nodes. The strategy is as follows. Each generator is considered as a S1. The branching S1 is selected ---- according to the generators ordering rule -- described in /13/. The reference row is --- created according to the periods ordering -- rule described in /13/. The estimation of a node is based on the pseudo-cost of a S1/6/ /16/. The branching node is the node with the best estimation between the two candidates nodes just created if any, or among -- all the candidates. If a given node is not fathomed by relaxation R2, relaxation R1, is used for analizing if the optimal - power generation cost in the successor nodes will be greater than the fathoming --- bound.

The relaxation R2 (that is, the LP problem) associated with a given node of the branch-and-bound phase is solved by using, as the - initial basis, the optimal solution of its - predecessor. But at node 1, it is obtained by using the feasible solution obtained by - algorithm a1.

The relaxation R2 in algorithm a2 does not - include all logical constraints (2); in the current version the frozen time and precedence relations constraints /15/ are not explicitly included in the model; these constraints may amount to several hundreds of mathematical - constraints. Relaxation R2 requires small - CPU time when these constraints are not included and, on the other hand, they do not - strongly deteriorate the (LP) relaxation R2 optimal value; then, it is better to check - separately their feasibility.

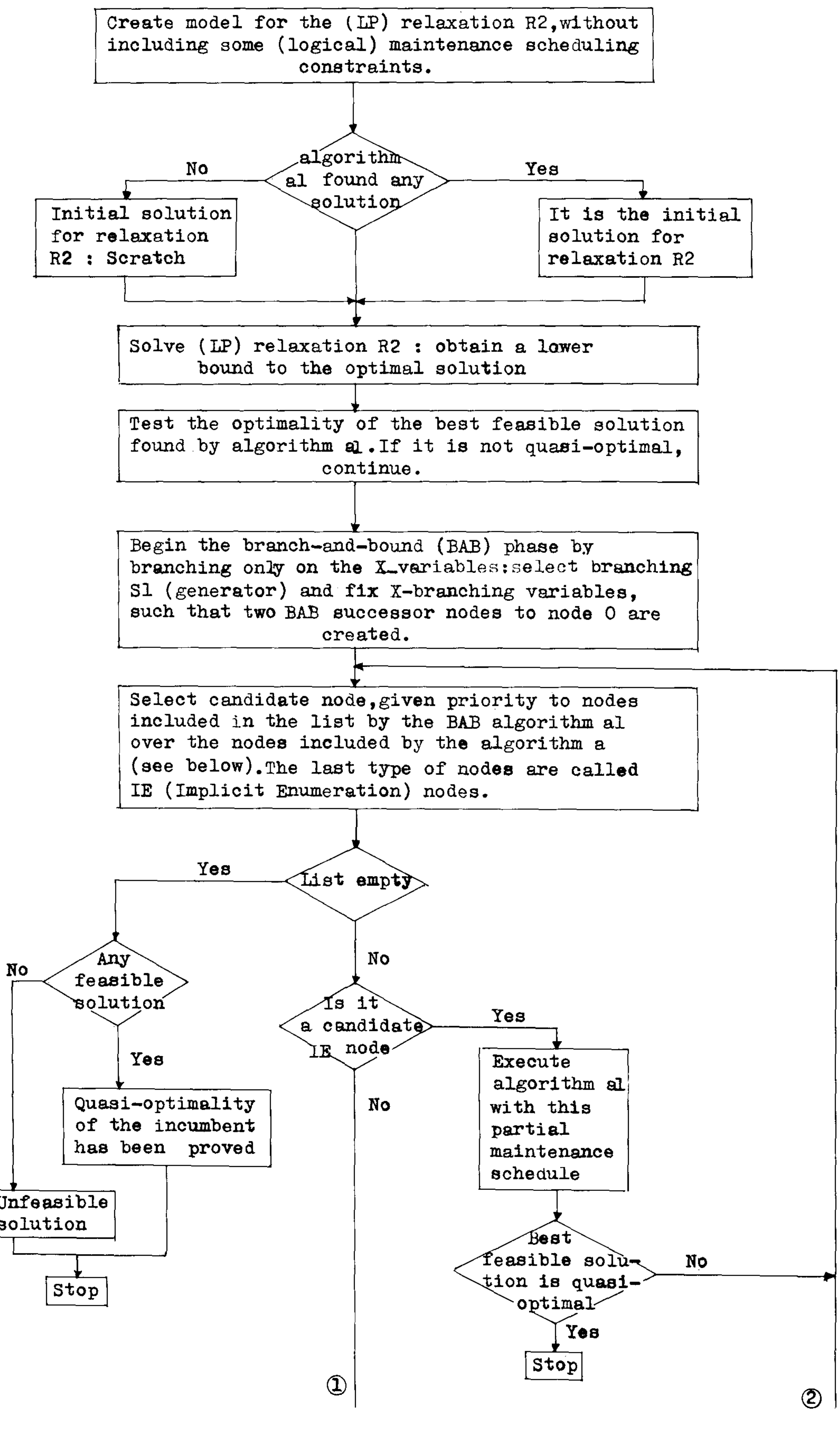
Algorithm a2 uses the implicit enumeration - strategy of algorithm a1 at each node (integer or not) of the branch-and-bound; so that the partial schedule in the implicit enumeration phase is included by the variables $\{X_{jt}\}$ that implicitly or explicitly have been ---- already branched by the given node or its -- predecessors. At a given node the general - procedure is a follows. Once problem R2 is solved and the node is not fathomed, the feasibility of the constraints not explicitly - included in the model is checked. If the -- node is not fathomed, the implicit enumeration search begins, and relaxation R1 is used. If no better feasible solution can be found, the node is fathomed. Once it finds a feasible solution or proves that there is not - any better feasible solution, the strategy - updates the incumbent solution parameters -- and returns to the branch-and-bound phase.

