LIFTING OF HOLOMORPHIC MAPPINGS ON LOCALLY CONVEX SPACES

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This note deals with the following question: If E and G are locally convex spaces, when can a holomorphic mapping f from E into a quotient space G/H be lifted to a holomorphic mapping $\hat{f}: E \to G$? In the present paper we shall characterize those locally convex spaces which have the holomorphic lifting property and the weak holomorphic lifting property, respectively. A locally convex space F is said to have the (weak) holomorphic lifting property if for each locally convex space E and each quotient space $G/H \simeq F$ for which each bounded subset in G/H can be lifted to a bounded subset in G, every holomorphic mapping $f: E \to G/H$ of locally uniform bounded type has a lifting $\hat{f}: E \to G$ (resp. $\hat{f}: E \to G_n''$) of the same kind where G_n'' denotes the bidual of G equipped with the natural topology. The holomorphic mappings $f: E \to F$ of locally uniform bounded type are just those holomorphic mappings for which there exist normed spaces E_o and F_o such that f has a factorization

$$E \xrightarrow{A} E_o \xrightarrow{\overline{f}} F_o \xrightarrow{B} F$$

where $\overline{f}$ is holomorphic and A,B are continuous and linear. We shall show that each locally complete co-$\mathcal{F}_1$-space (co-$\mathfrak{L}_1$-space) has the (weak) holomorphic lifting property. A locally convex space E is called co-$\mathfrak{L}_1$-space (resp. co-$\mathcal{F}_1$-space) if for each absolutely convex bounded subset B of E there exists another absolutely convex bounded subset C of E containing B such that the continuous extension $\tilde{J}_{BC}: \tilde{E}_B \to \tilde{E}_C$ to the completions of the canonical mapping $J_{BC}: E_B \to E_C$ is 1-factorable (resp. discretely 1-factorable). Under additional assumptions we shall prove that conversely a locally complete space with the (weak) holomorphic lifting property is a co-$\mathcal{F}_1$-space (co-$\mathfrak{L}_1$-space). As a corollary we obtain the following result: A (B)-space E is isomorphic to $\ell_1(I)$ for some index set I if and only if E has the holomorphic lifting property. Furthermore, we get the following characterization of $\mathfrak{L}_1$-spaces (in the sense of J. Lindenstrauss, A.

Pełczyński [18]): A (B)-space E is an $\mathcal{L}_1$-space if and only if E has the weak holomorphic lifting property. As a further consequence one obtains a dual version of the holomorphic extension theorem of P.J. Boland [3]: If E is a (DFM)-space and G is an (FN)-space, then every holomorphic mapping f from E into every quotient space G/H has a holomorphic lifting $\hat{f}: E \to G$.

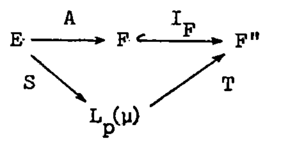
In the first section of this note we give some fundamental properties of co-$\mathcal{F}_1$-spaces and co-$\mathcal{L}_1$-spaces. In section 2 we deal with liftings of bounded linear mappings between locally convex spaces. Section 3 is devoted to locally convex spaces with the (weak) holomorphic lifting property. Applications to echelon and co-echelon spaces are given.

1. Co-$\mathcal{F}_1$- AND Co-$\mathcal{L}_1$-SPACES

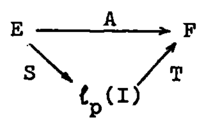
For a locally convex space E, $\mathcal{U}(E)$ will denote a 0-neighbourhood basis of closed absolutely convex 0-neighbourhoods and $\mathcal{B}(E)$ the system of all closed absolutely convex bounded subsets of E. If $U \in \mathcal{U}(E)$ and $B \in \mathcal{B}(E)$, then we denote by E_U and E_B the associated normed spaces and by $K_U: E \to E_U$ and $J_B: E_B \to E$ the canonical mappings. For $U, V \in \mathcal{U}(E)$ with $V \subset U$ and $B, C \in \mathcal{B}(E)$ with $B \subset C$, the canonical mapping $E_V \to E_U$ and $E_B \to E_C$ respectively, as well as its continuous extension $\tilde{E}_V \to \tilde{E}_U$ and $\tilde{E}_B \to \tilde{E}_C$ to the completions is denoted by K_{UV} and J_{BC}.

For locally convex spaces E and F, the vector space of all continuous linear mappings $E \to F$ is denoted by L(E,F).

Let E and F be (B)-spaces and let $I_F: F \to F''$ be the evaluation mapping. An operator $A \in L(E,F)$ is called p-factorable, $1 \leqslant p \leqslant \infty$, if the compose $I_F \circ A$ has a factorization



through some $L_p(\mu)$-space where $S \in L(E, L_p(\mu))$ and $T \in L(L_p(\mu), F'')$. An operator $A \in L(E,F)$ is said to be discretely p-factorable if A has a continuous linear factorization



through the (B)-space $\ell_p(I)$ for some index set I. The operator ideal of all p-factorable (resp. discretely p-factorable) operators is denoted by $\mathfrak{L}_p$ (resp. $\mathcal{F}_p$); see A. Pietsch [21], p. 272.

A locally convex space E is called ϵ-space (resp. π-space) if for each $U\epsilon\mathfrak{U}(E)$ there exists a $V\epsilon\mathfrak{U}(E)$ such that the canonical mapping $K_{UV}:\widetilde{E}_V\to\widetilde{E}_U$ is ∞-factorable (resp. 1-factorable). The ϵ- and the π-spaces were introduced and investigated by means of locally convex tensor products in R. Hollstein [9], [10].

Let $\mathcal{A}$ be any operator ideal. A locally convex space E is called co- $\mathcal{A}$ -space if for each bounded set $B\epsilon\mathfrak{B}(E)$ there is a bounded set $C\epsilon\mathfrak{B}(E)$ containing B such that the canonical mapping $J_{BC}:\widetilde{E}_B\to\widetilde{E}_C$ lies in $\mathcal{A}(\widetilde{E}_B,\widetilde{E}_C)$ (cf. H. Junek [13] and H. Jarchow [12], 21.5). By definition, a locally convex space E is a co- $\mathfrak{L}_1$-space (resp. co- $\mathcal{F}_1$-space) if for each $B\epsilon\mathfrak{B}(E)$ there is a $C\epsilon\mathfrak{B}(E)$ containing B such that $J_{BC}:\widetilde{E}_B\to\widetilde{E}_C$ is 1-factorable (resp. discretely 1-factorable).

