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Fernando Serrano (1957-1997)

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A biographical sketch (S. Xambó)

Fernando Serrano died on the second day of March 1997 after a short illness. Until the last moment he enjoyed life although he knew he was dying. He died as he always lived and worked, elegantly.

Fernando, who would have been 40 on the last day of July 1997, was at the apogee of a brilliant mathematical career, and it was almost two years since he and his wife, María Morrás Ruiz-Falcó, now Assistant Professor of Romance Philology at the "Universitat Pompeu Fabra" (Barcelona), were fathers of Álvaro, a child for whom Fernando felt a deep devotion.

Born in Barcelona on July 31 of 1957, in an impoverished but hard-working family of emigrants, his parents fought to give him and his two brothers the education they were barred from. The fruits of their struggle are evident: Fernando was not only a fine mathematician, but also a man with deep interest in culture.

If the schooling book reveals that his grades, including mathematics and physics, were always excellent, an unmistakable sign that we are in the presence of a boy with an exceptional talent is a personal diary, which was written during the term 1971-1972 (thus, he was fourteen years old) and in which he recorded with extraordinary care the reflections of all sorts that he was going through. As a matter of fact, a good part of these disquisitions have a mathematical character and their intellectual maturity suggests that he already knew that he would become a mathematician.

Even though his vocation was clear, Serrano explored other possibilities before taking the decision of devoting himself to mathematics. But he soon discovered that those were universes lying far away from his calling, that none of these disciplines would really satisfy his criteria of rigor and beauty. Therefore he enrolled in the five year program offered by the "Facultat de Matemàtiques" of the "Universitat de Barcelona", to obtain a bachelor's degree in mathematics. He joined in at the beginning of October 1974 and finished at the end of June 1979.

Those years were not easy for Serrano, for in order to contribute to the sustainment of his family, he worked for two years, in the evenings, in the comptability department of the "Corte Inglés" stores. He also taught many private tutorial classes. Yet these activities did not divert Serrano from his conscientious commitment to his studies, or from devoting some effort to "social" tasks at the university. In his fourth year he chose the option of "Fundamental Mathematics", and his interest in Algebra and Algebraic Geometry becomes an obvious fact when one considers the list of courses he took in his fifth year: Number Theory, Homological Algebra, Algebraic Geometry and Algebraic Curves. Moreover, in his fourth year he had taken Commutative Algebra, and had written (jointly with Joan-Miquel Sueiro) an article on Fermat's last theorem in Aleph, a magazine for the mathematics students.

In October of 1979, Ferran Serrano joined the Department of "Algebra i Fonaments" of the University of Barcelona as an assistant professor. He remained in this position for two years, in charge of the problem sessions of two of the key courses assigned to the Department: Algebra, which was a compulsory course in the third year, and Commutative Algebra, an optional course that could be taken in the fourth or fifth year. In that time, the mathematics students in the University of Barcelona could take, after graduation, an optional "revalidation examination". Passing this examination entitled them to a "degree", which was comparable to a Master's degree, and with it they could opt for the graduation Extraordinary Prize. Serrano took the examination in July of 1980 and with it he won the Extraordinary Prize.

In addition, Serrano had a very active participation in the Commutative Algebra and Algebraic Geometry Seminar that J. M. Giral and I had organized in the Department and which was focussed on the "homological conjectures", basically as formulated by M. Hochster. Not long after the Seminar had begun, Serrano found a remarkably simple proof of the formula p = d - r of Auslander and Buchsbaum relating the projective dimension p and the grade r of a finitely generated module M over a noetherian local ring A of grade d (see [1]¹).

Serrano also wrote two monographs [2, 3] on the subjects he prepared for the Seminar. The work [2], written in collaboration with J. M. Giral, contains a proof of the theorem of Auslander and Lichtenbaum according to which the Tor rigidity conjecture is true for regular rings and a proof of the theorem of Auslander (1961) according to which that conjecture implies the divisor of zero conjecture. The proof of this last theorem presented in the monograph does not use spectral sequences and is substantially more accessible than the original one. A far as [3], it is a quite lucid exposition of the work of M. Artin Algebraic approximation of structures

See Fernando Serrano's list of publications.

over complete local rings (Publ. Math. IHES, **36** (1969), 23-58), and it still can be considered a useful presentation of this subject.