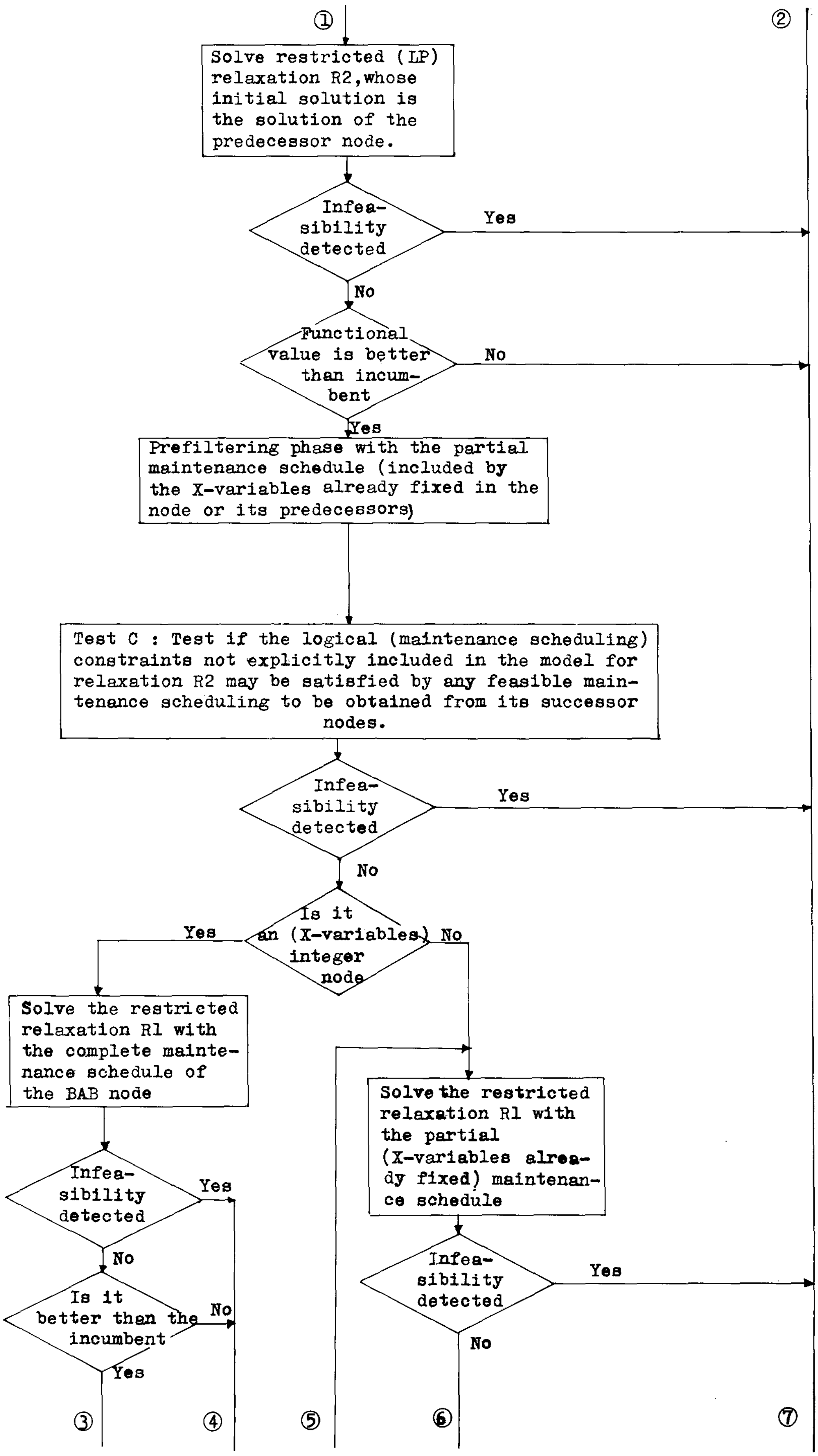
See en e.g. /1 ,p.60/;/25/ the main differences between branch-and-bound (jumptracking tactic) and implicit enumeration (backtracking tactic) methods. Algorithm a1 uses a backtracking -- tactic, and algorithm a2 a combination of --- both tactics and usually produces better results.

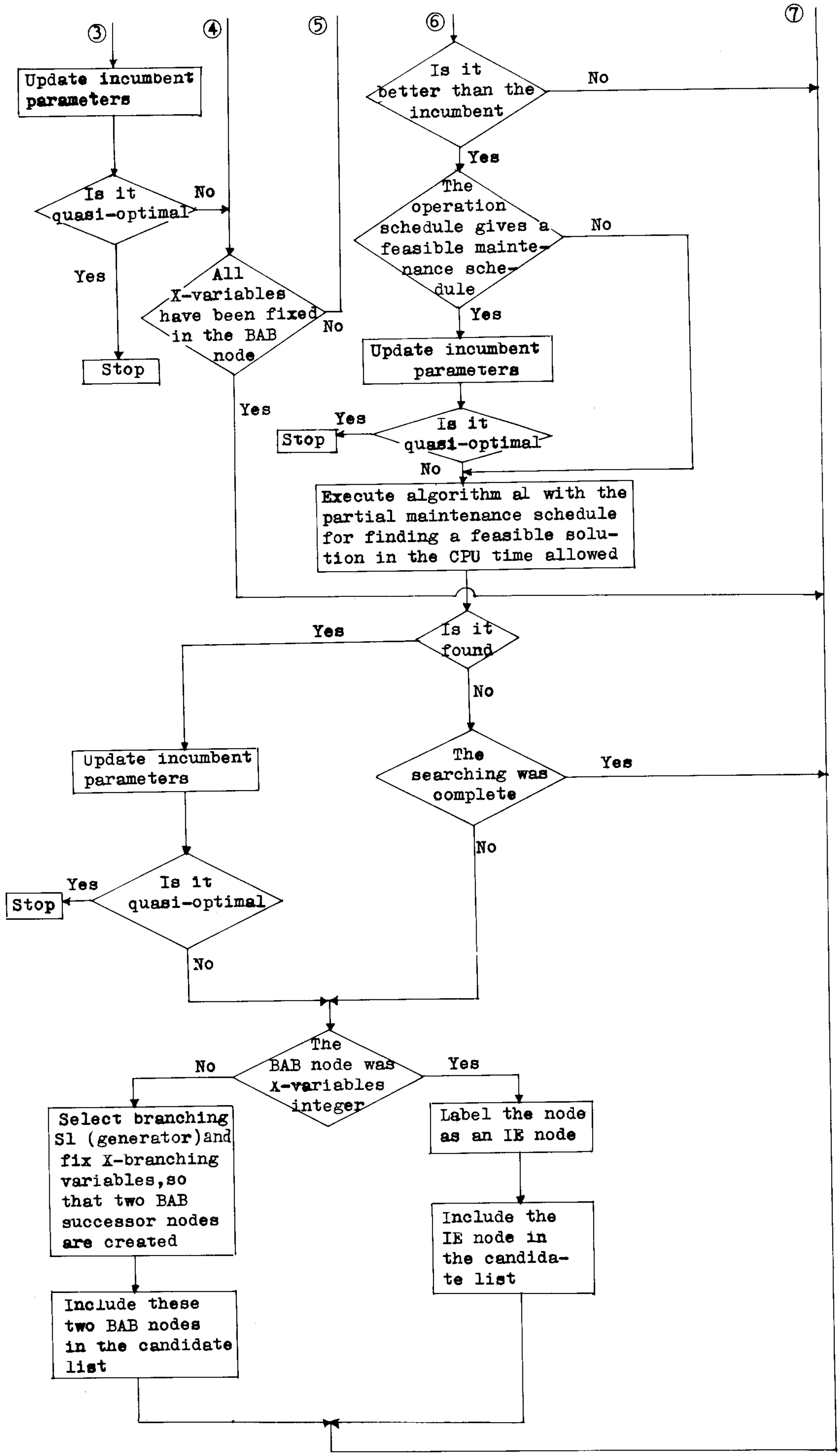
The prefiltering phase is used: (a) previously to executing any algorithm (b) at the beginning of the implicit enumeration search -- with the partial maintenance schedule obtained by the branch-and-bound node, and (c) at each branch of the implicit enumeration search in algorithms a1 and a2. The prefiltering phase eliminates redundant constraints and infeasible periods to begin maintenance, but it --

