

Functional Entities —and that ain't the half of it!*

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Abstract

Like many other languages, Spanish may specify fractionary nouns with definite articles (cf. *la mitad*). This fact seems to violate the presupposition of uniqueness generally assigned to the definite article, as no fraction may exist without another one (halves, in particular, come in pairs). But this violation is only apparent—at least if we assume that these fractionary nouns denote fractioning operations (rather than the results thereof) and occur in partitive (rather than attributive) constructions. These proposals can be justified independently, and extend to numeral noun constructions that did not survive into Contemporary Spanish.

Key words: definite article, presupposition of uniqueness, fractionary nouns, numeral nouns, partitive construction, attributive construction, logical types, mixed types, logical sorts, mixed sorts, sorted domains, Romance, Spanish, Catalan, Italian, French, Latin, Greek, German, Breton, English.

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1. Introduction

It is generally believed that the singular form of the definite article carries with it a presupposition of uniqueness—or that it may only combine meaningfully with nominals that denote singleton sets (Russell 1905, Strawson 1950, Strawson 1964, McCawley 1993). Take for example *half of the students*. Note that this nominal cannot denote a singleton, as halves come in pairs (things would be different if the

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nominal had been *top half of the students* instead). Consequently it cannot be specified by the definite article (1b); only by the indefinite one (1a).

- (1) a. A half of the students failed the course.
 b. *The half of the students failed the course.

Appealing as it is, this line of reasoning comes crashing down in Spanish, where both versions of (1) are acceptable (as well as similar in meaning).

- (2) a. Una mitad de los estudiantes suspendió el curso.
 ‘A half of the students failed the course.’
 b. La mitad de los estudiantes suspendió el curso.
 ‘[The] half of the students failed the course.’

As a matter of fact, native speakers even report a preference for (2b), which is the version with the definite article, over (2a), which is the version without it.

It should not escape the reader that the Spanish nominal *mitad de los estudiantes* ‘half of the students’ does not just denote two halves. It denotes two halves *for each way of splitting the set of students into two equal parts*. Obviously, there will be more than one way to do so any time there are more than two students to split. Thus, given ten students, there will be 126 ways to split them into two equal parts. Under such circumstances, the denotation of *mitad de los estudiantes* (‘half of the students’) is a set with $126 \cdot 2 = 252$ elements.¹

The point can be made more clearly with a sentence like *El Barcelona dominó durante la mitad del partido* (‘The Barcelona [soccer team] dominated half of the game’). For, note that there is a nondenumerable infinity of ways in which the duration of the game in question could be split in two equal parts (all but two of which would be temporally discontinuous); *mitad del partido* would refer to any one of these parts.

But the problems raised by (2b) are not peculiar to halves. They arise with other fractions as well. In fact, they become even more acute in such cases. For, thirds come in threes, fourths come in fours, and fifths come in fives. Yet, nominals headed by the corresponding fractionary nouns may combine, in Spanish, with both articles.

- (3) a. Una tercera/cuarta/quinta parte de los estudiantes suspendió el curso
 ‘A third/fourth/fifth of the students failed the course.’
 b. La tercera/cuarta/quinta parte de los estudiantes suspendió el curso
 ‘[The] third/fourth/fifth of the students failed the course.’

1. The number of combinations of n objects taken r at a time is $n!/r!(n-r)!$. Naturally, this number will correspond to the halves of a whole whenever $n = 2r$. Since complementary halves can be placed in a one-to-one correspondance, there will be twice as many halves as there are correspondances. Each of these correspondances represents a way to split a whole in halves.

And here again, the versions with the definite article are reportedly preferred, in actual speech, over the ones with the indefinite article.

Note that the problematic sentences above cannot be explained in terms of *contextual uniqueness*. For, although the presuppositions of the definite article are sometimes satisfied in contexts which identify one of many entities, the sentences in question can be used in contexts which have not identified any particular half of the students; they can be used as soon as the course from which a fraction is taken has been identified. Note also that the sentences in (2b) and (3b) cannot be explained in terms of *contextual prominence* either (where contextual prominence is a weaker form of contextual uniqueness). These sentences can be used in contexts in which no particular half of students is more prominent than any other. Note finally that one cannot explain the (b) sentences above by saying that their subjects refer to unique rational numbers—namely $1/2$, $1/3$, $1/4$, or $1/5$. For one thing, it makes no sense to say that a rational number failed a course. But even if the sentences in question could be about the academic performance of a number, then this number would have contradictory properties, as (2b) and (4) may easily be true at the same time.