In this period Serrano also wrote a short paper (unpublished) with the title A Note about the Grade of certain Ideals. In fact, he wrote it in two parts. In the first part, which is two pages long, he proves that the grade and the height of an ideal of finite homological dimension coincide if this ideal is the product of prime ideals (not necessarily distinct). In the second part, he gives a sufficient condition in order that the grade and the height of an ideal be the same, and then he proves that an ideal of finite projective dimension satisfies that condition if it is the intersection of finitely many ideals each of which is a product of finitely many prime ideals.

In this point of his life, Fernando Serrano decided to work for a doctoral degree at the University of Brandeis (Waltham, Massachusetts). He was offered a full tuition scholarship, a fellowship and a teaching assistantship, and he accepted them. He was also awarded a scholarship by the Spanish-Northamerican Joint Committee for Education and Culture, of the Friendship and Cooperation Treaty between both countries, which covered, among other items, the round-trip travel expenses.

He was enrolled in the Brandeis doctoral program in September 1981. At the end of the term 1981-1982, he received the Master of Arts degree and, at the end of the term 1984-1985, the Doctor of Philosophy degree in the field of Mathematics. His student record reveals that Serrano acquired, during these four terms, a solid and wide preparation in Algebraic Geometry, complemented with extensive studies in Commutative Algebra and Number Theory. His thesis [4] was defended on May 19, 1985, before a jury composed of D. Eisenbud, his thesis advisor, J. Harris, T. Matsusaka and P. Monsky. A year later, the Spanish Council of Universities awarded him the Doctor of Mathematics degree by ratifying the Brandeis degree.

This list of indisputable successes appears the more noteworthy the more one looks at diverse circumstances that concurred in Serrano's life during those years in Brandeis. A few months before his departure, he suffered a retina detachment. This mishap, which immobilized him for two weeks, and which led him to entertain serious doubts about his decision of moving to Brandeis, exposed the fact that his eyesight was quite delicate: often he had tired eyes and because of this he could not work for as many hours as he would have liked to. Another misfortune occurred at the beginning of his second term in Brandeis. His father fell severely ill and Fernando decided to return to Barcelona to stand close to his family in that adversity. He remained until he could see, a few weeks later, that his father's life was out of danger and that he would recover appreciably.

Thus the Brandeis period, with much lonely and hard work, was quite difficult for Serrano. But in those years he honed his capacity to find and select interesting problems, and he developed and tuned a combination of intuitions and techniques suitable to attack them. It is most likely that the experiences in those times were the yeast that produced his definitely individualistic attitude toward work and life,

his esteemed independence of thought and action. It was on those years that he adopted Hemingway's saying "to keep grace under pressure" as life's *motto*.

During the academic years 1985-1986 and 1986-1987, Serrano was Visiting Professor of the University of California at Berkeley, at the Department of Mathematics. If at Brandeis he had taught several courses on Precalculus and Calculus, and a class of Applied Linear Algebra in the Summer School, he only acquired full teaching responsibilities in this period, with a teaching load of two courses per semester, each course meaning three hours a week of class.

Among the teaching responsibilities that were entrusted to him was the course of Algebraic Geometry of the Berkeley doctoral program and which he imparted during the term 1986-1987. Besides, he was included immediately in the research team led by R. Hartshorne and A. Ogus. One of the first activities in this connection was his participation in Berkeley Algebraic Geometry Seminar, where in October 1985 he gave two lectures with the title Surfaces with a d-gonal hyperplane section.

