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On some geometric properties in $C(K,X)$ and $C(K,X)^*$ spaces

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Abstract

In this paper we relate the Bade property with some new geometric properties, similar to the λ–property (but certainly different), on $C(K,X)$ spaces. We also study the Bade property and the λ–property in the $C(K,X)^*$ spaces.

1. Introduction

Given a normed space X, B_X denotes its closed unit ball, S_X the unit sphere of X, and $\mathrm{Ext}B_X$ the set of extreme points of B_X. The space X is said to have the Bade–property (B.P.) if $\overline{\mathrm{Co}(\mathrm{Ext}B_X)} = B_X$. The following concepts were introduced by R.M. Aron and R.H. Lohman [2]: if $x \in B_X$, a triple (e,y,λ) is said to be *amenable* to x if $e \in \mathrm{Ext}B_X$, $y \in B_X$, $0 < \lambda < 1$ and $x = \lambda e + (1-\lambda)y$. In this case, we define $\lambda(x) = \sup\{\lambda : (e,y,\lambda) \text{ is amenable to } x\}$. X is said to have the λ*–property* if each $x \in B_X$ admits an amenable triple. If, in addition, $\lambda(X) = \inf\{\lambda(x) : x \in B_X\} > 0$, then X is said to have the *uniform* λ*–property*.

Let K be a compact Hausdorff space and let X be a normed space. By $C(K,X)$ we denote the Banach space of all continuous X–valued functions f on K, endowed with the uniform norm. The space $C(K,\mathbb{R})$ will be denoted by $C(K)$. Bade's theorem ([4]) states that $C(K)$ has the *Bade property* if and only if K is 0–dimensional. In [2], it is shown that if X has the λ–property then X has the Bade property, but it's also shown that the converse assertion is false by means of $C(K,\mathbb{C})$ where K is the unit ball of $\mathbb{C}$. In [5], it's shown that if K is a compact Hausdorff space, then

$C(K)$ has the λ–property if and only if K is 0–dimensional and, in this particular case, $C(K)$ has the uniform λ–property and $\lambda(C(K)) = \frac{1}{2}$. The λ–property on the sum of normed spaces is studied in [8] and [9]. In [3], it is shown that X has the λ–property if and only if B_X is the *sequentially–convex hull* of its extreme points; that is to say, for every $x \in B_X$, there exist sequences $(\alpha_n)_n$ of positive reals and $(e_n)_n \subset \mathrm{Ext}B_X$ with $x = \sum_{i=1}^{\infty} \alpha_i e_i$ and $\sum_{i=1}^{\infty} \alpha_i = 1$. As a consequence, if K is a compact Hausdorff space, then $B_{C(K)}$ is the sequentially–convex hull of its extreme points if and only if K is 0-dimensional. This result also appears in [12].

Given a normed space X, the space of convergent sequences in X, endowed with the supreme norm, will be denoted by $c(X)$.

2. The λ–property in $C(K, X)$ spaces

It is well known that if K is a compact Hausdorff space and X is a Banach space, then for every closed subset M of K and every continuous function $g : M \to X$ there exists a continuous function $f : K \to X$ such that $f|_M = g$ ([10]). From this result, we obtain:

Lemma 2.1

Let K be a 0–dimensional compact Hausdorff space and let X be a Banach space. For every closed subset M of K and every continuous function $g : M \to S_X$ there exists a continuous function $f : K \to S$ such that $f|_M = g$.

Proof. By the previous remark, there exists a continuous function $h : K \to X$ such that $h|_M = g$. Let A be a clopen set in K such that $\{t \in K : \ h(t) = 0\} \subset A$ and $M \subset A^c$. Let $y \in X$ be such that $\|y\| > \|h\|$. The function $f : K \to X$ defined by $f(t) = \frac{\chi_A(t)y+h(t)}{\|\chi_A(t)y+h(t)\|}$ verifies that $f(K) \subset S_X$ and $f|_M = g$. $\square$

Remark 2.2. A similar result, when K is a compact metric space and X an infinite–dimensional normed space, appears in [7].

The following results, when K is a compact metric space instead of a 0–dimensional compact space, are essentially, proved in [2], and can be proved by using similar techniques.