Diagram 3 : algorithm a2

(Quasi-optimality proof of the best solution).




①
②
Solve restricted (LP) relaxation R2,whose initial solution is the solution of the predecessor node.
Infea-sibility detected
Yes
No
Functional value is better than incum-bent
No
Yes
Prefiltering phase with the partial maintenance schedule (included by the X-variables already fixed in the node or its predecessors)
Test C : Test if the logical (maintenance scheduling) constraints not explicitly included in the model for relaxation R2 may be satisfied by any feasible main-tenance scheduling to be obtained from its successor nodes.
Infea-sibility detected
Yes
No
Is it an (X-variables) integer node
Yes
No
Solve the restricted relaxation R1 with the complete mainte-nance schedule of the BAB node
Solve the restricted relaxation R1 with the partial (X-variables alrea-dy fixed) maintenan-ce schedule
Infea-sibility detected
Yes
No
Is it better than the incumbent
No
Yes
Infea-sibility detected
Yes
No
③
④
⑤
⑥
⑦


③
④
⑤
⑥
⑦
Update incumbent parameters
Is it quasi-optimal
No
Yes
Stop
All X-variables have been fixed in the BAB node
No
Yes
Is it better than the incumbent
No
Yes
The operation schedule gives a feasible maintenance schedule
No
Yes
Update incumbent parameters
Yes
Stop
Is it quasi-optimal
No
Execute algorithm al with the partial maintenance schedule for finding a feasible solution in the CPU time allowed
Yes
Is it found
No
Update incumbent parameters
The searching was complete
Yes
No
Yes
Stop
Is it quasi-optimal
No
No
The BAB node was X-variables integer
Yes
Select branching Sl (generator)and fix X-branching variables,so that two BAB successor nodes are created
Label the node as an IE node
Include these two BAB nodes in the candidate list
Include the IE node in the candidate list

also detects the infeasibility of partial -- schedules; this last task is very important in the fathoming process of the branch-and-bound nodes.

Relaxation R1 is used (a) at each partial -- branch of the implicit enumeration search in algorithm a1, and (b) at each branch-and---- bound node in algorithm a2, once the prefiltering phase has eliminated infeasible pe--- riods to begin maintenance. Recall that if $C_1 > F$ the partial maintenance solution is fathomed. Note that if relaxation R1 corres-- ponds to a complete feasible maintenance --- schedule, C_1 is the cost of its optimum po-- wer generation schedule; so that if $C_1 < Z$ it will be the new incumbent solution.

The algorithms was written in ECL /20/ and - Fortran and they use the MPSX-MIP/370 system. An interactive graphics interface is used - and it is allowed the optimization interruption at a given number of iterations, at a - given CPU time, or when it is considered --- that the incumbent solution, compared with - the lower bound of the optimal solution, is good enough.

6. EXTENSIONS

Problem P2 as formulated by eqs.(1)-(3) and (8)-(10) can be extended by adding the following data and constraints.

6.1 Derating of generators capacity

The maximum power generation level RK(j,NK-- (j)) for $j \in J$ is only a theoretical capacity. The real available capacity is smaller; in - fact, it is different over the planning horizon. Let $d_{j\ell}$ denote the estimated derating corresponding to generator j in period ℓ. Then RK(j,NK(j)) for $j \in J$ may be sustituted - by $(1-d_{j\ell})$RK(j,NK(j)) for $\ell \in T$. Note that it could be possible that NK(j) must be reduced as a result of the value of the new maximum power generation level.

This extension of problem P2 affects the --- maintenance and operation scheduling, but -- the algorithm requires small modifications.

6.2 Estimate hourly distribution for the --- weekly power demand level

In problem P2, eqs.(3),(8) and (9) ensure -- that the power generation level $Q_{j\ell}$ satisfies the weighted power demand level E_ℓ for the peak load hour at week ℓ for $\ell \in T$. The extension described below allows to include a --- better representation of the power demand -- level to be satisfied, since it includes the level of more than one hour per week.

Let H_ℓ denote the set of 'typical' hours h= 1,2,..., $/H_\ell/$ whose demand $E_{\ell h}$ must be considered in week ℓ ; $H_{\ell h}$ denotes the weight of hour h in the empirical distribution of the power demand level in week ℓ.