On occasion of the International Congress of Mathematicians held at Berkeley in the Summer of 1986, H. Rossi assigned him to present V. V. Shokurov and to chair the session on *Numerical geometry of algebraic 3-folds* which this distinguished invited speaker was supposed to address. Unfortunately, Shokurov and other mathematicians of the then Soviet Union were not allowed to go to Berkeley, and so Serrano could not fully enjoy the honor which had been conferred to him.

It deserves a special mention his teaching job at Berkeley. In the teaching evaluation of the students that attended his courses, the high or very high marks are the rule. Rankings such as "excellent instructor", "organized and clear in his lectures", "ability to communicate with students", "care, concern and willingness to answer questions", "positive attitude towards his class", "effectiveness of his use of examples", "very well prepared" are recurring. In the Abstract Algebra course, many students said that Serrano was "the best math teacher they have encountered at Berkeley", and that he was "well prepared and gave exceptionally clear lectures". Also, in a reference letter of T. Y. Lam written at the beginning of 1987, in addition to many statements that are similar to those that we have already commented, some adduce views of other aspects. Of the students teaching evaluation he says that they place Serrano "into the top group of teachers in our Department". And with regard to the algebraic geometry course he says that "this is an extremely difficult course to teach, even for the senior faculty".

When the Berkeley contract was coming to its end, Serrano asked for a scholarship of the Spanish program to reincorporate people with a Ph.D. degree from a foreign university. He was awarded it, but he renounced it in favor of the University of Utah offer of a visiting position for the academic year 1987-1988. In the Utah contract, the teaching load was two courses per quarter. As far as research goes, he took an active part in the Algebraic Geometry Seminar, in which he gave two lectures on *The Picard group of a quasi-bundle* on April 1988.

In the Fall of 1987, Serrano won the position of "Professor titular" (a tenure position comparable to an Associate Professorship) at the Department of Algebra and Geometry of the University of Barcelona, but he did not actually take office in this job until the term 1988-1989. From this moment on, however, it is in this position where he carried out his main activities, be it the teaching of a number of courses, the involvement in academic responsibilities proper for a department, or the research in the field of algebraic geometry. In any case, Serrano considered that he had found an ideal place for his career.

Much of his teaching activity of this period belongs to algebraic geometry proper. For example, in each of the first five academic years he taught a doctoral course closely related to the topics of his specialty: Algebraic surfaces (1988-1989), Theory of schemes (1989-1990), Algebraic Geometry Seminar (1990-1991 and 1991-1992) and Projective varieties (1992-1993). He also taught the course Algebraic Geometry II, of the fifth undergraduate year, for three terms (1991-1992 to 1993-1994; in the academic years 1988-1989 and 1989-1990, he had already been in charge of the problem sessions of this same course).

In the last two terms, there is shift in the subjects of his teaching: he is entrusted twice the Differential Geometry of Curves and Surfaces course, once the Projective Geometry course (1994-1995) and once the Algebra II course (1995-1996). In fact, this trend can already be perceived in the academic year 1993-1994: if on one hand he teaches the Algebraic Geometry II course, on the other he does not teach a doctoral course, so that he completed his load with Projective Geometry and the problem sessions of Algebra II. These changes are related, most probably, with the new mathematics curriculum at the University of Barcelona, structured in four terms since 1992-1993, maybe also with the enrollment of new members by the department, or just with his desire to explore and study in depth subjects that he thought were complementary to his education in algebraic geometry.

In 1993 he organized the third annual meeting of the project Geometry of algebraic varieties, of the SCIENCE program of the European Community. This meeting was held in Barcelona on the 10 and 11 December of that year. Serrano was a member of other research projects: two of the DGICYT, with the title Families of algebraic varieties, I and II, and which were developed in the periods 1991-1994 and 1994-1997, respectively, and of the European project Algebraic Geometry in Europe, to be carried out in the period 1994-1997.

He was also one of the organizers of the Algebraic Geometry Seminar of the Department of Algebra and Geometry for two terms: 1990-1991 and 1991-1992. Serrano was also one of the organizers of the conference Europroj-94, which was held in September of 1994, 23-26.