Proposition 2.3

Let K be a compact Hausdorff space and let X be a strictly convex Banach space.

1. *If $f \in B_{C(K,X)}$ has an amenable triple, then $\lambda(f) \leq \frac{1+m_f}{2}$, where $m_f = \inf\{\|f(t)\| :\ t \in K\}$.*
2. *If $f \in B_{C(K,X)}$ and $m_f > 0$, then f has an amenable triple, $\lambda(f) = \frac{1+m_f}{2}$ and $\lambda(f)$ is attained.*
3. *If K is 0–dimensional, then $C(K,X)$ has the uniform λ–property and $\lambda(f) \leq \frac{1+m_f}{2}$, for every $f \in B_{C(K,X)}$.*

Remark 2.4. From proposition 2.3.3 we deduce that $c(X)$ has the uniform λ–property when X is a strictly convex Banach space, since $c(X)$ is isometric to $C(\gamma\omega, X)$, where $\gamma\omega$ is the Alexandroff compactification of the discrete space ω. In [1], it is proved that $c(X)$ has the Bade property if and only if X has it. We shall see, in 3.8, that this result does not hold for the λ–property.

Remark 2.5. Let K be a compact Hausdorff space and let X be a normed space. Let S be the subspace of $C(K,X)$ of the finite-valued functions. A function $f \in S$ can be written as $f = \sum_{i=1}^{n} x_i \chi_{A_i}$ where $\{x_1, \ldots, x_n\} \subset X$ and $\{A_1, \ldots, A_n\}$ is a family of disjoint clopen sets in K whose union is K. It is well known that when $X = \mathbb{R}$, K is 0–dimensional if and only if S is dense in $C(K, \mathbb{R})$. From this result it follows that, for an arbitrary normed space X, K is 0-dimensional if and only if S is dense in $C(K,X)$. In this case we also have that $S \cap S_{C(K,X)}$ is dense in $S_{C(K,X)}$.

3. Definition and study of a new geometric property

Let X be a normed linear space such that $\mathrm{Ext} B_X \neq \emptyset$. We denote by T_X, or when no doubt exists by T, the subset of B_X of the points that have an amenable triple. For every $\alpha \in (0,1]$, we denote by T_α the set of points $x \in T$ such that $\lambda(x) > \alpha$. Since $B_X \setminus S_X \subset T$, T is dense in B_X.

Definition 3.1. Let X be a normed space. We shall say that:

1. X has the *dense λ–property* (DλP) if $T \cap S_X$ is dense in S_X.
2. X has the *uniform dense λ–property* (UDλP) if, for some $\alpha \in (0,1]$, $T_\alpha \cap S_X$ is dense in S_X.
3. X has the *weak uniform dense λ–property* (WUDλP) if for some $\alpha > 0$ T_α is dense in B_X. In this case we shall denote $\lambda W = \sup\{\alpha > 0 : T_\alpha \text{ is dense in } B_X\}$.

It is straightforward to prove that if X has the UDλP, then X has the WUDλP. We don't know if the WUDλP implies the UDλP.

Proposition 3.2

If X is a normed space with the WUDλP then X has the Bade property.

Proof. Let $f : X \to \mathbb{R}$ a continuous linear form and let $\epsilon > 0$. We know that, for some $\beta > 0$, T_β is dense in B_X and hence there exist a $z \in T_\beta$ such that $\sup_{x\in B_X} f(x) - \beta\epsilon < f(z)$ where $z = \beta e + (1-\beta)y$, $e \in \mathrm{Ext}B_X$ and $y \in B_X$. Hence $\sup_{x\in B_X} f(x) - \beta\epsilon < f(z) \le \beta f(e) + (1-\beta)\sup_{x\in B_X} f(x)$ and $\sup_{x\in B_X} f(x) - \epsilon < f(e)$. This result clearly implies that $\sup_{x\in B_X} f(x) = \sup_{x\in \mathrm{Ext}B_X} f(x)$. □

The Bade property is not equivalent to the WUDλP (see 3.7) the following result does display the difference.