In the new problem, eq. (3) is substituted - by

$$\sum_{j \in J} Q_{j\ell h} \geq E_{\ell h} \qquad \text{for } h \in H_\ell, \ell \in T \tag{13}$$

where the variable $Q_{j\ell h}$ corresponding to the power generation level in hour $h \in H_\ell$ substitutes to variable $Q_{j\ell}$. The equivalence of - variable $Q_{j\ell h}$, in a similar way to the expression in eq.(9a) for variable $Q_{j\ell}$, can be -- written

$$Q_{j\ell h} \equiv \sum_{k=1}^{NK(j)} RK(j,k) x Y_{j\ell h}^{(k)}$$

$$\text{for } h \in H_\ell, E_j \leq \ell \leq L_j + D_j - 1, \quad j \in I \tag{14a}$$

where $Y_{j\ell h}^{(k)} \in \{0;1\}$ gives the same representation for hour h than variable $Y_{j\ell}^{(k)}$ does for week ℓ in problem P2. Eq.(9b) must be substituted by

$$\sum_{k=1}^{NK(j)} Y_{j\ell h}^{(k)} + \sum_{t=\ell-D_j+1}^{\ell} X_{jt} = 1 \tag{14b}$$

for the same h,ℓ,j. Note that eqs.(14) re-- present the case for which $Q_{j\ell h}$ is semicontinuous; for the continuous case, eqs.(8) must be substituted in an analogous way. Finally, the power generation cost function (10) must be substituted by

$$\min . C = \sum_{j \in J} \sum_{\ell \in T} \sum_{h \in H_\ell} H_{\ell h} \sum_{k=1}^{NK(j)} PC(j,\ell,k) x Y_{j\ell h}^{(k)} \tag{15}$$

Considering that in the above formulation variable $Y_{j\ell h}^{(k)}$ has the same special character than variable $Y_{j\ell}^{(k)}$ in the original problem P2, let extended problem P2 denote the new - problem. Note that if in the original pro-- blem P2, E_ℓ is obtained such that $E_\ell = \max_{(h)}$--

$\{E_{\ell h}\}$ it results that any of its feasible -- solutions is also feasible in the extended - problem P2. The only reason for this extension is to take account of the hourly distribution of the power demand level when optimizing the power generation cost function.

Relaxation R2

We may see that the dimensions of the extended problem P2 are very large; the CPU time required for obtaining the optimal solution of its relaxation R2 makes it impractical; - recall that the relaxation R2 of a problem - is its LP relaxation. Then we need to modify the extended problem P2 but only for the optimization of its relaxation R2, so that - the new formulation is still a good lower -- bound to the optimal solution of the extended problem P2 and, at the same time, its dimensions are smaller.

This goal may be acomplished by substituting constraints (13)-(14) (and in a similar way for the constraints related to the continuous variables $Q_{j\ell h}$) by certain type of surrogate constraints (see e.g./18/), so that the new relaxation is as follows.

$$\sum_{j\in I} \sum_{k=1}^{NK(j)} RK(j,k) \sum_{h\in H_\ell} H_{\ell h} Y_{j\ell h}^{(k)} \geq E_\ell \qquad (16a)$$

$$\sum_{k=1}^{NK(j)} \sum_{h\in H_\ell} H_{\ell h} Y_{j\ell h}^{(k)} + a_\ell \sum_{t=\ell-D_j+1}^{\ell} X_{jt} = a_\ell \qquad (16b)$$

for $j\in I$; for $E_j \leq \ell \leq L_j+D_j-1$, where

$$a_\ell = \sum_{h\in H} H_{\ell h} \quad \text{and} \quad E_\ell = \sum_{h\in H_\ell} H_{\ell h} E_{\ell h} \qquad (17)$$

and E_ℓ and a_ℓ represent, respectively, the - total power demand level and the number of hours in week ℓ. We may see that eqs.(16) are weaker than eqs.(13)-(14) because $S(H)\supset S$, where --- $S(H)$ and S are the sets of binary solutions, respectively to eqs.(16) and (13)-(14).

Since the special character of binary variables $Y_{j\ell h}^{(k)}$ is relaxed in relaxation R2 (in a similar way to binary variables $Y_{j\ell}^{(k)}$ described in Sec. 3) so that $0\leq Y_{j\ell h}^{(k)} \leq 1$ and continuous, eqs. (16) are equivalent to

$$0\leq Y_{j\ell}^{(k)} \leq 1 \text{ and continuous} \qquad (18a)$$

$$\sum_{j\in I} \sum_{k=1}^{NK(j)} RK(j,k) \times Y_{j\ell}^{(k)} \geq E_\ell / a_\ell \qquad (18b)$$

$$\sum_{k=1}^{NK(j)} Y_{j\ell}^{(k)} + \sum_{t=\ell-D_j+1}^{\ell} X_{jt} = 1 \qquad (18c)$$

The following remarks on formulation (18) -- are in order:

(a) $Y_{j\ell}^{(k)} a_\ell$ represents the number of hours that generator j will be working at ca-- pacity state k in week ℓ.

(b) The right-hand-side of eq.(18b) is the - average hourly demand level E_ℓ/a_ℓ in --- week ℓ; then E_ℓ is also satisfied.

(c) Assuming that a_ℓ is the same for $\forall \ell\in T$, the objective function (15) is substituted by a function that has the mathema-- tical expression of function (10) times a_ℓ. If a_ℓ is different for $\ell\in T$, then the power generation cost function can - be written

$$\min . C = \sum_{j\in J} \sum_{\ell\in T} a_\ell \sum_{k=1}^{NK(j)} PC(j.\ell,k) \times Y_{j\ell}^{(k)} \qquad (19)$$

(d) The average hourly modification of the - extended problem P2 is a relaxation of - this problem, because it satisfies E_ℓ -- but it is not guaranteed that $E_{\ell h}$ for -- $\forall h\in H_\ell$ is also satisfied.

(e) The optimal value of the new relaxation R2 is a lower bound to the optimal value of the extended problem P2.

(f) In anycase, relaxation R2 cannot be used alone in the algorithms described in --- Sec.5.