Another indication of the high scientific consideration in which Serrano was held is that he was invited to serve on the jury of a good number of doctoral dissertations on algebraic geometry in several universities all over Spain, and his participation in them was reputed to be especially effective.

Fernando Serrano was the Secretary of the Department of Algebra and Geometry for nearly three years: from April 10, 1992, till March 13, 1995. This fact, and the activities we have been describing in the foregoing paragraphs, show the good disposition of Serrano to cooperate in the collective tasks of his department or of the algebraic geometry community at large. His colleagues at the department considered that he was very sociable, in good terms with everybody, and that he had a calm stance in the face of the problems, not always easy, that come up in the life of a large department.

Serrano's mind was logical, but also adaptable, resilient, unconventional, capable of finding unsuspected shortcuts to arrive at the solution of problems. He had creativity and intuition, much energy and good judgment, and an innate aesthetic sense. He was concentrated, lucid, independent and reserved. He paid attention to details. In spite of the fact that for him feelings were most important, he preferred to set a distance, both physical and emotional, between himself and the others. At the same time, however, he was extroverted, enthusiastic and generous. His legacy to the mathematical community is something more than just a scientific one: his absolute integrity, his serene, courageous, godly example, will remain alive in the mind of all those that knew him.

Fernando had a deep religious sensibility, an unfaltering internal discipline, that supplied him the strength with which he faced, without complaining, physical or moral hardships. Not even in the last stages of his illness, which inflicted him frightful sufferings, was he tripped to whine: he thought that pain eventually leaves not traces in memory and that therefore it is preferable not to manifest it. Serrano knew well the *Mémoires d'Hadrien*, of Marguerite Yourcenar, and his fortitude is not unlike the emperor's when uttering his last sentence: "Tâchons d'entrer dans la mort les yeux ouverts...". This resolute health of his spirit entailed that many of his colleagues and friends were mistaken about the health of his body, so that they though he still had energies that in fact had abandoned him.

He delivered his life to the angel of death that was claiming it within the moral conditions that he had wanted: with a full conscience and with all his intellectual faculties intact. He even took scrupulously the steps that he deemed most suitable to prepare the people close to him for the denouement that was drawing up, for he wished that nobody collapsed, that nobody relinquished any bit of his life because of him. There is no doubt that he appreciated the amenities of life (a good meal, thoughtfully chosen clothing, going to the movies, a stroll in the evening, or the shelter of his house), and that he valued friendship much (this is why he felt so happy and grateful to all those that visited him), but at the end maybe he muttered to himself, as Hadrianus did:

Animula vagula, blandula, Hospes comesque corporis, Quae nunc abibis in loca Pallidula, rigida, nudula, Nec, ut soles, dabis iocos.

Carme Pedro Olivé, the woman doctor in charge of the medical treatment of Fernando's illness, found in him a person that was prepared to "die at ease with himself", and that she "did not recall anybody that died with his serenity". His profound dignity through his last months suggests that Fernando's personality had already reached a higher state of being.

A few remarks on his mathematical evolution

We have included [1, 2, 3], to which we referred earlier, because they shed some light onto Serrano's mathematical evolution. They are a presentation card of a brilliant young man which has just graduated and has already entered terrains of a considerable complexity. But they also are like an open parenthesis which will not close until he publishes, five years later, the results he obtained in his thesis and of which we will speak below.

During these five years, he first had to follow the regular course of the Brandeis doctoral program, but for anyone in Serrano's situation, this may appear as somewhat of a waste of time, as most of the subjects in the first year were already known to him, or did not seem quite relevant for his purposes. In any case, however, it is clear that this long parenthesis raises considerably his mathematical education and opens for him opportunities, such as being accepted as a visiting professor in Berkeley's Evans Hall, which otherwise he would hardly been able to reach.

There are still other factors that one may take into account. For example, of all his works, [2] and [19] are the only ones written jointly with someone else. This indicates a fact: Serrano liked to conduct research alone. He spent many hours a day at his desk, in a serene and reflective attitude, pondering the problems that interested him, and writing his findings with a most characteristic calligraphy.