Proposition 3.3

The following properties on a normed space X are equivalent:

(i) X has the WUDλP.

(ii) There exists $\alpha > 0$ such that for every $x \in B_X$ and $\epsilon > 0$ there exists an element $\sum_{i=1}^n a_i e_i \in \mathrm{Co}\,(\mathrm{Ext}B_X)$ such that $\|\sum_{i=1}^n a_i e_i - x\| < \epsilon$ and $a_i > \alpha$ for some $i \in \{1, 2, \ldots, n\}$.

Proof. Suppose i), let $\alpha > 0$ be such that T_α is dense in $B_X = \overline{\mathrm{Co}\,(\mathrm{Ext}B_X)}$. For every $x \in B_X$ and $\epsilon > 0$ there exists a $y \in T_\alpha$ such that $\|x - y\| < \frac{\epsilon}{2}$. We can write $y = \alpha e + (1-\alpha)z$ for some $e \in \mathrm{Ext}B_X$ and $z \in B_X$.

Choose $\sum_{i=2}^n b_i e_i \in \mathrm{Co}\,(\mathrm{Ext}B_X)$ such that $\|z - \sum_{i=2}^n b_i e_i\| < \frac{\epsilon}{2(1-\alpha)}$. If we take $a_1 = \alpha$, $e_1 = e$ and $a_i = (1-\alpha)b_i$, for $i > 1$, then $\sum_{i=1}^n a_i e_i \in \mathrm{Co}\,(\mathrm{Ext}B_X)$ and $\|x - \sum_{i=1}^n a_i e_i\| < \epsilon$.

Conversely, suppose ii). For a given $x \in B_X$ and $\epsilon > 0$, choose $\sum_{i=1}^n a_i e_i \in \mathrm{Co}\,(\mathrm{Ext}B_X)$ such that $a_1 > \alpha$ and $\|x - \sum_{i=1}^n a_i e_i\| < \epsilon$, then (e_1, z, α) is an amenable triple for $\sum_{i=1}^n a_i e_i$ where $z = \frac{1}{1-\alpha}(a_1 - \alpha)e_1 + \frac{1}{1-\alpha}\left(\sum_{i=1}^n a_i e_i\right)$. □

Proposition 3.4

Let K be a compact Hausdorff space and let X be a normed space.

(a) If X has the λ–property, then there exists an amenable triple for every finite–valued function $f \in B_{C(K,X)}$. If $\lambda(X) > \beta$ then $\lambda(f) > \beta$.

(b) If K is 0–dimensional and X has the λ–property (resp. the uniform λ–property), then $C(K, X)$ has the DλP (resp. the UDλP).

Proof. a) Let $f = \sum_{i=1}^n x_i \chi_{A_i}$ where $A_1, \ldots, A_n$ are disjoint clopen sets with $A_1 \cup \ldots \cup A_n = K$ and $\{x_1, \ldots, x_n\} \subset B_X$. Let $0 < \lambda < \min_{1\le i\le n} \lambda(x_i)$. For every $i \in \{1, \ldots, n\}$ there exists an amenable triple (e_i, y_i, λ) for x_i. Hence (e, g, λ) is an amenable triple for f, where $e = \sum_{i=1}^n e_i \chi_{A_i}$ and $g = \sum_{i=1}^n y_i \chi_{A_i}$. The proof of b) is obvious. □

Remark 3.5. If X has the DλP (resp. the UDλP), it is straightforward to prove that $C(K,X)$ has the DλP (resp. UDλP), for any 0–dimensional compact Hausdorff space K.

EXAMPLES 3.6: There exists a normed space X with the DλP which lacks the λ–property:

Let X_1 (resp. X_2) be a normed space without (resp. with) the λ–property. Let $X = X_1 \times X_2$ endowed with the norm $\|(x_1,x_2)\| = \|x_1\| + \|x_2\|$. It is clear that $\mathrm{Ext}B_X = \{(e_1,0):\ e_1 \in \mathrm{Ext}B_{X_1}\} \cup \{(0,e_2):\ e_2 \in \mathrm{Ext}B_{X_1}\}$. Since X_1 does not have the λ–property, X does not have it either. Nevertheless, we are going to prove that X has the DλP. Let $x = (x_1,x_2) \in S_X$ and $\epsilon > 0$.

