A NOTE ON THE DIMENSION OF THE RING OF ENTIRE FUNCTIONS

by

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Aguiló Fuster in [1] states that the dimension of the ring of entire functions is 1. The purpose of this note is to correct this statement.

Let A be the ring of entire functions. Then we have:

Prop. dim $A \geq 2$.

Proof. Let $\mathcal{U}$ be a non-trivial ultra filter on $\mathbf{C}$ (the set of complex numbers) such that $S \in \mathcal{U}$ for some denumerable set S. Then if $\mathcal{M}$ is the set of $f \in A$ such that $Z(f) \in \mathcal{U}$ (where $Z(f)$ is the set of zeros of f) then as was shown by Aguiló Fuster, $\mathcal{M}$ is a maximal ideal of A. Now let $\mathcal{P}$ be the set of $f \in \mathcal{M}$ such that $Z(f, n) \in \mathcal{U}$ for each $n \geq 0$ where $Z(f, n) = \{z \mid z \text{ is a zero of } f \text{ of order } \geq n\}$. Then $\mathcal{P}$ is a prime ideal. For if $f \cdot g \in \mathcal{P}$ suppose $Z(f, n) \notin \mathcal{U}$. Then $\mathbf{C}\text{-}Z(f, n) \in \mathcal{U}$. Now $Z(f \cdot g, 2n) \in \mathcal{U}$ so clearly $Z(g, n) \in \mathcal{U}$. Thus for each n, either $Z(f, n) \in \mathcal{U}$ or $Z(g, n) \in \mathcal{U}$. Clearly one or the other holds for each n so $f \in \mathcal{P}$ or $g \in \mathcal{P}$.

Note $\mathcal{P} \neq \mathcal{M}$, 0. For obviously $\mathcal{P} \neq \mathcal{M}$ since there is an element of $\mathcal{M}$ with $Z(f, 2) = \emptyset$ (we only need use the Mittag-Leffler theorem on S). For the same type reasoning $\mathcal{P} \neq 0$.

This says dim $A \geq 2$. It seems to be an open question whether dim $A = 2$.

REFERENCES

AGUILÓ FUSTER, RAFAEL. — *Estudio de los ideales del anillo de las funciones enteras*, Collect. Math. 17 (1965), 105-134.