- (4) La mitad de los estudiantes no suspendió el curso.
 '[The] half of the students did not fail the course.'

But nothing may have contradictory properties—let alone a number.

Related to this last point, note that one cannot claim either that the noun *mitad* ('half') itself refers to the unique rational number $1/2$. For, the name of this number is 'medio', not 'mitad'. Yet, only *mitad* may occur in this construction (2); not *medio* (5).

- (5) a. *Un medio de los estudiantes suspendió el curso.
 b. *El medio de los estudiantes suspendió el curso.

As to the fractionary nouns in (3), the names of the corresponding rationals are *tercio*, *cuarto*, *quinto*. Yet, none of these numerals may occur with the definite article in this construction:

- (6) a. Un tercio/cuarto/quinto de los estudiantes suspendió el curso.
 'A third/fourth/fifth of the students failed the course.'
 b. *El tercio/cuarto/quinto de los estudiantes suspendió el curso.

In short, numeral names do not behave at all as fractionary nouns for the purposes at hand.

2. A diachronic solution

Although these challenges to the received analyses of the definite article have not been discussed in the current literature, they have been known for a long time.

Thus, more than a hundred years ago, Meyer-Lübke (1890-1902, III, §153) remarked on a «noteworthy peculiarity» of the definite article of Romance languages—namely that it can specify entities that are neither named nor determined, but only removed, partitively, from a larger set. His examples were drawn from Italian (7a), French (7b), and Spanish (7c), to which we can add (7d), the Catalan equivalent of (2b):

- (7) a. Questo era più que i due terzi del tutto.
 ‘This was more than [the] two thirds of the whole.’
- b. Un vaste bâtiment brûlait sur les trois quarts de sa longueur.
 ‘A large building burned on [the] three quarters of its length.’
- c. No haber podido enseñar más que a las dos terceras partes de la colonia española
 ‘Not to have been able to teach to more than [the] two thirds of the Spanish colony.’
- d. La meitat dels estudiants va suspendre el curs.
 ‘[The] half of the students failed the course.’

Given that the problems in (2b) and (3b) extend to Romance languages other than Spanish, a solution that is equally general to them must be found. One such solution was offered by Gamillscheg (1966). According to this Romanist, the forms involved in these constructions are not definite articles, but only partitive pronouns that have become homophonous with definite articles in Romance. Consequently, *la mitad* is semantically equivalent to ‘half of them’ rather than to ‘the half’.

But Gamillscheg’s solution calls for pronouns to become articles in their morphosyntax while remaining pronouns in their semantics. Thus, if Latin *medietas de illi* (‘half of them’) is to become Spanish *la mitad* (‘the half’), then it is required (i) that postposed forms become preposed, (ii) that arguments become specifiers, (iii) that stressed forms become unstressed, (iv) that full forms become reduced, (v) that oblique forms lose their case marking, (vi) that meaningful forms become redundant, and (vii) that forms which agreed with wholes, end up agreeing with parts (so that *illi*, which agreed with its antecedent, ends up agreeing with *medietas* instead). Yet, throughout these rather dramatic changes, the forms in question must still preserve their original value as partitive pronouns. For if they don’t, then we end up with articles, thus losing all our ability to explain the violations of uniqueness that motivated the solution in the first place.

And then there is the question of why some partitive pronouns failed to follow this reanalysis. For *medietas de illi* has indeed survived as (*la*) *mitad de ellos* (‘([the]) half of them’) in Modern Spanish.

But the main problem with Gamillscheg’s analysis is that it is not general enough, as the problems in (2), (3), (7) can be found beyond Romance. Consider for example the equivalents of (2b) in German (8a), Greek (8b), and Breton (8c).

- (8) a. Die Hälfte der Studenten im Kurs sind durchgefallen.
 the half of.the students in.the course are failed
- b. I misi apo tus fitites apetihan
 the.m.pl.nom half.m.pl.nom from the.m.pl.acc students.acc failed.3pl
 sto mathima.
 to.the course
- c. C'hwita ar rummad-kentelioù a rae an hanter eus an studerien.
 Fail the course PRT did the half of the students
 '[The] half of the students failed the course.'