Relaxation R1

The algorithms dealing with the relaxation - R1 of the extended problem P2 as formulated in eqs.(13)-(14) have not strong modifica--- tions in comparison with problem P2. In the new relaxation, the knapsack problem (11) - corresponding to a given week ℓ is substituted by $/H_\ell/$ knapsack problems. These pro--- blems only differ among them and from pro--- blem (11) for week ℓ in the original problem P2, in the right-hand-side $E_{\ell h}$ (then, they - have identical objective function, knapsack constraint and special ordered sets). Then if the knapsack problems corresponding to -- each hour $h \in H_\ell$ are ordered in increasing order of $E_{\ell h}$ in week ℓ , it is not difficult - to solve the other problems $h \in H_\ell$ while solving the problem with $\max\{E_{\ell h}\}$ in each week ℓ. Let $C_{1\ell h}$ denote the minimum power generation cost of knapsack problem h in week ℓ; then the lower bound C_1 of the optimal solution of the extended problem P2 can be written

$$C_1 = \sum_{\ell \in T} \sum_{h \in H_\ell} H_{\ell h} \, C_{1\ell h} \qquad (20)$$

Recall that C_1(20) is the power generation - cost of a feasible solution in the extended problem P2, if this operation schedule co--- rresponds to a feasible maintenance schedule.

7. SOME COMPUTATIONAL EXPERIENCE

See in /14/ an extensive validation of the - algorithm. Here we report the computational experience obtained with one real-life problem. Data parameters: /J/= 25, /I/= 22, -- /T/= 52, quasi-optimality tolerance q=1%, 21 pairs of generators with in-between frozen - time constraints, no precedence relations -- constraints, no special classes, 4 plants, - weighted power demand level of the peak load hour of each period and piecewise linear power generation cost function with 2 generators with semicontinuous power generation.

Table 2 gives the maintenance scheduling --- data for the 22 generators to be maintened; where for each generator R is the rating for the gross reserve constraint, D is the maintenance outage duration, E and L are the earliest and latest available periods to begin - maintenance, Pl is the plant to which it belongs, and U is the frozen time; see /15/ -- for additional details on the maintenance -- constraints (2). Table 3 gives the number - of capacity states and the maximum power generation level in each capacity state for -- each of the 25 generators in the system. -- Table 4 gives the cost related data: energy class, fuel cost and incremental heat rate - for each capacity state for each generator ; see /13/. Table 5 gives the gross reserve, power demand level at the peak load hour and periods's group for each of the 52 periods - of the planning horizon; the energy classes are c=1 (nuclear), 2(oil), 3(coal) and 4(natural gas); the yearly increase rate of power generation cost in each class is Y(1)=0%, -- Y(2)=9%, Y(3)=10% and Y(4)=7%; see in /13/ - the meaning of these data (they are used for obtaining the cost function).

The 21 pairs of generators with in-between - maintenance frozen time (see table 2) are the following: (4,5), (4,6),(4,7),(4,8),(4,9),-- (5,6), (5,7), (5,8), (5,9), (6,7), (6,8), -- (6,9), (7,8), (7,9), (8,9), (11,12),(11,13), (11,14),(12,13),(12,14), and (13,14).

Generators j=1,2 and 3 have in advance fixed maintenance periods; then E_j=L_j so that E_j to E_j+D_j-1 are the periods for which these - generators are not available for the production system. Note that j=1 and 2 are not to be maintained by the system (then R_j=0).

System parameters; relaxations R1 and R2 on - problem P2; ordering rules r1 and p1 as described in /13 Sec.6/; algorithm a1 till finding the first feasible solution and algorithm a2 for improving it. The case was run in an IBM 370/158 with 1.5 megabytes of real storage and 2 megabytes of virtual storage, and using the system VM/CMS. The time reported is CPU time from the beginning of the -- running.

Except for the unusual case in which the --- chosen capacity state for all generators in

Table 2

Data of generators to be maintained

I	NAME	R	D	E	L	P1	P2	U
1	GEN-1	0	4	40	40	0	0	0
2	GEN-2	0	4	9	9	0	0	0
3	PL3-6	646	8	1	1	3	0	0
4	PL2-8	210	4	1	49	2	0	2
5	PL2-7	200	5	1	48	2	0	2
6	PL2-3	85	4	14	49	2	0	2
7	PL2-6	95	4	1	40	2	0	2
8	PL2-4	95	4	1	34	2	0	2
9	PL2-5	100	5	1	12	2	0	2
10	PL3-5	850	10	1	26	3	0	0
11	PL3-1	80	5	1	48	3	0	4
12	PL3-4	100	5	1	48	3	0	4
13	PL3-2	85	5	1	38	3	0	4
14	PL3-3	85	9	1	36	3	0	4
15	PL1-1	95	4	6	49	1	0	0
16	PL1-2	95	4	14	49	1	0	0
17	PL1-3	190	7	1	12	1	0	0
18	PL1-4	205	4	24	29	1	0	0
19	PL4-1	100	3	1	50	4	0	0
20	PL4-2	100	3	1	50	4	0	0
21	PL4-3	100	6	1	47	4	0	0
22	PL4-4	100	3	1	47	4	0	0

production is the capacity state 1, the total power generation level in a given period is exactly the power generation level at the -- peak hour. The level of each generator will be the maximum power generation level of the chosen capacity state, except for, at most, one generator whose level will be between the maximum and minimum levels of its chosen capacity state (if this is not the capacity -- state 1).

The dimensions of problem P2 are; 19 special ordered sets of type 1 (with 670 0-1 varia-- bles) defined by eq.(1), corresponding to generators j=4 to 22; 396 special ordered sets of type 2 defined by eqs.(8), corresponding to generator 23 and the known periods in which ge-- rators j=1 to 22 must be in operation (see - Table 2); 784 modified S2 defined by eqs.(9), corresponding to generators j=4 to 22; and - 104 modified S2. The last set corresponds- to generators j=24 and 25; they are not owned by the utility and, then, in any period the power generation level is zero ($Q_{j\ell}=0$) or, at least, $m_j \leq Q_{j\ell}$ where m_j is the minimum -- level allowed. Its LP relaxation has in total 2319 constraints and 6806 0-1 continuous variables; but the relaxation R2 to be used by algorithm a2 has 1622 constraints and --- 6806 0-1 continuous variables with 29616 non zero elements (and a 0.21% matrix density), since the 697 maintenance frozen time cons-- traints defined in eqs.(2) (see /15/) are -- not explicitly included.