(a) If $\|x_1\| = 1$, then $x_2 = 0$. Let $e_2 \in \mathrm{Ext}B_{X_2}$ and let $y = \big((1-\frac{\epsilon}{3})x_1, \frac{\epsilon}{3}x_2\big)$. We have that $\|x-y\| = \frac{2\epsilon}{3} < \epsilon$, $\|y\| = 1$ and $y = \frac{\epsilon}{3}(0,e_2) + (1-\frac{\epsilon}{3})(x_1,0)$.

(b) If $\|x_1\| < 1$ then $x_2 \neq 0$ and if $x_2 \in \mathrm{Ext}B_{X_2}$ then $x \in \mathrm{Ext}B_X$. When $x_2 \notin \mathrm{Ext}B_{X_2}$ it is straightforward to prove that there exists an amenable triple (e_2,y_2,λ) for x_2 such that $\|x_2\| = \lambda + (1-\lambda)\|y_2\|$. Hence $(x_1,x_2) = \lambda(0,e_2) + (1-\lambda)(\frac{1}{1-\lambda}x_1, y_2)$ and x has an amenable triple.

Let us observe that, in this case, if B_{X_1} does not have extreme points then X does not have the Bade property since $\overline{\mathrm{Co}\,(\mathrm{Ext}B_X)} \subset \{0\} \times X_2$. Hence the D$\lambda$P does not imply the Bade property.

EXAMPLES 3.7: It is well know that ℓ_1 has the λ–property (hence, the Bade property) but lacks the uniform λ–property (Cf. [2]). We now prove that ℓ_1 lacks the WUDλP.

If there exists an $\alpha \in (0,1)$ such that T_α is dense in B_{ℓ_1}, then we can choose $n \in \mathbb{N}$ such that $\frac{2n-1}{n^2} < \alpha$. Let $X = (x_i)_{i\in\mathbb{N}}$ be the sequence defined by

$$x_i = \begin{cases} \dfrac{1}{n} & \text{if} \quad i \le n \\ 0 & \text{if} \quad i > n\,. \end{cases}$$

Let $z \in T_\alpha$ be such that $\|x - z\| < \frac{1}{n^2}$ and let (e,y,α) be an amenable triple for z. Since $e = (e_i)_{i\in\mathbb{N}} \in \mathrm{Ext}B_{\ell_1}$, there exists $p \in \mathbb{N}$ such that

$$e_i = \begin{cases} \delta & \text{if} \quad i = p \\ 0 & \text{if} \quad i \neq p. \end{cases}$$

where δ is either $+1$ or -1.

Since $\|x - z\| < \frac{1}{n^2}$, $\sum_{i\neq p} \left|\frac{1}{n} - (1-\alpha)y_i\right| < \frac{1}{n^2}$ and hence $y_i > \frac{n-1}{n^2(1-\alpha)}$, for every $i \neq p$. This contradiction proves our assertion.

EXAMPLES 3.8: There exists a normed space with the WUDλP which lacks the λ–property.

The normed space that we use is considered in [2]. Let C^1 denote the convex hull of the union of the sets $\{(x, y, 0) : |x|, |y| \leq 1\}$ and $\{(x, 0, z) : x^2 + z^2 = 1, z \geq 0\}$ in $\mathbb{R}^3$. Let $C = (0, 0, 1) + C^1$ and let $\|\cdot\|$ denote the norm on $\mathbb{R}^3$ whose unit ball is $B = \mathrm{Co}(C \cup -C)$. Let $X = (\mathbb{R}^3, \|\cdot\|)$. The space X has the uniform λ–property since X is finite dimensional. Nevertheless, Navarro [11] proved that $c(X)$ does not have the λ–property (the sequence $(x_n)_{n\in\mathbb{N}}$, where $x_{2n} = \left(\cos\frac{\pi}{2n}, 0, 1 + \sin\frac{\pi}{2n}\right)$ and $x_{2n-1} = (1, \frac{1}{n}, 1)$, does not have an amenable triple). As a consequence of 3.4 we can say that $c(X)$ has the UDλP and hence, the WUDλP.