The $\mathfrak{L}_1$-spaces in the sense of J. Lindenstrauss and A. Pełczyński [18] are just the co- $\mathfrak{L}_1$-Banach spaces; see Y. Gordon et al. [8], p. 355. Furthermore, a (B)-space E is a co- $\mathcal{F}_1$-space if and only if E is isomorphic to $\ell_1(I)$ for some index set I (cf. 2.7).

Clearly, each co- $\mathcal{F}_1$-space is a co- $\mathfrak{L}_1$-space. On the other hand, a co- $\mathfrak{L}_1$-space need not be a co- $\mathcal{F}_1$-space in general; consider e.g. the Banach space $L_1(\lambda)$ where λ is the Lebesgue measure on [0, 1].

A locally convex space E is called co-Schwartz space if for each $B\epsilon\mathfrak{B}(E)$ there is a $C\epsilon\mathfrak{B}(E)$ containing B such that $J_{BC}:\widetilde{E}_B\to\widetilde{E}_C$ is compact. Now we prove

1.1. Proposition: A co- $\mathfrak{L}_1$-space E is a co- $\mathcal{F}_1$-space if E is a co-Schwartz space.

Proof: Let $B\epsilon\mathfrak{B}(E)$ be given. There exist $C,D\epsilon\mathfrak{B}(E)$ with $B\subset C\subset D$ such that $J_{BC}:\widetilde{E}_B\to\widetilde{E}_C$ is 1-factorable and $J_{CD}:\widetilde{E}_C\to\widetilde{E}_D$ is compact. The adjoint $J_{BC}':\widetilde{E}_C'\to\widetilde{E}_B'$ has a factorization

$$\widetilde{E}_C' \xrightarrow{\ S\ } L_\infty(\mu) \xrightarrow{\ T\ } \widetilde{E}_B'$$

through an $L_\infty(\mu)$-space where $S\epsilon L(\widetilde{E}_C',L_\infty(\mu))$ and $T\epsilon L(L_\infty(\mu),\widetilde{E}_B')$. Since $J_{CD}':\widetilde{E}_D'\to\widetilde{E}_C'$ is compact, J_{CD}' admits a continuous linear factorization

$$\widetilde{E}_D' \xrightarrow{\ K\ } H \xrightarrow{\ L\ } \widetilde{E}_C'$$

through a closed subspace H of c_o (cf. G. Köthe [16], p. 226). Because $L_\infty(\mu)$ has the extension property, there exists an extension $R\epsilon L(c_o,L_\infty(\mu))$ of $S\circ L\ \epsilon\ L(H,L_\infty(\mu))$. It follows that the adjoint $J_{BD}' = J_{BC}'\circ J_{CD}'$ has a factorization

$$\widetilde{E}_D' \xrightarrow{K} c_o \xrightarrow{T \circ R} \widetilde{E}_B'$$

through c_o, hence the double adjoint $J_{BD}'':\widetilde{E}_B''\to\widetilde{E}_D''$ factors through ℓ_1. There is furthermore a $B' \in \mathfrak{B}(E)$ with $D \subset B'$ such that $J_{DB'}:\widetilde{E}_D\to\widetilde{E}_{B'}$ is compact. Since $J_{DB'}''$ maps $\widetilde{E}_D''$ into $\widetilde{E}_{B'}$, the mapping $J_{BB'}:\widetilde{E}_B\to\widetilde{E}_{B'}$ is discretely 1-factorable. This completes the proof.

The co-$\mathcal{N}$-spaces are called co-nuclear where $\mathcal{N}$ denotes the operator ideal of nuclear operators. By definition, each co-nuclear space is a co-$\mathcal{F}_1$-space, in particular a co-$\mathfrak{L}_1$-space. On the other hand, a nuclear space need not be a co-$\mathfrak{L}_1$-space. In fact, let E be a (B)-space which is not an $\mathfrak{L}_1$-space, i.e. the identity Id:E→E is not 1-factorable. If E_σ denotes the vector space E endowed with the weak topology, then E_σ is a nuclear space which cannot be a co-$\mathfrak{L}_1$-space since E and E_σ have the same bounded sets. If, however, E is a nuclear (DF)- or a nuclear (F)-space, then E is co-nuclear and hence a co-$\mathcal{F}_1$-space and a co-$\mathfrak{L}_1$-space, respectively.

Next we consider permanence properties of co-$\mathfrak{L}_1$- and co-$\mathcal{F}_1$-spaces. First we note that closed subspaces and quotients of co-$\mathfrak{L}_1$-spaces and co-$\mathcal{F}_1$-spaces respectively are generally not of the same kind, e.g. for each infinite-dimensional $\mathfrak{L}_1$-space (resp. $\ell_1(I)$-space) E there exist a closed subspace and a quotient of E which are not $\mathfrak{L}_1$-spaces (resp. $\ell_1(I)$-spaces).

An injective inductive limit $\underset{\alpha\to}{\text{ind}}\, E_\alpha$ of locally convex spaces E_α is said to be regular if every bounded set in $\underset{\alpha\to}{\text{ind}}\, E_\alpha$ is contained and bounded in some E_β. Every regular inductive limit $\underset{\alpha\to}{\text{ind}}\, E_\alpha$ of co-$\mathfrak{L}_1$-spaces (resp. co-$\mathcal{F}_1$-spaces) E_α is again a co-$\mathfrak{L}_1$-space (resp. co-$\mathcal{F}_1$-space). This follows from the following proposition which can easily be proved.

1.2. ***Proposition:*** *Let $\mathcal{A}$ be any operator ideal. Each regular inductive limit $\underset{\alpha\to}{\text{ind}}\, E_\alpha$ of co-$\mathcal{A}$-spaces E_α is also a co-$\mathcal{A}$-space.*