The initial solution 3039372 (obtained at -- 0.36 m. by algorithm a1) is only 9.53% quasi -optimal with C_1=2310674; C_1(11) is in this case a week lower bound of the optimal solution. But C_2=3031604 was obtained at 15.21 m. (and at 2210 LP iterations); then the initial solution is 1.23% quasi-optima. Since q=1%, the branch-and-bound phase is required; it finds another 4 better feasible solutions (although they are very similar), so that at node 22 (and at 56.11 m. with in total 4560 LP iterations) a feasible solution was found with $C^{(\delta)}$= 3056314; it is 0.80% quasi-optima. Table 6 gives the maintenance and operation scheduling of this solution; tables 7 gives the generators that at each period will have its power generation level between the bounds of their chosen capacity state, so that the total level exactly satisfies the power de-- mand level of the peak hour.

A crucial algorithm in the system is the special convex/nonconvex knapsack algorithm that optimizes relaxation R1; it does not consider the maintenance scheduling constraints, but

Table 3
Power generation levels and capacity states for each generator

J	NAME	NK(J)	PRODUCTION LEVELS RK(J,K)					
1	GEN-1	4	45.0	171.0	174.0	176.0		
2	GEN-2	4	45.0	171.0	174.0	176.0		
3	PL3-6	5	190.0	266.0	380.0	494.0	646.0	
4	PL2-8	4	90.0	154.0	200.0	205.0		
5	PL2-7	3	90.0	154.0	195.0			
6	PL2-3	5	30.0	45.5	64.0	79.0	82.0	
7	PL2-6	5	30.0	44.7	68.0	85.3	92.0	
8	PL2-4	5	30.0	50.0	71.5	85.5	91.0	
9	PL2-5	5	30.0	45.5	67.0	82.5	94.0	
10	PL3-5	5	250.0	350.0	500.0	650.0	850.0	
11	PL3-1	5	30.0	32.0	56.0	76.0	80.0	
12	PL3-4	5	30.0	60.0	72.5	83.5	95.0	
13	PL3-2	5	30.0	32.0	56.0	76.0	80.0	
14	PL3-3	5	30.0	40.0	63.2	81.5	85.0	
15	PL1-1	5	30.0	45.5	67.0	82.5	91.0	
16	PL1-2	5	30.0	45.5	67.0	82.5	91.0	
17	PL1-3	4	80.0	88.0	154.0	190.0		
18	PL1-4	4	80.0	88.0	154.0	195.0		
19	PL4-1	5	45.0	52.5	72.5	87.5	97.0	
20	PL4-2	5	45.0	52.5	72.5	87.5	97.0	
21	PL4-3	5	45.0	52.5	72.5	87.5	97.0	
22	PL4-4	5	45.0	52.5	72.5	87.5	97.0	
23	GEN-3	4	300.0	400.0	500.0	610.0		
24	GEN-4	5	.0	44.5	89.0	133.5	178.0	
25	GEN-5	6	.0	21.0	42.0	84.0	126.0	168.0

it obtains the optimum operation schedule of a given partial or complete maintenance schedule. Then if this schedule is feasible, the optimal solution in relaxation R1 is a feasible solution of the whole problem. We may see in Sec. 4 that if the maintenance schedule constraints are relaxed, the problem is converted in /T/=52 independent knapsack problems. The algorithm /12/ dealing with these problems is very fast. It requires about 0.10 m. of CPU time to obtain the solution to the 52 periods problems. Each knapsack problem corresponds to a period; it has about 23 special ordered sets (each set corresponds to a generator that must not be in maintenance in this period) and each set has about 5 variables (they corrrespond to the capacity states); an additional variable is needed per generator if it is not required to be in production in the given period. The knapsack constraint in each period is the weighted power demand level to be satisfied at the peak hour. It is interesting to note that the whole set of the /T/ problems is not usually solved at each node. The two reasons are the following: (1) By using the

Table 4

Cost-related data

J	NAME	C	F(J)	INCREMENTAL HEAT RATE IHR(J,K) k=1	k=2	k=3	k=4	k=5	k=6
1	GEN-1	3	3.10	11.060	8.807	9.470	9.496		
2	GEN-2	3	3.10	11.050	8.832	9.525	9.554		
3	PL3-6	3	4.10	15.130	8.093	8.320	8.554	8.800	
4	PL2-8	2	1.60	9.260	8.262	9.354	10.100		
5	PL2-7	2	1.60	9.260	8.262	9.284			
6	PL2-3	2	1.97	12.210	10.156	10.672	11.430	11.975	
7	PL2-6	2	1.97	10.830	8.631	9.000	9.559	10.041	
8	PL2-4	2	1.97	11.410	9.274	9.984	10.960	11.739	
9	PL2-5	2	1.97	10.680	8.156	8.480	8.989	9.599	
10	PL3-5	3	3.50	13.400	8.093	8.320	8.554	8.800	
11	PL3-1	3	3.50	11.710	9.629	10.025	11.150	12.242	
12	PL3-4	3	3.50	10.780	8.450	8.850	9.375	10.233	
13	PL3-2	3	3.50	11.710	9.629	10.025	11.150	12.242	
14	PL3-3	3	3.50	9.860	9.188	9.684	10.599	11.234	
15	PL1-1	2	1.50	10.680	8.156	8.480	8.989	9.514	
16	PL1-2	2	1.50	10.680	8.156	8.480	8.989	9.514	
17	PL1-3	2	1.50	9.450	7.769	8.249	9.214		
18	PL1-4	2	1.50	9.450	7.769	8.249	9.284		
19	PL4-1	3	3.15	9.620	8.527	8.791	9.300	9.914	
20	PL4-2	3	3.15	9.620	8.527	8.791	9.300	9.914	
21	PL4-3	3	3.15	9.620	8.527	8.791	9.300	9.914	
22	PL4-4	3	3.15	9.620	8.527	8.791	9.300	9.914	
23	GEN-3	1	1.10	10.520	9.515	9.540	9.565		
24	GEN-4	4	2.37	0.000	13.255	13.265	13.275	13.285	
25	GEN-5	4	2.37	0.000	14.612	14.617	14.625	14.635	14.645