4. The Bade and the λ–property in $C(K, X)^*$ spaces

In [5], it is shown that the following properties on a compact Hausdorff space are equivalent:

a) K is dispersed; b) $C(K)^*$ has the λ–property; c) $C(K)^*$ has the Bade property.

Now we investigate the Banach spaces X such that a similar result holds by considering the spaces $C(K, X)$ instead of the spaces $C(K)$.

Let T be a set and let X be a normed space. We consider the space $\ell_1(T, X) = \{(x_t)_{t\in T} : \sum_{t\in T} \|x_t\| < \infty\}$ endowed with the norm $\|(x_t)_{t\in T}\| = \sum_{t\in T} \|x_t\|$. If $T = \mathbb{N}$ we write $\ell_1(X)$ instead of $\ell_1(\mathbb{N}, X)$.

The following result, in the case $T = \mathbb{N}$, appears in [2] and [8]. It can be proved by the same techniques.

Lemma 4.1

(a) $(x_t)_{t\in T} \in \mathit{Ext}B_{\ell_1(T,X)}$ if and only if there exists a $t_0 \in T$ such that $x_{t_0} \in \mathit{Ext}B_X$ and $x_t = 0$ for $t \in T \setminus t_0$; (b) $\ell_1(T, X)$ has the λ–property if and only if X has it; (c) If T is an infinite set $\ell_1(T, X)$ does not have the uniform λ–property.

Theorem 4.2

The space $\ell_1(T, X)$ has the Bade property if and only if X has the Bade property.

Proof. X has the Bade property if and only if for every continuous linear form $h : X \to \mathbb{R}$

$$\sup_{x\in B_X} h(x) = \sup_{x\in \mathrm{Ext}B_X} h(x)\,.$$

Suppose X has the Bade property. For every $f \in \ell_1(T,X)^*$ let $f_{t_0} : X \to \mathbb{R}$ denote the continuous linear form defined by $f_{t_0}(x) = f(x^{t_0})$, where $x^{t_0} \in \ell_1(T,X)$ is given by $x_t^{t_0} = 0$ if $t \neq t_0$ and $x_t^{t_0} = x$ if $t = t_0$. We have $f\left((x_t)_{t\in T}\right) = \sum_{t\in T} f_t(x_t)$ for every $(x_t)_{t\in T} \in \ell_1(t,X)$. Let $M = \sup\left\{f\left((x_t)_{t\in T}\right) :\ (x_t)_{t\in T} \in B_{\ell_1(T,X)}\right\}$ and, for every $t \in T$, let $M_t = \sup\{f_t(x) :\ x \in B_X\}$. Clearly $M = \sup\{M_t :\ t \in T\}$. For a given $\epsilon > 0$, there exists a $p \in T$ such that $M_p + \frac{\epsilon}{2} \geq M$ and, since X has the Bade property, there exists an $e \in B_X$ such that $f_p(e) + \frac{\epsilon}{2} \geq M_p$. We have $f(e^p) + \epsilon \geq M$ and $e^p \in \mathrm{Ext}B_{\ell_1(T,X)}$. This proves that $\ell_1(T,X)$ has the Bade property.

Suppose $\ell_1(T,X)$ has the Bade property. Let $g : X \to \mathbb{R}$ be a continuous linear form. We choose $t_0 \in T$ and we define $f : \ell_1(T,X) \to \mathbb{R}$ by $f\left((x_t)_{t\in T}\right) = g(x_{t_0})$.