For a sequence $a = (\alpha_k)$ of numbers $\alpha_k \geqslant 0$ let

$$\ell_p(a) := \{ (x_j) \in \mathbb{K}^{\mathbb{N}} : (|x_j| \alpha_j) \in \ell_p \}, \qquad 1 \leqslant p < \infty$$

and

$$c_o(a) := \{ (x_j) \in \mathbb{K}^{\mathbb{N}} : (|x_j| \alpha_j) \in c_o \}$$

Let A be a monotonic increasing countable system $a^{(1)} \leqslant a^{(2)} \leqslant \ldots$ of sequences $a^{(k)} = (\alpha_{jk})_j$ of positive numbers α_{jk} and let V be a monotonic decreasing system $v^{(1)} \geqslant v^{(2)} \geqslant \dots$ of sequences $v^{(k)} = (\omega_{jk})_j$ of numbers $\omega_{jk} > 0$. The space $\lambda_p(A) := \bigcap_{k=1}^{\infty} \ell_p(a^{(k)})$ (resp. $\lambda_o(A) := \bigcap_{k=1}^{\infty} c_o(a^{(k)})$) is called echelon space of order p (resp. of order 0) which is an (F)-space with respect to the usual topology. The co-echelon space $\mathcal{K}_p(V)$ of order p is defined to be the inductive limit

$$\mathcal{K}_p(V) := \underset{k\to}{\mathrm{ind}}\ \ell_p(v^{(k)}) \qquad \text{for } 1 \leqslant p < \infty$$

and

$$\mathcal{K}_o(V) := \underset{k\to}{\mathrm{ind}}\ c_o\,(v^{(k)}).$$

For each $1 \leqslant p < \infty$ the inductive limit $\mathcal{K}_p(V)$ is regular, hence, by 1.1, every co-echelon space $\mathcal{K}_1(V)$ is a co- $\mathcal{F}_1$-space. Now we prove

*1.3. **Proposition:** If a co-echelon space $\mathcal{K}_p(V)$ of order $1 < p < \infty$ is a co- $\mathcal{L}_1$-space, then $\mathcal{K}_p(V)$ must be nuclear.*

Proof: Let $E = \mathcal{K}_p(V)$ be a co-echelon space of order $1 < p < \infty$ which is a co- $\mathcal{L}_1$-space and let $B \in \mathcal{B}(E)$ be given. There exist $C, D \in \mathcal{B}(E)$ with $B \subset C \subset D$ such that $J_{CD}: \tilde{E}_C \to \tilde{E}_D$ factors through ℓ_p and the compose $I_C \circ J_{BC}: \tilde{E}_B \to \tilde{E}_C''$ of J_{BC} and the evaluation mapping $I_C: \tilde{E}_C \to \tilde{E}_C''$ has a continuous linear factorization through an $L_1(\mu)$-space. The adjoint $J_{BD}' = J_{BC}' \circ J_{CD}': \tilde{E}_D' \to \tilde{E}_B'$ of J_{BD} is r-integral for some $r > \max(2,q)$ where $1/p + 1/q = 1$, since J_{BD}' is the compose of the $L_\infty(\mu)$-factorable operator J_{BC}' and the ℓ_q-factorable operator J_{CD}' (cf. A. Pietsch [21], 22.4.2). Now the compose of n r-integral operators is nuclear where n is any natural number with $2r \leqslant n$ (cf. [21], 20.2.4 and 29.7.2), hence one can find a set $M \in \mathcal{B}(E)$ such that the adjoint $J_{BM}': E_M' \to E_B'$ of J_{BM} is nuclear. Since the double adjoint $J_{BM}'': E_B'' \to E_M''$ is nuclear, the mapping $J_{BM}: E_B \to E_M$ is quasinuclear. This shows that $\mathcal{K}_p(V)$ is co-nuclear. Thus, $\mathcal{K}_p(V)$ is as a (DF)-space also nuclear.

Furtheron, we have

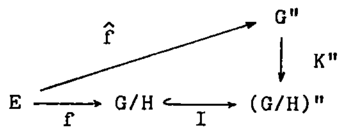
*1.4. **Proposition:** Every echelon space $\lambda_1(A)$ is a co- $\mathcal{L}_1$-space.*

Proof: Let $\lambda_1(A)$ be an echelon space with respect to a monotonic increasing system A of sequences $a^{(k)} = (\alpha_{jk})_j$ of positive numbers. Let E be the co-echelon

$\mathcal{K}_0(V)$ of order 0 where V is the associated decreasing system $v^{(1)} \geqslant v^{(2)} \geqslant \dots$ of sequences $v^{(k)} = (\omega_{jk})_j$ with $\omega_{jk} = 1/\alpha_{jk}$. By a result of K.–D. Bierstedt et al. [2], $\mathcal{K}_0(V)$ is topologically isomorphic to a dense subspace of the reduced projective limit $\operatorname{proj}_{\overleftarrow{\overline{v} \in \overline{V}}} c_0(\overline{v})$ where $\overline{V}$ consists of all sequences $\overline{v} = (\varphi_j)$ of non negative numbers φ_j which satisfy the condition $\sup_j(\varphi_j\alpha_{jk}) < \infty$ for all k. From this it follows that $E = \mathcal{K}_0(V)$ is an ϵ-space since each (B)-space $c_0(\overline{v})$ is an ϵ-space and since the ϵ-spaces are stable under the formation of reduced projective limits and dense subspaces (cf. [9], [10]). Thus for each $U \in \mathcal{U}(E)$ there exists a $V \in \mathcal{U}(E)$ contained in U such that $K_{UV}: \widetilde{E}_V \to \widetilde{E}_U$ is ∞-factorable, hence the adjoint $K'_{UV}: E'_{U^\circ} \to E'_{V^\circ}$ can be factored through an $L_1(\mu)$-space. Since $\lambda_1(A)$ is isomorphic to the strong dual $(\mathcal{K}_0(A))'_b$, $\lambda_1(A)$ is a co- $\mathfrak{L}_1$-space.

2. Liftings of bounded linear mappings in locally convex spaces

Let E be a locally convex space, let G/H be a quotient space of a locally convex space G and let K: G→G/H be the quotient mapping. A mapping $\hat{f}$:E→G is said to be a lifting of a mapping f:E→G/H if f=K∘$\hat{f}$. We say that f:E→G″ is a lifting to G" of f if the following diagram commutes



where I:G/H→(G/H)″ denotes the evaluation mapping.

Let E,F be locally convex spaces. A mapping $A \in L(E,F)$ is called bounded if A maps some 0-neighbourhood in E into a bounded subset of F. The vector space of all bounded linear mappings E→F is denoted by LB(E,F). If $A \in LB(E,F)$, then there exist $U \in \mathcal{U}(E)$ and $B \in \mathcal{B}(F)$ such that A admits a factorization

$$E \xrightarrow{K_U} E_U \xrightarrow{A_{UB}} F_B \xrightarrow{J_B} F$$

where $A_{UB} \in L(E_U, F_B)$.

The following definition plays an important role on the investigation of liftings of bounded linear mappings.