TABLE 5

GROSS RESERVE, PEAK LOAD AND GROUP (USUALLY, MONTH) PER PERIOD

ℓ	G	E	g	ℓ	G	E	g	ℓ	G	E	g
T= 1	1676	2607	1	T=19	2550	2297	5	T=36	2430	2415	8
T= 2	1606	2677	1	T=20	2620	2225	5	T=37	2280	2565	9
T= 3	1466	2817	1	T=21	2370	2475	5	T=38	2370	2475	9
T= 4	1706	2577	1	T=22	2540	2305	5	T=39	2400	2445	9
T= 5	1806	2477	1	T=23	2520	2325	6	T=40	2200	2465	9
T= 6	2192	2737	2	T=24	2310	2535	6	T=41	2370	2295	10
T= 7	2322	2607	2	T=25	2310	2535	6	T=42	2250	2415	10
T= 8	2452	2477	2	T=26	2120	2725	6	T=43	2230	2435	10
T= 9	2242	2507	2	T=27	2560	2285	6	T=44	2697	2231	10
T=10	2412	2337	3	T=28	2440	2405	7	T=45	2687	2241	11
T=11	2322	2427	3	T=29	2160	2685	7	T=46	2527	2401	11
T=12	2341	2411	3	T=30	2550	2295	7	T=47	2477	2451	11
T=13	2631	2301	3	T=31	2460	2385	7	T=48	2337	2591	11
T=14	2551	2381	3	T=32	2240	2605	8	T=49	2087	2841	11
T=15	2661	2271	4	T=33	2240	2605	8	T=50	1967	2961	12
T=16	2841	2091	4	T=34	2300	2545	8	T=51	1857	3071	12
T=17	2761	2171	4	T=35	1960	2885	8	T=52	2327	2601	12
T=18	2560	2287	4								

G= GROSS RESERVE, E= PEAK LOAD, G= GROUP (USUALLY , MONTH), PER PERIOD L solution of the knapsack problem correspon-- ding to the same period in the predecessor - node, a node may be fathomed if the power ge neration cost of the current solution of --- some periods plus the cost of the solution - of the problems corresponding to the rest of the periods in the predecessor node, is grea ter than the fathoming bound (note that a -- problem in a node is not more relaxed than - the problem corresponding to the same period in the predecessor node); and (2) It is not necessary to solve a problem corresponding - to a given period, if the problem correspon- ding to the same period in the predecessor - node is more relaxed than the current pro--- blem and its optima solution does not violate the tighter constraints of the current pro-- blem.

8. CONCLUSION

A methodology for solving the Generators Main tenance and Operation Scheduling has been des cribed. It allows to include an ample varie- ty of constraints and a nonlinear power gene- ration cost function. It contains several - types of branching rules to be used in a com bined implicit enumeration and branch-and- - bound method; and several optimizing strate- gies that may help the user to easily design the most suitable global strategy for his/her specific problem.

Four of the most interesting conclusions of - this work are: (1) Restating that in integer programs, it is always worthy to formulize - them so that the LP feasible set be as close as possible to the integer feasible set; (2) Introducing ad-hoc implicit enumeration algo rithms in the well-tested general purpose LP branch-and-bound methods seems to be a very promising area of research; (3) In sparse -- multiperiod-multicommodities integer problems, some exclusivity restrictions may amount to several hundreds of mathematical constraints; in these situations it is worthy to not ex-- plicitly introduce these constraints in the LP system, and at each node check the feasi- bility of this node and its successors; (4) In sparse multiperiod integer programs an -- adecuate selection of the branching integer variable may produce nodes whose LP subpro-- blems may be decomposed in as many independent problems as periods in the original formula- tion;in this situation it is worthy to solve separately these problems since their solu-- tion is the optimal solution of the succe--- ssors to the given node. In our case these independent problems are convex/nonconvex -- knapsack problems with a variety of Special Ordered Sets.

In the cases that we have run, the strategies that have better performance are: generators ordering rule r1 and periods beginning main- tenance ordering rule p1 as described in /13 Sec. 6/; relaxations R1 and R2 for obtaining - an strong lower bound to the optimal solution of the problem; algorithm a1 till finding -- the first feasible solution and algorithm a2 to improve it. The special knapsack algo---

Table 6

SCHEDULE NUM. 5 Power generation cost 3056314.64 BEST SCHEDULE QUASI-OPTIMALITY 0.80%