Terrain

There are in addition his "style norms". It is quite clear that he had a refined aesthetic sense that could be seen in the problems he selected, in the way he dealt with them, in the form he chose to present the results, and even in the journal where he decided to submit them. Furthermore, Serrano followed Gauss principle of pauca sed matura, because his writings have a high quality. For example, he postponed one year the publication of the main results of his thesis because F. Catanese acknowledged him, during the AMS Summer 1985 Algebraic Geometry Conference held at the Bowdoin College (Main, USA), of the existence of a vanishing theorem of Miyaoka, stronger than the one he had been using in his thesis, and which, in his own words, "when applied to the situations that interested me, yielded improved or new results", and so he spent the year following the Summer 1985 working out these ideas. Thus the papers [5, 6, 7], which are the result of this work, are not a faithful reflection of his thesis, but rather an augmented and improved version of it.²

Similar comments can be made of his participation in conferences, congresses and seminars, which he reduced to a minimum for the restriction he put on himself of repeating himself as little as possible.

As [19] is the only work not yet published, we include here a few lines, written by Serrano himself in September 1996, that describe its contents:

In the last months we have studied, jointly with Thomas Peternell (Bayreuth University), the geometrical consequences that can be derived from the assumption that a three dimensional variety has relative antidualizing sheaf which is numerically effective. If the fibration is the Albanese map, we know that all the fibers are smooth. We conjecture that this should hold for general fibrations (having the property stated above). We have a conjectural scheme of proof and we have worked out a few cases.

Acknowledgments

This biographical sketch would not have been possible without the help of many people, especially the close relatives of Fernando who, in spite of their affliction, still so recent, welcomed me on every occasion I had to call on them for informations that I could not have obtained in any other way. My gratefulness, in particular, to María Morrás Ruiz-Falcó, for having handed me several key documents and materials, and for her articulate appreciations on Serrano's human and moral constitution, which I have tried to reflect faithfully; and to Domitila García Montes, Fernando's mother, for similar reasons. A special place, among the people that have helped me to better understand the personality of Fernando Serrano, is occupied by Serrano's brothers, Luis and David, and by the woman doctor Carme Pedro Olivé and the nurses that attended Fernando at the Quirón Clinic.

It is also a pleasant duty to thank people whom I have borrowed words, especially T. Y. Lam and R. M. Miró, or who have helped me in other ways, especially

² [22], pàg. 2.

D. Eisenbud, R. Hartshorne and H. Bruce. Moreover, several members of the Department of Algebra and Geometry have eased my task with the documents, informations and comments they provided me: M. Alberich, C. Curràs, J. Elías, J. M. Giral, J. Guàrdia, F. Guillén, R. M. Miró, J. C. Naranjo, V. Navarro, G. Welters and S. Zarzuela. I have to make extensive this acknowledgment to M. A. Barja, of the Department of Applied Mathematics I of the Universitat Politècnica de Catalunya, of whom Serrano was the thesis advisor and who, for this reason, is in charge of the custody of a body of materials.

Finally I would like to thank all the people that were kind enough to read some version, or maybe several, of these biographical gatherings. It has been in this way that I have been able to correct many errors and inaccuracies, and to take into account innumerable suggestions which have improved considerably the text. But it is also an author obligation to declare that he is the sole responsible of the shortcomings that remain, and which, unfortunately, are always discovered when it is to late to remedy them. I feel some comfort, however, in believing that Fernando would have forgiven me.

Serrano's scientific work: an appreciation (A. Conte)

The scientific production of Fernando Serrano is characterized by a remarkable unity. Serrano was a very sound algebraic geometer from the technical point of view; he knew well both the geometry of projective varieties and the more advanced techniques of contemporary Algebraic Geometry. But he didn't like abstract generalities on their own. Instead he was a problem solver, and a very elegant one. The problems he liked to solve were the ones historically determined, i.e. the ones coming from the great tradition of classical Algebraic Geometry, which he dealt with by the most updated technical tools. His beloved field of activity was the geometry of algebraic varieties of low dimension (curves, surfaces, threefolds); to it, in his too short scientific career, he brought substantial contributions which will long last for their importance and geometrical beauty.

