Some Late Medieval Tables in Hebrew for Planetary Equations

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Abstract: In this paper the focus is on tables for planetary equations in three sets of astronomical tables in Hebrew and Judeo-Arabic (Arabic written in Hebrew characters) that depart from the standard tradition represented by the zij of al-Battānī (d. 929). The *Persian Tables* in Hebrew by Solomon ben Elijah of Thessalonika (late fourteenth century) is an adaptation of the *Īlkhānī* zij by Naṣīr al-Dīn al-Ṭūsī (d. 1274) with a Byzantine Greek intermediary. An anonymous set of tables in Hebrew is an adaptation of the *Šāmil* zij the author of which is generally assumed to be Athīr al-Dīn al-Abharī (fl. 1240). And Joseph b. Isaac Ibn Waqār (fourteenth century) composed a set of astronomical tables in Judeo-Arabic in which he referred to Ibn al-Kammād (fl. 1116). These tables illustrate the ingenuity of medieval astronomers in offering new presentations of tables, without changing the models or the underlying parameters. They also reveal the ongoing interest by members of the Jewish community in developments in Islamic astronomy.

Keywords: Ibn Waqār, *Persian Tables*, Naṣīr al-Dīn al-Ṭūsī, *Šāmil* zij, Ibn al-Kammād

Introduction

One of the main goals of medieval astronomy was the determination of planetary longitudes at any time, past or future. Generally, this was accomplished by appealing to a set of tables, some for mean motions and others for the equations, that is, the difference between a planet's mean longitude and its true longitude. In this paper we focus on some unusual tables for planetary equations in Hebrew and Judeo-Arabic (i.e., Arabic written in Hebrew characters) that depend on Islamic sources and that have not been discussed in any detail. Similar tables in Latin are found in

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the Alfonsine tradition, and there will be occasional references to discussions of them (Chabás and Goldstein 2013; reprinted in Chabás and Goldstein 2015, chapter 6). After a preliminary section on the planetary tables of al-Battānī (Raqqa, d. 929), we turn to the tables of Joseph b. Isaac Ibn Waqār (Toledo, fl. second half of the fourteenth century) composed in Judeo-Arabic, the *Persian Tables* by Solomon ben Elijah (Saloniki [Thessalonika], late fourteenth century), and an anonymous set of tables, adapted from the *Šāmil* zij which is also anonymous but assumed to have been composed by Athīr al-Dīn al-Abharī (fl. 1240). Solomon's *Persian Tables* are an adaptation of the Byzantine Greek *Persian Syntaxis* which, in turn, is an adaptation of the Persian *Īlkhānī* zij by Naṣīr al-Dīn al-Ṭūsī (d. 1274); for both the Greek and the Persian versions there are many copies: see, e.g., Mercier 1984, p. 39, and Mozaffari 2018-2019, p. 233. All three sets of tables depart from the standard tradition represented by al-Battānī although, in almost all cases, the underlying parameters were unchanged (for a list of maximum equations in various other sets of tables, see Chabás and Goldstein 2015, p. 159). So the interest is in the variety of ways to present Ptolemy's planetary equation tables (for a general account of zijes, see King, Samsó, and Goldstein 2001). In the zij of al-Battānī, the argument in a given planetary equation table is not the same for all columns, but in these three sets of tables the argument is the same for all columns in a given table, which means there are several tables for the planetary equations of each planet rather than just one (for an example in Latin, see Chabás and Goldstein 2004; reprinted in Chabás and Goldstein 2015, chapter 8). In the *Persian Tables* there is the additional feature of displaced tables which does not occur in any earlier set of tables in Hebrew (for discussion of displaced tables, see section 3, below). There is only one known set of tables in Latin—in the Alfonsine tradition—that systematically used displaced tables but, based on the underlying parameters and the displacements, it is unrelated to the *Persian Tables* (Chabás and Goldstein 2013; reprinted in Chabás and Goldstein 2015, chapter 5). The goal of displaced tables was to reduce or eliminate subtractions, because the medieval rules for manipulating what are now understood to be negative numbers were complicated.