2.1. Definition: A quotient space F/H *of a locally convex space* F *is said to have the* (BL)*-property if each bounded subset* B *of* F/H *can be lifted to a bounded subset* C *of* F, i.e. B ⊂ K(C).

It is well-known that for a (DFM)-space or a normed space F every quotient F/H has the (BL)–property. This, however, need not be true for an (FM)-space F; consider e.g. an echelon space λ_1 which is an (FM)-space and has a closed subspace H such that $\lambda_1/H \simeq \ell_1$ (cf. G. Köthe [15], §31,5.). By a result of A.E. Merzon [20] an (F)-space H is quasi normable if and only if for each locally convex space F containing H as a topological linear subspace F/H possesses the (BL)-property. In particular, if F is an (FS)-space, then each quotient F/H has the (BL)-property.

For a locally convex space E let $E_{\mathfrak{B}}$ be denote the Cartesian ℓ_1-product of all (B)-spaces $\widetilde{E}_B$, $B \in \mathfrak{B}(E)$, i.e. $E_{\mathfrak{B}}$ consists of all families $x = (x_B)$ where $x_B \in \widetilde{E}_B$ for all $B \in \mathfrak{B}(E)$ such that $(\|x_B\|)_B$ is absolutely summable. $E_{\mathfrak{B}}$ is a (B)-space with respect to the norm

$$\|x\| := \sum_{B \in \mathfrak{B}(E)} \|x_B\|.$$

The following proposition gives a connection between the (BL)-property and liftings of bounded linear mappings.

2.2. ***Proposition:*** *Let* E = F/H *be a locally complete quotient space such that each mapping* $A \in LB(E_{\mathfrak{B}}, F/H)$ *has a lifting* $\hat{A} \in LB(E_{\mathfrak{B}}, F)$. *Then* F/H *has the* (BL)*-property.*

Proof: For $B \in \mathfrak{B}(E)$ let $I_B: E_B \to E_{\mathfrak{B}}$, $P_B: E_{\mathfrak{B}} \to E_B$, $J_B: E_B \to E$ and $K: F \to F/H$ be the canonical mappings. By assumption, the compose $J_B \circ P_B \in LB(E_{\mathfrak{B}}, F/H)$ has a lifting $S \in LB(E_{\mathfrak{B}}, F)$. Because of $J_B \circ P_B = K \circ S$ one has $J_B = K \circ S \circ I_B$. The set $C := S \circ I_B(B)$ is bounded in F with $K(C) = B$. This completes the proof.

For every locally complete bornological space F one can define an associated quotient space F_1/H in the following way: Let $\{B_\alpha : \alpha \in I\}$ be the system of all Banach disks B_α in F. For each $\alpha \in I$ the mapping

$$K_\alpha : \ell_1(B_\alpha) \to F_{B_\alpha}, \ (\lambda_x)_{x \in B_\alpha} \to \sum_{x \in B_\alpha} \lambda_x x$$

is a surjective homomorphism. Since F is bornological, the mapping

$$S: \bigoplus_{\alpha \in I} \ell_1(B_\alpha) \longrightarrow F, \quad (z_\alpha)_\alpha \to \sum_\alpha J_\alpha \circ K_\alpha (z_\alpha)$$

from the locally convex direct sum $\bigoplus_\alpha \ell_1(B_\alpha)$ into F is a surjective homomorphism, too, where $J_\alpha : F_{B_\alpha} \to F$ denotes the canonical embedding for each $\alpha \in I$.

Setting $F_1 := \underset{\alpha}{\oplus} \ell_1(B_\alpha)$, the space F is isomorphic to the quotient space F_1/H where H = Ker S.

Later we need

*2.3. **Lemma:** For every locally complete bornological space* F *the associated quotient space* F_1/H *has the* (BL)*-property.*

Proof: Let N be a bounded subset of F_1/H. Let $\check{S}: F_1/H \to F$ be the canonical topological isomorphism and let $K: F_1 \to F_1/H$ be the natural mapping. Since $\check{S}(N)$ is bounded in F, there exists a Banach disk B_β in F and a bounded subset N_β in F_{B_β} with $J_\beta(N_\beta) = \check{S}(N)$ where $J_\beta : F_{B_\beta} \to F$ denotes the canonical embedding. Furthermore, there is a bounded subset M_β in $\ell_1(B_\beta)$ with $K_\beta(M_\beta) = N_\beta$. The set $M := I_\beta(M_\beta)$ is then bounded in $F_1 = \underset{\alpha}{\oplus} \ell_1(B_\alpha)$ where $I_\beta . \ell_1(B_\beta) \to \underset{\alpha}{\oplus} \ell_1(B_\alpha)$ denotes the canonical embedding. Because of

$$\check{S}(K(M)) = S(M) = J_\beta \circ K_\beta (M_\beta) = \check{S}(N)$$

we have K(M) = N.

Now we prove

*2.4. **Proposition:** Let* F *be a locally complete co-* $\mathfrak{L}_1$*-space* [*resp. co-* $\mathcal{F}_1$*-space*]. *Then for every locally convex space* E *and every quotient space* $G/H \simeq F$ *having the* (BL)*-property, every mapping* $A \in LB(E,G/H)$ *has a lifting* $\hat{A} \in LB(E,G''_n)$ [*resp.* $\hat{A} \in LB(E,G)$].

Proof: Let F be a locally complete co- $\mathfrak{L}_1$-space and let $A \in LB(E,G/H)$ be given. There exists a $U \in \mathfrak{U}(E)$ and a $B \in \mathfrak{B}(G/H)$ such that A is the compose of continuous linear mappings

$$E \xrightarrow{K_U} E_U \xrightarrow{A_{UB}} (G/H)_B \xhookrightarrow{J_B} G/H$$