To begin with, let us consider the problem solved in [7].

The classical "Noether–Enriques–Petri Theorem" says that the canonical curve C of genus g in \mathbb{P}^{g-1} is always the intersection of the quadric hypersurfaces going through it unless C is trigonal or isomorphic to a plane quintic, in which exceptional cases the intersection W(C) of all quadric hypersurfaces going through C is respectively equal to a rational normal scroll of degree g-2 in \mathbb{P}^{g-1} or the Veronese surface $V_4 \subseteq \mathbb{P}^5$.

In 1972 B. Saint-Donat extended this theorem by considering any irreducible nonsingular curve C of genus g embedded in \mathbb{P}^r by a complete linear system |L| and showing that, if deg $L \geq 2g + 2$, then the homogeneous ideal I_C of $C \subseteq \mathbb{P}^r$ is

generated by quadrics and, if deg L=2g+1, I_C is generated by quadrics and cubics. In his paper Serrano considers the borderline situation deg L=2g+1 and shows that W(C) consists either of C plus (possibly) a line and finitely many isolated points, or is a rational normal scroll of dimension two. Moreover, in this last case he shows that W(C) can be only one of three types that he determines completely. The proof is a very good instance of Serrano's elegance and makes a creative use of a classical theorem of Castelnuovo on the postulation of points.

Serrano's Ph.D. thesis written at Brandeis University in 1985 under the direction of D. Eisenbud was dedicated to a particular case of a very classical problem: to determine all surfaces having a hyperplane section with some definite property (in this case, containing a special pencil). An expanded version of it was published in [6].

Let S be a smooth projective surface, K its canonical divisor and C a very ample divisor on S. The "adjunction mapping" associated to C is the one determined by the complete linear system |C + K| and is denoted by $\Phi_{|C+K|}$.

In the introduction to [6] Serrano says:

"The adjunction mapping $\Phi_{|C+K|}$ has proved to be a very useful tool for the projective classification of surfaces. Another important motivation for the study of $\Phi_{|C+K|}$ is the following. By the adjunction formula, the restriction of $\Phi_{|C+K|}$ to C turns out to be the projection of the canonical image of C from q points, q denoting $\dim H^1(S,\mathcal{O}_S)$. Inasmuch as the canonical image of C reflects very well the linear systems on C, the map $\Phi_{|C+K|}$ will allow, in some cases, to "extend" the linear systems on C to the whole surface. In particular, it is clear that $\Phi_{|C+K|}$ fails to be an embedding provided there exists a hyperelliptic divisor in |C|. One is thus led to a classical piece of research carried out by some Italian geometers about a hundred years ago. Castelnuovo studied surfaces in \mathbb{P}^n whose general hyperplane section is hyperelliptic, but Theorem 5.10 shows that the classification he proposed is incomplete (see Remark 5.11). Castelnuovo overlooked Case (4) of Theorem 5.10, which is precisely the only one for which the hyperelliptic involutions of the sections are not "geometrically evident", that is, not obtained by restricting a fibration of the surface. Our methods will allow to attack a more general problem than Castelnuovo's, namely the structure of surfaces in \mathbb{P}^n having at least one hyperelliptic hyperplane section. That was settled in most cases by the work of Sommese, van de Ven and Ein. The remaining cases are treated here, and they provide the above mentioned counterexample to Castelnuovo's statement. The complete list of surfaces having at least one hyperelliptic hyperplane section is given in Theorem 5.10."