MAINTENANCE TO BEGINNING PERIOD FOR EACH GENERATOR TO BE MAINTAINED

49 45 12 29 34 24 9 40 16 49 48 10 27 39 15 50 47 41 37 40 9 1

PERIODS STATUS OF EACH GENERATOR

NOTE.0:IN MAINTENANCE; OTHERWISE:PRODUCTION CAPACITY STATE

DUNKIRK1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	3	4	5	5	5	0	0	0	0
DUNKIRK2	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	3	0	0	0	0	5	5	5	5
DUNKIRK3	4	4	4	4	4	4	4	4	4	4	4	0	0	0	0	0	0	0	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	3	4	4	4	4	4	4	4	4
DUNKIRK4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	3	4	0	0	0	0	4	4	4	4	4	4	4	4	4	4	4	3	3	4	4	4	4	4	4	4
HUNTLE63	5	5	5	5	1	5	5	1	1	1	1	5	1	1	3	3	5	5	3	1	3	1	1	5	1	5	1	1	5	1	1	5	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	4	5	5	5	5
HUNTLE64	5	5	5	5	3	5	5	3	2	1	1	5	1	1	3	3	5	5	4	3	3	1	1	0	0	0	0	1	5	1	1	5	2	1	5	1	2	1	1	1	1	1	2	1	1	1	1	4	5	5	5	5
HUNTLE65	5	5	5	5	5	5	5	5	0	0	0	0	0	4	5	5	5	5	5	5	5	3	3	5	3	5	1	1	5	1	4	5	4	4	5	1	4	1	1	4	1	4	4	1	1	2	4	5	5	5	5	5
HUNTLE66	5	5	5	5	4	5	5	4	3	1	2	5	3	3	5	5	5	5	5	4	5	1	2	5	2	5	1	1	5	1	3	5	3	3	5	1	4	1	1	0	0	0	0	1	1	1	2	5	5	5	5	5
HUNTLE67	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	0	0	0	0	0	3	3	3	3	3	3	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	2	3	3	3	3	3	3	3
HUNTLE68	4	4	4	4	4	4	4	4	4	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	2	2	4	3	4	4	4	4	4	2	4	3	3	4	2	4	4	2	2	3	4	4	0	0	0	0
OSWEGO1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
OSWEGO2	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
OSWEGO3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
OSWEGO4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	2	1
OSWEGO5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	3	4	1
ALBANY1	2	2	2	2	1	2	2	1	1	1	1	2	1	1	1	1	2	2	1	1	1	1	1	2	1	2	1	1	2	1	1	2	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	3	0	0	0
ALBANY2	2	2	2	1	1	2	2	1	1	1	1	1	1	1	1	1	2	2	1	1	1	1	1	2	1	2	1	1	2	1	1	2	1	1	2	1	1	1	1	1	1	1	1	1	1	1	0	0	0	3	4	2
ALBANY3	2	2	2	1	1	2	2	1	1	1	1	1	1	1	1	1	2	2	1	1	1	1	1	2	1	2	1	1	2	1	1	2	1	1	2	1	1	1	1	1	0	0	0	0	0	0	1	1	3	3	4	2
ALBANY4	2	2	2	1	1	2	2	1	1	1	1	1	1	1	1	1	2	2	1	1	1	1	1	2	1	2	1	1	2	1	1	2	1	1	2	1	0	0	0	1	1	1	1	1	1	1	1	1	3	3	4	2
ROSETON1	2	2	2	1	1	2	2	1	1	1	1	1	1	1	1	1	2	2	1	1	1	1	1	2	1	2	1	1	2	1	1	2	1	1	2	1	1	1	1	0	0	0	0	1	1	1	1	1	2	2	4	2
ROSETON2	1	1	2	1	1	2	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	4	2
OSWEGO6	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9MILE	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
ALBANYGT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ROTTERGT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 7

Capacity state not fully used in 0.80% quasi-optima solution

T	J	NAME	OTHERS	PROD	K	MAX
1	20	GEN-1	2561.0	46.0	2	171.0
2	20	GEN-1	2561.0	116.0	2	171.0
3	21	GEN-2	2687.0	130.0	2	171.0
4	16	PL4-1	2531.0	46.0	2	52.5
5	6	PL2-4	2426.3	50.7	3	71.5
6	21	GEN-2	2687.0	50.0	2	171.0
7	20	GEN-1	2561.0	46.0	2	171.0
8	6	PL2-4	2426.3	50.7	3	71.5
9	6	PL2-4	2460.0	47.0	2	50.0
10	9	PL2-7	2176.0	161.0	3	195.0
11	8	PL2-6	2392.0	35.0	2	44.7
12	16	PL4-1	2362.0	49.0	2	52.5
13	8	PL2-6	2247.0	54.0	3	68.0
14	8	PL2-6	2329.5	51.5	3	68.0
15	5	PL2-3	2224.5	46.5	3	64.0
16	5	PL2-3	2029.5	61.5	3	64.0
17	20	GEN-1	2116.0	55.0	2	171.0
19	6	PL2-4	2212.0	85.0	4	85.5
20	6	PL2-4	2171.3	53.7	3	71.5
21	5	PL2-3	2414.5	60.5	3	64.0
22	7	PL2-5	2247.0	58.0	3	67.0
23	8	PL2-6	2284.0	41.0	2	44.7
24	20	GEN-1	2410.0	125.0	2	171.0
25	8	PL2-6	2504.0	31.0	2	44.7
26	20	GEN-1	2660.0	65.0	2	171.0
27	3	PL1-3	2127.0	158.0	4	190.0
29	20	GEN-1	2526.0	159.0	2	171.0
30	10	PL2-8	2097.0	198.0	3	200.0
31	8	PL2-6	2324.5	60.5	3	68.0
32	20	GEN-1	2526.0	79.0	2	171.0
33	6	PL2-4	2557.5	47.5	2	50.0
34	8	PL2-6	2489.5	55.5	3	68.0
35	21	GEN-2	2795.0	90.0	2	171.0
36	9	PL2-7	2251.0	164.0	3	195.0
37	8	PL2-6	2494.5	70.5	4	85.3
38	10	PL2-8	2277.0	198.0	3	200.0
39	10	PL2-8	2247.0	198.0	3	200.0
40	7	PL2-5	2392.0	73.0	4	82.5
41	9	PL2-7	2131.0	164.0	3	195.0
42	7	PL2-5	2347.0	68.0	4	82.5
43	6	PL2-4	2399.5	35.5	2	50.0
44	10	PL2-8	2111.0	120.0	2	154.0
45	3	PL1-3	2059.5	181.5	4	190.0
46	7	PL2-5	2356.0	45.0	2	45.5
47	7	PL2-5	2375.7	75.3	4	82.5
48	5	PL2-3	2512.5	78.5	4	79.0
49	15	PL3-5	2479.5	361.5	3	500.0
50	15	PL3-5	2479.5	481.5	3	500.0
51	15	PL3-5	2564.5	506.5	4	650.0
52	21	GEN-2	2498.5	102.5	2	171.0

rithms /12/ for obtaining the optimum operation schedule of a given maintenance schedule are quite satisfactory.

We have experimented with the estimate hourly distribution of the weekly power demand level (Sec.6.2) in some test problems and the results are encouraging. It seems to be a promising area of future experimentation with - real-life problems.

The system outlined in the previous sections may find the optimal solution; but more important is that, by using the lower bound -- strategies on the optimal solution and an -- interactive graphics interface, it is possible to find in a very small CPU time a variety - of quasi-optinal solutions with very different schedules in the maintenance of the generators.

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