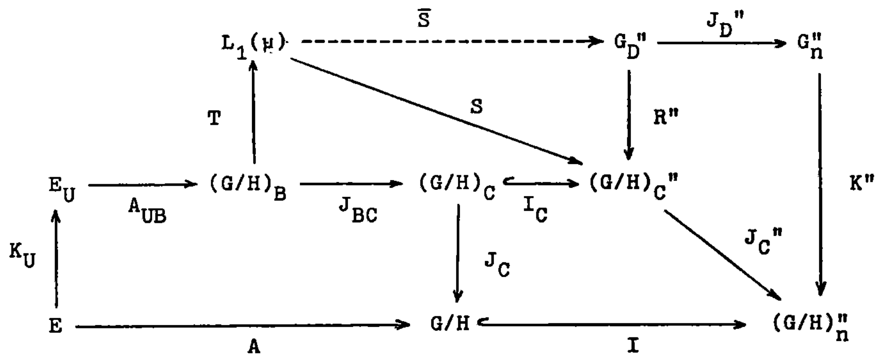
By assumption, there is a $D \in \mathfrak{B}(G)$ with $C := K(D) \supset B$ such that the canonical mapping $J_{BC}: (G/H)_B \to (G/H)_C$ is 1-factorable. Thus the compose $I_C \circ J_{BC}$: $(G/H)_B \to (G/H)_C''$ of J_{BC} and the evaluation mapping $I_C: (G/H)_C \to (G/H)_C''$ has a continuous linear factorization

$$(G/H)_B \xrightarrow{T} L_1(\mu) \xrightarrow{S} (G/H)_C''$$

through some $L_1(\mu)$-space. The linear mapping

$$R: G_D \longrightarrow (G/H)_C, \; x \longmapsto K(J_D(x))$$

is a surjective metric homomorphism. Now the mapping $S \in L(L_1(\mu), (G/H)_C'')$ has a lifting $\overline{S} \in L(L_1(\mu), G_D'')$ with $S = R'' \circ \overline{S}$ (cf. H.E. Lacey [17], p. 178). This leads to the following commutative diagram



where I: $G/H \to (G/H)_n''$ denotes the evaluation mapping. Because of

$$I \circ A = K'' \circ J_D'' \circ \overline{S} \circ T \circ A_{UB} \circ K_U \quad .$$

the mapping $\hat{A} \in LB(E, G_n'')$, defined by

$$\hat{A} := J_D'' \circ \overline{S} \circ T \circ A_{UB} \circ K_U$$

is a lifting of A to G''.

The lifting theorem for co- $\mathcal{F}_1$-spaces F can be proved with the same factorization method using the well-known fact that every continuous linear mapping from a (B)-space into a quotient $X/H \simeq \ell_1$ (I) of a (B)-space X can be lifted to X.

Now we shall show the converse of 2.4 under additional assumptions.

*2.5. **Theorem:** Let* F *be a* (B)*-space or a bornological reflexive space. The following assertions are equivalent*

(1) F *is a co-* $\mathcal{L}_1$*-space.*

(2) *Every mapping* $A \in LB(F_{\mathfrak{B}}, F_1/H)$ *has a lifting* $\hat{A} \in LB(F_{\mathfrak{B}}, (F_1)_n'')$.

(3) *For every locally convex space* E *and for every quotient space* $G/H \simeq F$ *having the* (BL)*-property, every* $A \in LB(E, G/H)$ *has a lifting* $\hat{A} \in LB(E, G_n'')$.

Proof: The implication (1) ⇒ (3) was proved in 2.4 and (3) ⇒ (2) follows by 2.3. It remains to show (2) ⇒ (1).

Let $\mathfrak{B} = \{ B_\alpha : \alpha \in \Gamma \}$ be the system of all Banach disks in F and let $B \in \mathfrak{B}$ be given. Let $P_B : F_{\mathfrak{B}} \to F_B$ and $J_B : F_B \to F$ be the canonical mappings. The compose $A = J_B \circ P_B$ lies in $LB(F_{\mathfrak{B}}, F)$ and has, by assumption, a lifting $\hat{A} \in LB(F_{\mathfrak{B}}, (F_1)''_n)$ such that $S'' \circ \hat{A} = I \circ A$ where $S: F_1 \to F$ and $I: F \to F''_n$ denote the canonical mappings. Let M be the unit ball in $F_{\mathfrak{B}}$. Since $\hat{A}(M)$ is bounded in $(F_1)''_n = \underset{\alpha \in \Gamma}{\oplus} (\ell_1(B_\alpha))''$ there exists a finite set $N \subset \Gamma$ such that the bidual space $F''_o = \underset{\alpha \in N}{\oplus} (\ell_1(B_\alpha))''$ of $F_o := \underset{\alpha \in N}{\oplus} \ell_1(B_\alpha)$ contains $\hat{A}(M)$ as a bounded subset. Thus, $I \circ A$ admits a factorization

$$F_{\mathfrak{B}} \xrightarrow{\hat{A}_1} F''_o \xrightarrow{I''_o} F''_1 \xrightarrow{S''} F''_n$$

where $\hat{A}_1 \in LB(F_{\mathfrak{B}}, F''_o)$ and $I_o: F_o \to F_1$ denotes the canonical embedding. Let $\hat{A}_o \in LB(F_B, F''_o)$ be the restriction mapping of $\hat{A}_1$ to F_B. We obtain the following commutative diagram

$$\begin{array}{ccccc} F''_o & & \xrightarrow{\quad I''_o \quad} & & F''_1 \\ \uparrow{\scriptstyle \hat{A}_o} & & & & \downarrow{\scriptstyle S''} \\ F_B & \xrightarrow[J_B]{} & F & \xrightarrow[I]{} & F''_n \end{array}$$

The (B)-space F''_o is as the bidual of $F_o = \underset{\alpha \in N}{\oplus} \ell_1(B_\alpha)$ an $L_1(\mu)$-space, hence the compose $I \circ J_B : F_B \to F''_n$ can be factored through an $L_1(\mu)$-space.

If F is a (B)-space, then the identity Id: $F \to F$ is 1-factorable, thus F is an $\mathfrak{L}_1$-space.

Now let F be a reflexive space. Then the mapping $J_B: F_B \to F$ has a continuous linear factorization

$$F_B \xrightarrow{R_1} L_1(\mu) \xrightarrow{R_2} F.$$

Furthermore, there exists a $C \in \mathfrak{B}(E)$ containing B such that R_2 can be decomposed into

$$L_1(\mu) \xrightarrow{T} F_C \xhookrightarrow{J_C} F$$

where $T \in L(L_1(\mu), F_C)$. Now we have $J_C \circ J_{BC} = J_B = J_C \circ T \circ R_1$, hence $J_{BC} = T \circ R_1$, since J_C is injective. It follows that J_{BC} is 1-factorable. This completes the proof.

For co- $\mathcal{F}_1$-spaces the following lifting theorem holds

*2.6. **Theorem:** Let* F *be a locally complete bornological space. The following assertions are equivalent*
(1) F *is a co-* $\mathcal{F}_1$*-space.*
(2) *Every mapping* $A \in LB(F_{\mathcal{B}}, F_1/H)$ *has a lifting* $\hat{A} \in LB(F_{\mathcal{B}}, F_1)$.
(3) *For every locally convex space* E *and for every quotient space* $G/H \simeq F$ *with the* (BL)*-property, every* $A \in LB(E,G/H)$ *has a lifting* $\hat{A} \in LB(E,G)$.