In his classical paper of 1890 on surfaces whose hyperplane sections are hyperelliptic curves Castelnuovo claimed that such a surface, if it is not a scroll, must be rational and ruled by conics. Serrano's counterexample referred to above shows that also some particular geometrically ruled surfaces of irregularity q=1 can occur. All the paper is full of many beautiful and deep geometrical results. I was not surprised in finding Castelnuovo quoted again by Serrano in this paper. As a matter of fact, among the classical Italian algebraic geometers, Castelnuovo is the one who reminds me mostly of Serrano. Both of them wrote a paper only when there was a problem to be solved. And they did solve it in a masterly and elegant way.

In the same vein, the paper [11] studies elliptic surfaces with an ample divisor of genus two, completing the classification of a particular class of surfaces and refining previous results of Beltrametti, Lanteri, Palleschi and Sommese.

Serrano wrote many other papers on surfaces: [5], on the extension of morphisms defined by a divisor; [9], [12], [13], [15], [18], on various kinds of fibrations on them; [10], on the Picard group of a quasi-bundle; [14], on the sheaf of relative differentials; [16], on the projectivized cotangent bundle. All of them contain interesting and deep results. But his most beautiful paper on surfaces seems to me to be [8].

In it he aims to describe the Picard group of the bielliptic (or hyperelliptic, as they were classically called) surfaces and to show that some of these surfaces can be embedded into \mathbb{P}^4 . Bielliptic surfaces are defined to be minimal algebraic surfaces of Kodaira dimension 0 and irregularity 1 and play a special role in the birational classification of surfaces.

The classical result on them was proved in 1907 by Bagnera and De Franchis, who showed that they fall into seven distinct families.

Serrano gives an explicit description of the Picard group for a bielliptic surface S and, "as pleasant reward of such a study" (as he says), is able to construct smooth models in \mathbb{P}^4 for some of these surfaces. Surfaces in \mathbb{P}^4 are in general quite hard to study, and several important questions remain undiscovered. Besides Serrano's example, only three more were known at the time: elliptic quintic scrolls and Horrocks-Mumford and Ellingsrud-Peskine abelian surfaces.

In his "In Memoriam" published in the SCM/Notícies, n. 6, Juliol 1997, p. 3, János Kollár says:

"After graduation he went to Berkeley. We met again three years later when we were both at the University of Utah. It is around that time that he worked out the example of a hyperlliptic surface of degree 10 in \mathbb{P}^4 . This is a beautiful example of his research. Many people thought that the study of ample divisors on hyperelliptic surfaces has very few surprises or difficulties. Fernando first discovered that the problem is much more subtle than people had thought, and then gave a complete answer."

I suggest that from now on this surface be called "Serrano Surface", thinking that the best way of honoring his memory would be to link his name to one of the geometrical objects he studied most passionately.

Serrano was from the beginning a member of the European Networks "Geometry of Algebraic Varieties" and "AGE - Algebraic Geometry in Europe" coordinated by

me and of both of which Barcelona was a node, under the direction of Prof. Gerald Welters. Serrano was at his home inside the Network. We are a group of friends, most of whom have been working together since almost 25 years, who share the same attitude towards Algebraic Geometry, and this was the same as Serrano's. I am sure that his interest on algebraic varieties of dimension 3 (shortly, three-folds) was stimulated by AGE, because among us there are some of the best specialists in this field.

His paper [17] is a very nice one. A divisor D on an algebraic variety X is said to be numerically effective (shortly nef) if $DC \geq 0$ for all curves $C \subseteq X$. It is said to be strictly nef if DC > 0 for all curves $C \subseteq X$. The most important problem in this area is to figure out how far are strictly nef divisors from being ample. It has been known for some time that strictly nef non-ample divisors do exist. Mumford first gave an example for surfaces by taking $S = \mathbb{P}(E)$, where E is a stable bundle of rank 2 and degree 0 over a nonsingular curve of genus $g \geq 2$ defined over \mathbb{C} and D is the divisor corresponding to $\mathcal{O}_S(1)$. Ramanujam, starting from Mumford's example, was able to construct a strictly nef non ample effective divisor D on a complete non-singular threefold X. Serrano's paper deals with the following questions (here K_X denotes the canonical divisor on a smooth algebraic variety X):

- (I) Given a strictly nef divisor L on X, is any smooth deformation of L "in the direction of K_X " ample, i.e. is $L + \epsilon K_X$ an ample \mathbb{Q} -divisor for sufficiently small rational numbers $\epsilon > 0$?
- (II) If the anticanonical divisor $-K_X$ is strictly nef, does it follow that $-K_X$ is ample (i.e. is X a Fano manifold)?