1. Al-Battānī

The tables of the planetary equations in the widely diffused Arabic zij of al-Battānī can be considered the standard presentation of Ptolemy's planetary theory in the Middle Ages because they had such a profound influence on tables in Latin and

Hebrew, as well as in Arabic. These tables were intended to avoid computing planetary positions directly from Ptolemy's models. There is a surprising variety of ways to present these tables without changing the model, while allowing some parameters to be modified. Figure 1 displays Ptolemy's model for an outer planet, where O is the observer, D is the center of the deferent circle AC, E is the equant point about which mean motion takes place such that OD = ED, OV is the direction to Aries 0°, A is the apogee of the planet's deferent, whose longitude λ_A is angle VOA, C is the center of the epicycle, $\bar{A}_e$ is the mean epicyclic apogee, A_e is the true epicycle apogee, $O\bar{P}$ is the direction to the mean planet, P is the position of the true planet, and $O\bar{S}$ is the direction to the mean Sun. Note that $O\bar{P}$ is parallel to $EC\bar{A}_e$ and CP is parallel to $O\bar{S}$. Angle $\bar{\varkappa}$ is the mean center (counted from the planet's apogee), $\bar{\alpha}$ is the mean anomaly (counted from the mean epicyclic apogee), and λ is the true longitude (counted from Aries 0°). Both $\bar{\varkappa}$ and $\bar{\alpha}$ are linearly dependent on time, i.e.,

$$\bar{\varkappa} = \bar{\varkappa}_0 + \mu_1 \cdot t$$

and

$$\bar{\alpha} = \bar{\alpha}_0 + \mu_2 \cdot t$$

where $\bar{\varkappa}_0$ is the mean center at epoch (the starting date), μ_1 is the mean motion of center (for example, in degrees per day), and t is the time since epoch (for example, in days). Similarly, $\bar{\alpha}_0$ is the mean anomaly at epoch, μ_2 is the mean motion of anomaly. The goal is that, given $\bar{\varkappa}$ and $\bar{\alpha}$, to find angle $\bar{P}OP$, the difference between mean and true position of the planet at time t. The first step is to find q_1 (= angle ECO), for $\varkappa = \bar{\varkappa} - q_1$, where $\varkappa$ is the true center, and $\alpha = \bar{\alpha} + q_1$, where α is the true anomaly (angle A_eCP). Then, given $\varkappa$ and α, to find q_2 = angle A_eOP. Hence, the true position of the planet from its apogee is

$$AOP = \bar{\varkappa} - q_1 + q_2. \qquad [1]$$

Given the eccentricity OD and the radius of the epicycle CP, where the radius of the deferent is 60, one can find angles q_1 and q_2 by means of trigonometry. However, the tables of the planetary equations make it much easier for the user, requiring only addition, subtraction, and multiplication.

The tables of the planetary equations in al-Battānī's zij are arranged in the same way for each of the five planets (Nallino 1903-1907, 2:108-137). We introduce

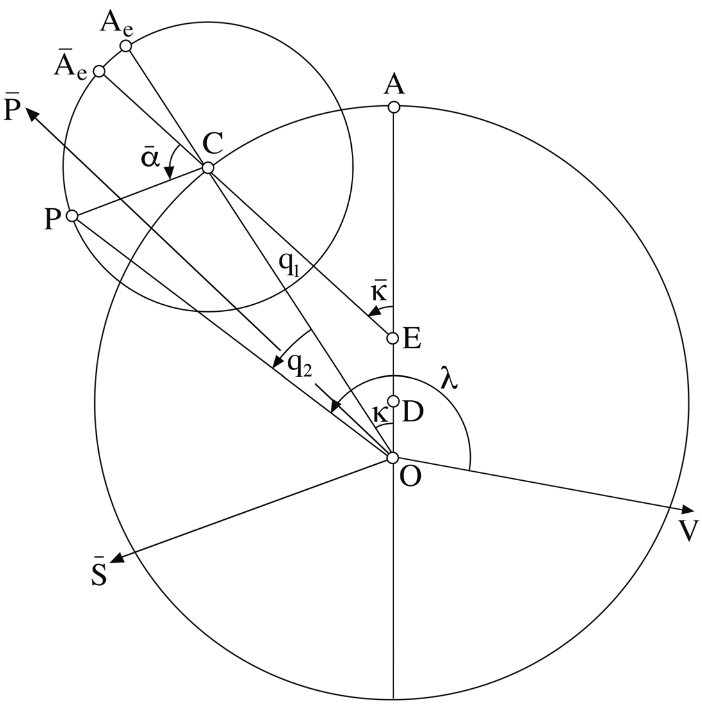

FIGURE 1: Ptolemy's model for an outer planet

the convention that an entry in the *i*th column in the tables of the planetary equations in al-Battānī's zij is denoted c_i, e.g., c_3 represents an entry in the third column. In these tables