Incidentally, definite halves can be found also in English, albeit marginally, in the colloquialism in (9) used in the title of this paper.

- (9) And that ain't the half of it!

3. A synchronic solution

Instead of attempting to salvage portions of Gamillscheg's diachronic proposal, I will try to develop a synchronic solution to the problems in (2), (3), (7), (8), (9). This solution involves two proposals. The first is to interpret the noun *mitad* ('half') used here, not in terms of halves, but rather in terms of the operation that produces them; the second is to propose a partitive analysis of constructions like *la mitad de los estudiantes* ('[the] half of the students') (as opposed to an attributive one).

To be more specific, let h be a function that assigns, to each element x of a universe of discourse, the set $h(x)$ consisting of all of the halves into which x can be split. Thus, if x is a collection of four students a, b, c, d , then $h(x)$ is the set of pairs of students defined in (8).

- (10) $h(x) = \{ab, ac, ad, bc, bd, cd\}$.

Since x is a collection of four students, there are $4!/2!(4-2)! = 24/4 = 6$ elements in $h(x)$; if x were instead a collection of ten students, then there would be 210.

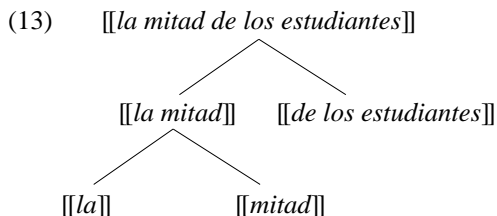
Having availed ourselves of h , we will allow the Spanish noun *mitad* ('half') to denote the singleton of this function:

- (11) $[[mitad]] = \{h\}$

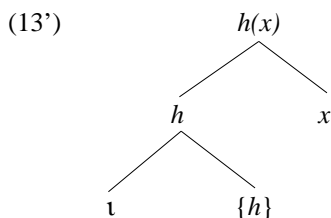
Denoting a singleton, *mitad* may now be specified by the definite article without violating the presupposition of uniqueness it bears. But will the definite article get a chance to combine with this noun? Only if *la mitad de los estudiantes* ('[the] half of the students') has the partitive structure in (12a) rather than the attributive structure of (12b).

- (12) a. [la mitad] de los estudiantes.
 b. la [mitad de los estudiantes]

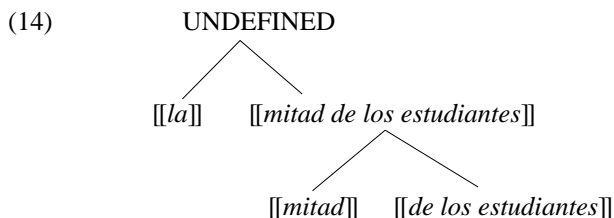
For, if the structure is as indicated in (12a), then the simplest compositional interpretation of the phrase will proceed as indicated in (13),



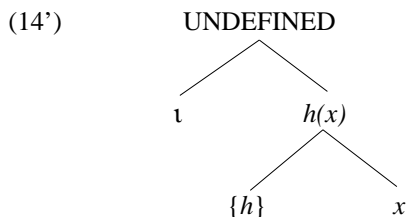
This interpretive process will succeed. For, if $[[mitad]]$ is indeed the singleton $\{h\}$ of the halving function h as claimed in (11), then the iota operator ι contributed by $[[la]]$ will combine meaningfully with that singleton. And the result $h = [[la\ mitad]]$ may subsequently apply to $[[de\ los\ estudiantes]]$, which is a collection x of students, in order to yield $h(x) = [[la\ mitad\ de\ los\ estudiantes]]$ —namely the desired set of halves of that collection:



But, if the structure of the problematic phrase was instead attributive as indicated in (12b), then its simplest compositional interpretation would (attempt to) proceed as follows.



Note that the interpretive process in (14) fails to yield a defined denotation. For, if *mitad de los estudiantes* is meaningful, then it would have to denote a set of two or more halves, and the effect of $[[la]]$ on such a set would be undefined because of the presupposition of uniqueness:



4. Independent evidence for the proposal

Independent evidence that fractionary nouns like *mitad* denote uniquely in sentences like (2b) comes from the fact that these nouns cannot be modified by the adjective *otra* ('another'). Thus, (4') is not a viable alternative to (4), even when the latter is true.