Proof: In view of 2.3 and 2.4 it remains to prove the implication (2) ⇒ (1).

Let $\mathcal{B} = \{B_\alpha : \alpha \in \Gamma\}$ be the system of all Banach disks in F and let $B \in \mathcal{B}$ be given. Similar as in the proof of 2.5 one can show that there exists a finite set $N \subset \Gamma$ such that the mapping $J_B: F_B \to F$ can be factored through the $\ell_1(I)$-space $F_o = \bigoplus_{\alpha \in N} \ell_1(B_\alpha)$. Now one can find a $C \in \mathcal{B}(F)$ with $B \subset C$ such that $J_{BC}: F_B \to F_C$ factors through F_o, hence J_{BC} is discretely 1-factorable. We conclude that F is a co- $\mathcal{F}_1$-space. This completes the proof.

If F is a (B)-space such that every bounded linear mapping A from a (B)-space E into a quotient $G/H \simeq F$ of a (B)-space G has a bounded linear lifting $\hat{A}: E \to G$, then F is isomorphic to $\ell_1(I)$ for some index set I (cf. G. Köthe [14], p. 188). Thus, by 2.6 we get

*2.7. **Proposition:** A* (B)*-space* E *is a co-* $\mathcal{F}_1$*-space if and only if* E *is isomorphic to* $\ell_1(I)$ *for some index set* I.

By a result of K. Floret [7], p. 110, a bounded linear mapping A from a $\mathcal{L}_1$-space E into a quotient G/H of a locally convex space G has a bounded linear lifting $\hat{A}: E \to G_n''$ if the image A(B) of the unit ball B in E is contained in the closure $\overline{K(C)}$ for some bounded subset C of G where $K: G \to G/H$ denotes the quotient mapping. We mention without proof that this result can be generalized to locally convex π-spaces by using the above factorization method. Let us further remark that liftings of compact linear mappings between locally convex spaces were investigated in R. Hollstein [9] with tensor product methods.

3. Locally convex spaces with the (weak) holomorphic lifting property

Throughout this section all locally convex spaces are assumed to be complex vector spaces.

Let E and F be locally convex spaces. A continuous mapping f: E→F is said to be holomorphic in E if for each $y' \in F'$ the function $y' \circ f$ is Gâteaux-holomorphic, i.e. for all $a,b \in E$ the function $z \to y'(f(a+zb))$ is holomorphic in $\mathbb{C}$. The vector space of all holomorphic mappings from E into F is denoted by H(E,F).

*3.1. **Definition:** A mapping* $A \in H(E,F)$ *is said to be of locally uniform bounded type if there exist* $U \in \mathfrak{U}(E)$ *and* $B \in \mathfrak{B}(F)$ *such that for all* $x \in E$ *there are* $\lambda_x > 0$ *and* $\mu_x > 0$ *with*

$$f(x + \lambda_x U) \subset \mu_x B.$$

By $H_{lub}(E,F)$ we denote the linear subspace of H(E,F) consisting of all mappings of locally uniform bounded type. By definition, every mapping $f \in H_{lub}(E,F)$ is locally bounded, i.e. each point $x \in E$ has a neighbourhood whose image under f is bounded.

Now we prove

*3.2. **Proposition:** Let* E *and* F *be locally convex spaces. A mapping* $f \in H(E,F)$ *is of locally uniform bounded type if and only if there exist* $U \in \mathfrak{U}(E)$ *and* $B \in \mathfrak{B}(F)$ *such that* f *can be decomposed into*

$$E \xrightarrow{K_U} E_U \xrightarrow{\overline{f}} F_B \xhookrightarrow{J_B} F$$

where $\overline{f} \in H(E_U,F_B)$.

Proof: Let $f \in H_{lub}(E,F)$. There exist $U \in \mathfrak{U}(E)$ and $B \in \mathfrak{B}(F)$ such that for all $x \in E$ there are $\lambda_x > 0$ and $\mu_x > 0$ with $f(x + \lambda_x U) \subset \mu_x B$. If $x - y$ lies in the null space of U, then one has $f(x) = f(y)$, since by the Liouville theorem the bounded entire function $\alpha \to y'(f(x+\alpha(y-x)))$ in $\mathbb{C}$ is constant for all $y' \in F'$. The mapping $\overline{f}: E_U \to F_B$, defined by $\overline{f}(K_U(x)) = f(x)$ is well-defined and holomorphic with $J_B \circ \overline{f} \circ K_U = f$.

Now let $f \in H(E,F)$ be a holomorphic mapping which can be decomposed into

$$E \xrightarrow{K_U} E_U \xrightarrow{\overline{f}} F_B \xhookrightarrow{J_B} F$$

where $\overline{f} \in H(E_U,F_B)$. Since $\overline{f}$ is continuous there exist, for each $x \in E$, numbers $\lambda_x > 0$ and $\mu_x > 0$ such that $\overline{f}(K_U(x) + \lambda_x K_U(U)) \subset \mu_x B$. Thus, we have $f(x+\lambda_x U) \subset \mu_x B$.

From 3.2 it follows immediately that a mapping $f: E \to F$ is holomorphic of locally uniform bounded type if and only if there exist normed spaces E_o and F_o such that f admits a factorization

$$E \xrightarrow{A} E_o \xrightarrow{\bar{f}} F_o \xrightarrow{B} F$$

where $\bar{f}$ is holomorphic and A,B are continuous and linear.

For normed spaces E and F a mapping f ϵ H(E,F) is said to be of bounded type if it is bounded on all bounded subsets of E. Following, J.F. Colombeau, J. Mujica [5] a holomorphic mapping f between locally convex spaces E and F is said to be of uniform bounded type if there exist U $\epsilon\mathcal{U}(E)$ and B $\epsilon\,\mathcal{B}(F)$ such that f can be decomposed into

$$E \xrightarrow{K_U} E_U \xrightarrow{\bar{f}} F_B \xhookrightarrow{J_B} F$$

where $\bar{f}$ is holomorphic of bounded type. The vector space of all holomorphic mappings f: E→F of uniform bounded type is denoted by $H_{ub}(E,F)$.