It can be proved that

$$\sup\{g(x) :\ x \in B_X\} = \sup\left\{f\left((x_t)_{t\in T}\right) :\ (x_t)_{t\in T} \in \mathrm{Ext}B_{\ell_1(T,X)}\right\}$$

and $\sup\{g(x) :\ x \in \mathrm{Ext}B_X\} = \sup\left\{f\left((x_t)_{t\in T}\right) :\ (x_t)_{t\in T} \in \mathrm{Ext}B_{\ell_1(T,X)}\right\}$. Since $\ell_1(T,X)$ has the Bade property we deduce that

$$\sup\{g(x) :\ x \in B_X\} = \sup\{g(x) :\ x \in \mathrm{Ext}B_X\} \quad \square$$

Remark 4.3. If X is a Banach space and K a compact Hausdorff space, it is well known ([6]) that $C(K,X)^* \simeq \mathrm{rcabv}\,(\Sigma, X^*)$ where Σ is the σ–field of Borel subsets of K and $\mathrm{rcabv}\,(\Sigma, X^*)$ is the Banach space of all regular countably additive measures F on Σ with values in X^* and of finite variation on K, endowed with the total variation norm ($\|F\| = |F|(K)$). It can be proved that $\mathrm{Ext}B_{\mathrm{rcabv}(\Sigma,X)} = \{x\delta_t :\ x \in \mathrm{Ext}B_X, t \in K\}$, where $x\delta_t$ denote the measure $x\delta_t : S \to X$ defined for $A \in \Sigma$ by $x\delta_t(A) = x$ if $t \in A$ and $x\delta_t(A) = 0$ if $t \notin A$.

Theorem 4.4

Let X be a Banach space and K a compact Hausdorff space. The following properties are equivalent: a) $C(K)^*$ and X^* have the Bade property; b) $C(K,X)^*$ has the Bade property.

Proof. That a) implies b) is a consequence of the fact that if K is dispersed then $\mathrm{rcabv}\,(\Sigma, X^*)^* \sim \ell_1(K, X^*)$. In order to prove that b) implies a) we note that if K is not dispersed then there exist an atomless measure $\mu : \Sigma \to [0,1]$ such that $\mu(K) = 1$. We choose an $x^* \in S_{X^*}$ and let $F : \Sigma \to X^*$ be the measure defined for $A \in \Sigma$ by $F(A) = \mu(A)x^*$. We have that $|F| = \mu$. If $\epsilon \in (0, \frac{1}{2})$, there exists $\sum_{i=1}^n \alpha_i x_i^* \delta_{t_i} \in \mathrm{Co}\left(\mathrm{Ext} B_{\mathrm{rcabv}(\Sigma,X^*)}\right)$ such that $\|F - \sum_{i=1}^n \alpha_i x_i^* \delta_{t_i}\| < \epsilon$. Hence $\mu\left(K \setminus \{t_1, \ldots, t_n\}\right) < \epsilon$, which implies that $\mu(K) < \epsilon$. This contradiction proves that K must be dispersed. Let $x^* \in B_{X^*}$ and $\epsilon > 0$. Let $t_0 \in K$ be arbitrary. Since $C(K, X^*)$ has the Bade property there exists $\sum_{i=1}^n \alpha_i x_i^* \delta_{t_i} \in \mathrm{Co}\left(\mathrm{Ext} B_{\mathrm{rcabv}(\Sigma,X^*)}\right)$ such that $\|x^* \delta_{t_0} - \sum_{i=1}^n \alpha_i x_i^* \delta_{t_i}\| < \epsilon$. Therefore $\|x^* \delta_{t_0}(K) - \sum_{i=1}^n \alpha_i x_i^* \delta_{t_i}(K)\| = \|x^* - \sum_{i=1}^n \alpha_i x_i^*\| < \epsilon$. $\Box$

Corollary 4.5

Let K be a compact Hausdorff space and X a Banach space such that X^* has the λ–property. Then the following properties are equivalent: a) K is dispersed; b) $C(K, X)^*$ has the Bade property; c) $C(K, X)^*$ has the λ–property.

Remark 4.6. If K is an infinite set $C(K, X)^*$ does not have the uniform λ–property.

Corollary 4.7

Let K be a compact Hausdorff space and X an arbitrary Banach space. The following properties are equivalent: a) $C(K, X)^*$ has the λ–property; b) $C(K)^*$ and X^* have the λ–property.

Remark 4.8. By means of the former results we can conclude that the Banach space $X = C(\gamma\omega, Y)$, where Y is the space of the example 3.8. is such that X has the Bade property, X^* has the λ–property but X fails to have the λ–property. In the bibliography we have not found any Banach space with these characteristics

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