(4') *La otra mitad de los estudiantes no suspendió el curso.

The problem with (4') is that there can only be one *mitad* (when denoted by a noun specified by the definite article). And similar facts hold for the other fractionary nouns in (3b).

The partitive analysis in (12a) can be motivated independently of the issues at hand. For, consider the exchange in (15).

(15) A: ¿Cuántos de los estudiantes suspendieron el curso?
'How many of the students failed the course?'

B: La mitad.
'[The] half.'

Notice that the question in (15) contains an interrogative pronoun whose answer (or antecedent) in the relevant states of affairs is *la mitad*. Since only constituents can serve as answers (or antecedents), and since only constituents can occur as complete statements, *la mitad* must be a constituent, as claimed by (12a).

Notice also that *cuántos* ('how many'), the interrogative pronoun of (15), is a pronoun of quantity. As such, it could be answered by a specifier *la mitad*, not by a head (*la*) *mitad*. For, consider the noun phrase in (16).

(16) una caja de madera.
'a box of wood'

This noun phrase is ambiguous in the same way that its English gloss is. It may refer either to a box made of wood (the attributive reading) or to a boxful of wood (the partitive reading). Yet, if (16) occurs as the answer to the question in (17),

(17) ¿Cuánto de madera compraste?
how much of wood bought.you.past
'How much wood did you buy?'

then its ambiguity is resolved in favor of the partitive reading. *La mitad* must therefore be allowed to be a specifier, as indicated in (12a).

Finally, consider again the Greek example in (8b):

- (18) I misi apo tus fitites apetihan
 the.m.pl.nom half.m.pl.nom from the.m.pl.acc students.acc failed.3pl
 sto mathima.
 to.the course
 ‘[The] half of the students failed the course’

As can be seen from the morpheme-by-morpheme gloss of this sentence, *i misi* ‘the halves’ is plural. Yet, as can be gathered from its idiomatic gloss, (18) states only that *one* half of the students failed the course; it does not state that both did.

Notice that the plurality of *i misi* (‘the halves’) in (18) is due to agreement with *tus fitites* (‘the students’). For, if we replaced it with the singular *ton fititi* (‘the student’), then we would have the singular *o misos* (‘[the] half’) instead:

- (19) o misos apo ton fititi
 the.m.sg.nom half.m.sg.nom from the.m.sg.acc student.m.sg.acc
 ‘[the] half of the student’

As a matter of fact, *misos* and *fititi* agree not only in number, but also in gender. For, if we replaced the masculine *ton fititi* (‘the student’) in (19) with the neuter *to ghala* (‘the milk’), as done in (20), then we would have the neuter *to miso* (‘[the] half’) instead:

- (20) to miso apo to ghala
 the.neut.sg.nom half.neut.sg.nom from the.neut.sg.acc milk.neut.sg.acc
 ‘[the] half of the milk’

This remarkable agreement can be explained only if *misi* (‘halves’) in (18) is in the specifier position of *i misi apo tus fitites* (‘the halves of the students’), as claimed in (12a). It could not be explained if it were in its head, as claimed in (12b). For, while specifiers may agree with their nominal heads, nominal heads do not agree with the nouns that modify them from attributive position. Compare in this light (19) with the clearly attributive construction in (21).

- (21) o dhaskalos ton fititon
 the.sg.nom teacher.sg.nom the.pl.gen student.pl.gen
 ‘the teacher of the students’

Here *dhaskalos* (‘teacher’) is singular, and therefore does not agree with *fititon* (‘students’), which is plural.

Further evidence for a syntactic difference between (18) and (21) comes from the fact that *tus fitites* is accusative in (18) whereas *ton fititon* is genitive in (21).

What's more, the functional analysis of *misi* ('halves') we espouse may even explain why the partitive construction in (18) contains an accusative (governed by a preposition) whereas the attributive construction in (21) contains a genitive. For, while the accusative *tus fitites* ('the students') in (18) names an entity that undergoes a (halving) operation, the genitive *ton fititon* ('of the students') in (21) involves an entity (the collectivity of students) that has a teacher.