For all locally convex spaces E and F the following inclusions hold

$$H_{ub}(E,F) \subset H_{lub}(E,F) \subset H(E,F).$$

If E and F are normed spaces, then one has

$$H(E,F) = H_{lub}(E,F)\,,$$

but even $\mathbb{C}$-valued holomorphic mappings on locally convex spaces are generally not of locally uniform bounded type. For example, let E be the (FN)-space $H(\mathbb{C})$ of all entire functions on $\mathbb{C}$ equipped with the topology of uniform convergence on the compact subsets of $\mathbb{C}$. The mapping f: $H(\mathbb{C}) \to \mathbb{C}$, $\varphi \to \varphi(\varphi(0))$ is holomorphic but is not of locally uniform bounded type (cf. J.F. Colombeau [4], 2.7.2).

In general, $H_{ub}(E,F)$ is a proper subspace of $H_{lub}(E,F)$. Consider e.g. a holomorphic mapping between (B)-spaces E and F which is not of bounded type.

If E is a (DFM)-space and F is a metrizable locally convex space, then by a result of J.F. Colombeau, J. Mujica [5] one has

$$H_{ub}(E,F) = H_{lub}(E,F) = H(E,F).$$

Let us remark that R. Meise and D. Vogt [19] have investigated necessary and sufficient conditions for (FN)-spaces E satisfying the relation $H(E,\mathbb{C}) = H_{ub}(E,\mathbb{C})$.

*3.3. **Definition:** A locally convex space* F *is said to have the holomorphic lifting* *property [resp. weak holomorphic lifting property] if for every locally convex space* E *and for every quotient* $G/H \simeq F$ *having the* (BL)*-property, every mapping* $A \in H_{lub}(E,G/H)$ *has a holomorphic lifting* $\hat{A} \in H_{lub}(E,G)$ *[resp.* $\hat{A} \in H_{lub}(E,G''_n)$*].*

Now we prove

3.4. ***Proposition:*** *The following assertions hold*

(1) *Every locally complete co-* $\mathfrak{L}_1$*-space has the weak holomorphic lifting property.*

(2) *Every locally complete co-* $\mathfrak{F}_1$*-space has the holomorphic lifting property.*

Proof: (1) Let F be a locally complete co- $\mathfrak{L}_1$-space, let $G/H \simeq F$ be a quotient space with the (BL)-property, let E be any locally convex space and let $f \in H_{lub}(E,G/H)$. There exist $U \in \mathfrak{U}(E)$ and $B \in \mathfrak{B}(G/H)$ such that f admits a factorization

$$E \xrightarrow{K_U} E_U \xrightarrow{\overline{f}} (G/H)_B \overset{J_B}{\hookrightarrow} G/H$$

where $\overline{f} \in H(E_U,(G/H)_B)$. Applying 2.4, $J_B \in LB((G/H)_B,G/H)$ has a lifting $\hat{J}_B \in LB((G/H)_B\ G''_n)$with $I\circ J_B = K''\circ\hat{J}_B$ where $I: G/H \to (G/H)''$ and $K: G \to G/H$ denote the canonical mappings. Thus we have

$$I\circ f = I\circ J_B\circ\overline{f}\circ K_U = K''\circ\hat{J}_B\circ\overline{f}\circ K_U ,$$

hence $\hat{f}: = \hat{J}_B\circ\overline{f}\circ K_U \in H_{lub}(E,G''_n)$ is a lifting of f to G''_n. This proves that F has the weak holomorphic lifting property.

(2) Using 2.4, the assertion (2) can be shown in a similar way.

Now we shall prove the converse of 3.4(1) under the assumptions of 2.5.

3.5. ***Theorem:*** *Let* F *be a* (B)*-space or a bornological reflexive space. The following assertions are equivalent*

(1) F *is a co-* $\mathfrak{L}_1$*-space.*

(2) F *has the weak holomorphic lifting property.*

(3) *Each mapping* $f \in H_{lub}(F_{\mathfrak{B}}, F_1/H)$ *has a holomorphic lifting* $\hat{f} \in H(F_{\mathfrak{B}}, (F_1)''_n)$.

Proof: It remains to prove the implication (3) ⇒ (1). By 2.5 it suffices to show that each linear mapping $A \in LB(F_{\mathfrak{B}}, F_1/H)$ has a linear lifting $\check{A} \in LB(F_{\mathfrak{B}}, (F_1)''_n)$.

Let $A \in LB(F_{\mathfrak{B}}, F_1/H)$ be given. By assumption, there exists a holomorphic lifting $\hat{A} \in H(F_{\mathfrak{B}}, (F_1)_n'')$ with $K''\circ\hat{A} = I\circ A$ where $K: F_1 \to F_1/H$ and $I: F_1/H \to (F_1/H)_n''$ denote the canonical mappings. For each $x \in F_{\mathfrak{B}}$ there exists a power series $\sum_{m=o}^{\infty} \frac{\hat{d}^m\hat{A}(x)}{m!}(y-x)$ wich converges uniformly to $\hat{A}(y)$ in a neighbourhood of x where, for each $m \in \mathbb{N}$, $\frac{\hat{d}^m\hat{A}(x)}{m!}$ is a continuous m-homogeneous polynomial from $F_{\mathfrak{B}}$ into $(F_1)_n''$ (cf. S. Dineen [6], p. 55). Setting $\check{A} := \hat{d}^1\hat{A}(0) \in LB(F_{\mathfrak{B}}, (F_1)_n'')$, $\check{A}(x)$ is the directional derivative of $\hat{A}$ at 0 in direction to x. Thus, for each $x \in F_{\mathfrak{B}}$ one has

$$K''\circ\check{A}(x) = K''(\lim_{\lambda\to o}(1/\lambda)\,\hat{A}(\lambda x)) = I\circ A(x)$$

hence $\check{A} \in LB(F_{\mathfrak{B}}, \overline{(F_1)_n''})$ is the required linear lifting of A. This completes the proof.

Up to minor modifications, the proof of the following theorem is the same as in 3.5

*3.6. **Theorem:** Let* F *be a locally complete bornological space. The following assertions are equivalent*

(1) F *is a co-* $\mathfrak{F}_1$*-space.*

(2) F *has the holomorphic lifting property.*

(3) *Each mapping* $f \in H_{lub}(F_{\mathfrak{B}}, F_1/H)$ *has a holomorphic lifting* $\hat{f} \in H(F_{\mathfrak{B}}, F_1)$.