This paper deals with the above questions in dimension 2 and 3 (question (II) had been answered affirmatively for surfaces by Maeda in 1993).

Its main results are the following:

Theorem 1

Question (I) is answered positively for Gorenstein surfaces.

Theorem 2

Question (II) has an affirmative answer for smooth 3-folds.

Theorem 3

Let L be a strictly nef Cartier divisor on a smooth 3-fold X. Suppose we are not in one of the following situations:

- (i) either X is a Calabi-Yau 3-fold and $L \cdot c_2(X) = 0$, or
- (ii) X is of negative Kodaira dimension, and either the irregularity q(X) is ≤ 1 or else q(X) = 2 and $\chi(\mathcal{O}_X) = 0$.

Then $L + \epsilon K_X$ is an ample \mathbb{Q} -divisor for any $\epsilon \in \mathbb{Q}$ such that $0 < \epsilon < 1/4$.

Theorem 2 is particularly important, since it softens in a substantial way the condition on a threefold in order to be Fano. In its proof, Serrano makes use not only of Mori's technique of extremal rays, but also of Donaldson's results on the representations of the fundamental group, showing in this way how broad was his knowledge of the most advanced tools of contemporary Algebraic Geometry.

Question (II) had been posed by Campana and Peternell in 1991. Peternell is a member of the Bayreuth node of AGE. He started to work with Serrano in the spring of 1996 on what would have been Fernando's last scientific work. The paper is now finished (see [19]) and is published in this volume. In it the authors study the global structure of projective threefolds X whose anticanonical bundle $-K_X$ is nef. In differential geometric terms this means that we can find metrics on $-K_X = \det T_X$ (where T_X denotes the tangent bundle of X) such that the negative part of the curvature is as small as we want. The notion of nefness is weaker than the requirement of a metric of semipositive curvature and is the appropriate notion in the context of algebraic geometry. In 1993 Demailly, Peternell and Schneider (another dear friend and member of AGE who died untimely at the end of last August while climbing alone near Nice) had proved that the Albanese map $\alpha: X \longrightarrow Alb(X)$ is a surjective submersion if $-K_X$ carries a metric of semipositive curvature, or, equivalently, if X carries a Kähler metric with semipositive Ricci curvature. It was conjectured that the same holds if $-K_X$ is only nef, but there are very serious technical difficulties to extend the proof, because the metric of semipositive curvature has to be substituted by a sequence of metrics whose negative parts in the curvature converge to 0. The conjecture splits naturally into two parts: surjectivity of α and smoothness. Surjectivity was proved in dimension 3 already in 1993 by Demailly, Peternell and Schneider and in general by Qi Zhang in 1996, using characteristic p. The main result of the paper by Peternell and Serrano proves smoothness in dimension 3:

Theorem

Let X be a smooth projective 3-fold with $-K_X$ nef. Then the Albanese map is a surjective submersion.

At the end of the introduction Peternell writes: "This paper being almost finished modulo linguistical efforts, the second named author died in March 1997. Although we have never met personally, the first named author will always remember and gratefully acknowledge the fruitful and enjoyable collaboration by letters and electronic mail."

From these last papers of him it appears clearly that Serrano had entered into full maturity as a mathematician. His knowledge of the geometry of algebraic varieties of low dimension was deep and extensive and he was able to play around this difficult field of research at full ease. Everywhere in the world he was recognized and

appreciated as one of the best specialists in activity. By now we can only imagine how many beautiful results he could have proved in the future. And we deeply feel how big a loss his untimely death was not only for his relatives and friends but for all the international mathematical community.