5. Anticipating potential objections

It might be objected that the interpretation in (11) introduces a major complication in the interpretation of nouns. For, whereas nouns are usually interpreted as sets of entities, the interpretation in (11) makes the noun *mitad* ('half') denote a function from entities to sets of entities instead. Cast in terms of Montague's classic ontology, the proposal in (10) would make the denotation of *mitad* be an object of type (22a) rather than simply an object of type (22b).

(22) a. $\langle e, \langle e, t \rangle \rangle$

b. $\langle e, t \rangle$

Notice that the objection we are anticipating here is not just that the denotation of *mitad* is more complex than we thought; it is that noun denotations are of *mixed type*, as some would be of type (22a) while others, of type (22b).² And this would be undesirable because, if nouns were of mixed type, then all expressions that operate on them would likewise have to be of mixed type, as an operator on a type (22a) object cannot operate on a type (22b) object, and vice versa. Note that these expressions would have to include articles, quantifiers, possessives, demonstratives, numerals, adjectives, and even prepositional phrases, as all of these expressions are nominal operators. A split in the semantic type of nouns would therefore result in grammars with widespread ambiguity. If no independent evidence for such ambiguities can be found, such ambiguity would be spurious.

Fortunately, the proposal in (11) does not require such ambiguity. For, notice that, although semantics requires nouns to name entities, it does not place any constraints on what these entities can be. Thus, although nouns may be forced to name persons, places, or things, nothing prevents functions from being things—and hence from being among the entities that nouns can name.

Thus we can say that nouns like Spanish *mitad*, Italian *metà*, French *moitié*, Catalan *meitat*, German *Hälfte*, Greek *misos*, Breton *hanter*, and sometimes even English *half*, all refer to a particular set of entities, namely the set $\{h\}$ consisting of the function h that assigns, to each entity x of the universe of discourse, the set $h(x)$ of halves of x .

2. Even proper names may be assigned to type $\langle e, t \rangle$, as a proper name may be said to refer to the characteristic function of the singleton of an entity (i.e. to a function that assigns truth to one and only one entity).

Cast in terms of characteristic functions, all these nouns would refer to the function from entities to truth values that assigns truth to *h* and falsity to every other element of the universe of discourse. This means that all of the nouns in question are of the standard Montague type (22b), and that nouns do not have to be of mixed type because of them. Nouns need only be of *mixed sort*: they can name sets of entities which are either functional or entitive.

It might also be objected that our interpretation of *mitad* ('half') in terms of a singleton precludes its pluralization, as names of singletons do not take well to pluralization:

(23) ?The authors of Hamlet were brilliant.

Yet, *mitad* ('half') may easily pluralize as *mitades* ('halves'):

(24) Las dos mitades de la clase suspendieron el curso.

'The two halves of the class failed the course.'

It should be noticed, incidentally, that the plural in (24) is not a redundant plural of concordance like the one we saw in the Greek example in (18). It is rather a meaningful plural referring to more than one half.

To accommodate the meaningful pluralization of *mitad* (and its cognates) we will propose that these nouns are ambiguous between a *functional* reading (in which a halving operation is denoted) and an *entitive* reading (in which the results of applying said operation are denoted). For if we do, then the definite articles in (2), (3), (7), (8), (9) will select (or *coerce*) the functional readings of *mitad* and its cognates, while the pluralization in (24) will instead select for their entitive readings. As to the indefinite articles in (2), (3), and elsewhere, they will be consistent with both readings of these nouns, and would therefore preserve their ambiguity.

Admittedly, this solution to the problem in (22) represents a weakening of our analysis of *mitad* and its cognates. Yet, it seems to be called for by the facts. Moreover, it may also help explain the crosslinguistic variation in the data. For, fractionary nouns that combine with the definite article would be lexically ambiguous between functional and entitive readings; fractionary nouns that resist the definite article would be unambiguously entitive instead. This difference between, say, English and Spanish, would thus reduce to an ordinary difference in their lexical inventories: Spanish has two nouns *mitad* whereas English has only one noun *half* (that it can use productively).

6. Expanding the data

In its entry for EL/LA/LOS/LAS, the remarkable *Diccionario de Construcción y Régimen de la Lengua Castellana* points out that Spanish numerals preceded by the definite article could denote, at least until recently, «a part of a set previously designated with a number.» The examples mentioned in this entry, none of which are possible in contemporary Spanish, are given in (25).