By the above mentioned result of J.F. Colombeau and J. Mujica [5] each holomorphic mapping from a (DFM)-space into a metrizable locally convex space is holomorphic of uniform bounded type. From 3.4 it follows

*3.7. **Corollary:** Let* E *be a (DFM)-space and let* F *be a co-* $\mathfrak{F}_1$*-Fréchet space [resp. co-* $\mathfrak{L}_1$*-Fréchet space]. Then for each quotient space* $G/H \simeq F$ *having the (BL)-property, every mapping* $f \in H(E,G/H)$ *has a lifting* $\hat{f} \in H(E,G)$ *[resp.* $\hat{f} \in H(E,G_n'')$*].*

It is well-known that every Fréchet-valued holomorphic mapping on a closed subspace of a (DFN)-space G has a holomorphic extension to G (cf. J.F. Colombeau, J. Mujica [5], 7.4). From 3.7 we obtain a dual version of this holomorphic extension theorem

*3.8. **Corollary:** Let* E *be a (DFM)-space and let* F *be an* (FN)*-space. Then for every quotient* $G/H \simeq F$ *with the* (BL)*-property every holomorphic mapping* $f: E \to G/H$ *has a holomorphic lifting* $\hat{f}: E \to G$.

From 3.8 it follows that every holomorphic mapping f from a (DFM)-space E into a quotient G/H of an (FN)-space G has a holomorphic lifting $\hat{f}$: E→G since the quotient of an (FN)-space possesses always the (BL)-property.

Let us remark that a holomorphic mapping f from an (FN)-space E into a quotient G/H of an (FN)-space G need not be holomorphically liftable to G. For example, let G be the (FN)-space H($\mathbb{C}$). G has a continuous norm, hence there exists a closed subspace H of G which is not complemented in G (cf. G. Köthe [15], § 31.4(1)). We assume that the identity Id: G/H→G/H has a holomorphic lifting $\hat{\mathrm{Id}}$: G/H→G. The argument used in the proof of 3.5 shows that Id: G/H→G/H has a continuous linear lifting $\check{\mathrm{Id}}$: G/H→G. If K: G→G/H denotes the canonical mapping, then the mapping P: = $\check{\mathrm{Id}}$∘K is a continuous projection with Ker P = H, hence we have a contradiction.

For (B)-spaces, we get by 2.7 and 3.6

*3.9. **Proposition:** Let* F *be a (B)-space. The following assertions are equivalent*

(1) F *is isomorphic to* ℓ_1(I) *for some index set* I.

(2) F *has the holomorphic lifting property.*

(3) *For every (B)-space* E *and every quotient* G/H ≃ F, *every holomorphic mapping* f: E→G/H *has a holomorphic lifting* $\hat{f}$: E→G.

It is well-known that the $\mathcal{L}_1$-spaces are precisely those (B)-spaces F which have the compact lifting property, i.e. for every (B)-space E and every quotient G/H ≃ F, every compact linear mapping A: E→G/H has a compact linear lifting $\hat{A}$: E→G. By 3.5 we obtain the following characterization of $\mathcal{L}_1$-spaces by means of holomorphic liftings

*3.10. **Proposition:** Let* F *be a (B)-space. The following assertions are equivalent*

(1) F *is an* $\mathcal{L}_1$*-space.*

(2) F *has the weak holomorphic lifting property.*

(3) *For every (B)-space* E *and every quotient space* G/H ≃ F *of a (B)-space* G, *every holomorphic mapping* f: E→G/H *has a holomorphic lifting* $\hat{f}$: E→G″.

For (B)-spaces E and F, H_K(E,F) denotes the vector space of all holomorphic mappings f: E→F such that for each x ∈ E there is a neighbourhood V of x such that $\overline{f(V)}$ is compact in F. Let us remark that R.M. Aron [1] has proved with a tensor product method that for each quotient space F = G/H for which F′ is complemented in G′ and F has the approximation property, every f ∈ H_K(E,G/H) has a lifting $\hat{f}$ ∈ H_K(E,G).

By 3.4 every co-echelon space $\mathcal{K}_1$(V) has the holomorphic lifting property and by 1.4 and 3.4 every echelon space λ_1(A) possesses the weak holomorphic lifting property. For (co-) echelon spaces of order 1 < p < ∞, we get

*3.11. **Proposition:** Let* E *be a co-echelon space* $\mathcal{K}_p(V)$ *or an echelon space* $\lambda_p(A)$ *of order* $1 < p < \infty$. *The following assertions are equivalent*
(1) E *is nuclear.*
(2) E *has the weak holomorphic lifting property.*
(3) E *has the holomorphic lifting property.*

Proof: If $E = \mathcal{K}_p(V)$, $1 < p < \infty$, then the equivalence (1) $\Leftrightarrow$ (2) $\Leftrightarrow$ (3) follows from 1.3 and 3.6.

Now let E be an echelon space $\lambda_p(A)$ of order $1 < p < \infty$. It remains to show the implication (2) $\Rightarrow$ (1).

Suppose that E has the weak holomorphic lifting property. By 3.5 E is a co- $\mathcal{L}_1$-space. Let us show that the strong dual E'_b is an ϵ-space, i.e. for each $U \in \mathfrak{U}(E'_b)$ there is a $V \in \mathfrak{U}(E'_b)$ contained in U such that the canonical mapping $K_{UV}: \widetilde{(E'_b)}_V \longrightarrow \widetilde{(E'_b)}_U$ is ∞-factorable. Let $B \in \mathfrak{B}(E)$ be given. There exists a $C \in \mathfrak{B}(E)$ containing B such that the canonical mapping $J_{BC}: E_B \to E_C$ is 1-factorable. It follows that the adjoint $J'_{BC}: E_C' \to E_B'$ has a continuous linear factorization through an $L_\infty(\mu)$-space. Since E is reflexive, one has $E_B' = (E'_{B^\circ})''$ and $E_C' = (E'_{C^\circ})''$ isometrically, hence the double adjoint $K_{B^\circ C^\circ}'': (E'_{C^\circ})'' \to (E'_{B^\circ})''$ of the canonical mapping $K_{B^\circ C^\circ}: E'_{C^\circ} \to E'_{B^\circ}$ factors through $L_\infty(\mu)$. Thus, $K_{B^\circ C^\circ}$ is ∞-factorable and we conclude that $E'_b = \mathcal{K}_q(V)$ is an ϵ-space where $1/q + 1/p = 1$ and V is the associated matrix of A. By R. Hollstein [11], lemma 3, every co-echelon space $\mathcal{K}_q(V)$ of order $1 < q < \infty$ which is an ϵ-space must be nuclear. It follows that $\lambda_p(A)$ is nuclear. This completes the proof.

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