- (25) a. Entiendan esto [...] los que de sesenta años de vida, **los cuarenta** se les han pasado en pecado (M. de Chaide, *Magd.*).
 ‘Understand this those with a life of sixty years [the] forty of which have passed them by in sin.’
- b. Sería mala granjería [...] del hortelano que de cien árboles le faltasen **los noventa** [...] y de los diez que prendiesen al mejor tiempo, se le secasen **los cinco** (Venegas, *Agonía*).
 ‘It would be bad farming on the part of the grower that, of one hundred trees, [the] ninety would fail him, and that of the ten that would bear fruit at the same time, [the] five would dry up.’
- c. Constaba entonces [el ejército de Cortés] de novecientos hombres; **los ciento noventa y cuatro** entre arcabuces y ballestas; los demás de espada, rodela, y lanza; ochenta y seis caballos y diez y ocho piezas de artillería, **las tres** de hierro, gruesas, y **las quince** falconetes de bronce (Solís, *La Conquista de México*).
 ‘The army of Cortés consisted then of nine hundred men; [the] one hundred ninety four of harquebus and crossbow; the rest of sword, buckler and spear; eighty six horses and eighteen pieces of artillery, [the] three iron and thick, and [the] fifteen bronze falconets.’
- d. Estaba la caballería católica muy mal parada, tanto que de las tres partes de los soldados había **las dos** a pie (Coloma, *Guerras de los Estados Bajos*).
 ‘the Catholic cavalry was in very bad shape, so much so that, of the three parts of the soldiers, there were [the] two on foot.’

It should be clear that sentences (25) pose the same problem as sentences (2), (3), (7), (8), (9), as they ask us to take *the* forty years from sixty (25a), *the* ninety trees out of a hundred (25b), *the* five trees out of ten (25b), *the* hundred ninety four men from nine hundred (25c), *the* three (and *the* fifteen) artillery pieces out of eighteen (25c), and *the* two of the three parts of soldiers (25d). In short, all of the sentences in (25) involve taking one of several fractions from a total (yet regarding this fraction as definite).

As a consequence of this, we could give the problem posed by sentences (25) the same solution we gave to sentences (2), (3), (7), (8), (9). To be more specific, we could say that the numeral nouns in these sentences denote, not collections of a particular size, but rather operations which extract a fraction from a total—40/60 in (25a), 90/100 in (25b), and so on. We would also need to adopt partitive analyses of these constructions. Evidence for the latter comes from the fact that the Catalan glosses of these examples make use of the partitive pronoun *en*. Thus, (25a) involves *n’han passat quaranta en pecat*; (25b) involves *li’n faltessin noranta [...] se’n marcissin cinc*, and so on.³

3. I am indebted to an anonymous reviewer for this Catalan data.

Naturally, the recovery of these operations from the syntax is more complex here than in the cases considered thus far (up to now, nouns like *mitad* ('half') invoked fractions; here the relevant nouns name only their numerators; the denominators must be found elsewhere in the sentence). But even this complexity may be desirable, as it might explain why constructions like the ones in (25) did not survive into Contemporary Spanish.

References

- Cuervo, Rufino José (1994). *Diccionario de construcción y régimen de la lengua castellana por R.J. Cuervo; continuado por el Instituto Caro y Cuervo*. Nueva edición ordenada por el Gobierno de la República de Colombia. Santafé de Bogotá, Instituto Caro y Cuervo.
- Gamillscheg, Ernst (1966). «Zum spanischen Artikel und Personalpronomen». *Revue de Linguistique Romane* 30: 250-256.
- McCawley, James (1993). *Everything that linguists have always wanted to know about logic (but were ashamed to ask)*. Second Edition. Chicago, University of Chicago Press.
- Meyer-Lübke, Wilhelm (1890-1902). *Grammatik der Romanischen Sprachen*. Leipzig.
- Russell, Bertrand (1905). «On denoting». *Mind* 14: 479-493.
- Strawson, Peter (1950). «On referring». *Mind* 59: 320-344.
- Strawson, Peter (1964). «Identifying reference and truth values». *Theoria* 30: 96-118.