$$q_1 = c_3(\bar{\varkappa}) \qquad [2]$$

and

$$q_2 = c_6(\alpha) + c_4(\varkappa) \cdot c_5(\alpha), \text{ for } c_4 \leq 0 \qquad [3]$$

or

$$q_2 = c_6(\alpha) + c_4(\varkappa) \cdot c_7(\alpha), \text{ for } c_4 \geq 0, \qquad [4]$$

where $c_3(\bar{\varkappa})$ is the equation of center, $c_6(\alpha)$ is the equation of anomaly at mean distance of the epicycle, $c_4(\varkappa)$ is a coefficient of interpolation (called «minutes of proportion») that varies from $-60'$ to $+60'$, $c_5(\alpha)$ is the difference between the equation of anomaly at mean distance and its equation at greatest distance, and $c_7(\alpha)$ is the difference between the equation of anomaly at mean distance and its equation at least distance. Hence

$$\lambda = \lambda_A + \bar{\varkappa} + c_3(\bar{\varkappa}) + c_6(\alpha) + c_4(\varkappa) \cdot c_5(\alpha), \text{ for } c_4 \leq 0 \quad [5]$$

or

$$\lambda = \lambda_A + \bar{\varkappa} + c_3(\bar{\varkappa}) + c_6(\alpha) + c_4(\varkappa) \cdot c_7(\alpha), \text{ for } c_4 \geq 0. \quad [6]$$

In al-Battānī's tables for the planetary equations, the argument for c_3 is $\bar{\varkappa}$; the argument for c_4 is $\varkappa$; and the argument for c_5, c_6, and c_7 is α. Note that the argument is not the same for all columns. The entries in al-Battānī's tables of the planetary equations are the same as those in the corresponding tables in Ptolemy's *Handy Tables*, with the exception of the equation of center for Venus, where Ptolemy's maximum entry is 2;24° at arguments 85° to 89° (Stahlman 1960, p. 315), and al-Battānī's maximum entry is 1;59° at arguments 84° to 97°. This change was made because of the assumption that the equation of center of Venus should equal that of the Sun, and the equation of center of the Sun was changed on the basis of new observations in the ninth century (Goldstein and Sawyer1977, pp. 167-168; Caussin de Perceval 1804, p. 40).

Al-Battānī's tables of the planetary equations were included without any changes in the set of astronomical tables compiled in Hebrew by Abraham Bar Ḥiyya (d. ca. 1136) with the exception of the equation of center for Venus, where the maximum 2;23° at 88° to 99° agrees with Ptolemy's value for the maximum solar equation: see, e.g., Paris, Bibliothèque nationale de France [BnF], MS heb. 1046, 19a-26b. We note that this value, taken from *Almagest* III.3, differs very slightly from that used by Ptolemy in his *Handy Tables*. In Immanuel ben Jacob Bonfils's Tables for 1340, the tables of the planetary equations are the same as in al-Battānī, but for the equation of center for Venus where the maximum 2;23° agrees with Bar Ḥiyya (Goldstein and Chabás 2017, p. 89).

2. Ibn Waqār

Joseph b. Isaac Ibn Waqār (Toledo, fl. second half of the fourteenth century) was a member of a prominent Jewish family (see, e.g., Steinschneider 1893, p. 598; Baer 1961, 1:136-137, 404; Castells 1996). He composed an extensive set of astronomical tables in Arabic written in Hebrew script, dated 761 AH [= 1359-1360], uniquely extant in Munich, Bayerische Staatsbibliothek, MS heb. 230 (for the date, see 4b:13-14; for the author's name, see 13b:1). The first nine folios contain a Hebrew translation of the canons to the tables composed by the author himself 37 years later (798 AH : 5b:-4 and –3). The radices in this zij are given for 720 AH [= 1320-1321]. On 4a:10-11 the author refers to the zij *al-Kawr* by the scholar Yūsuf [Ibn] al-Kammād, that is, the zij *al-Kawr ʿalā al-dawr* by Abū Jaʿfar Aḥmad b. Yūsuf Ibn al-Kammād (Cordoba, fl. 1116), a zij that is only partially preserved in Arabic. On the other hand, Ibn al-Kammād's zij *al-Muqtabis* is extant in Latin and Hebrew versions, although the original Arabic is lost (Chabás and Goldstein 1994, pp. 2-3, 38; Chabás and Goldstein 2015, p. 578). On 53a there is a table for oblique ascensions for Toledo, on 54a there is a table for lunar parallax for Toledo «whose latitude is 39;55°», and on 81a there is a table for the length of daylight in Toledo, confirming the association of the author with this city. For Ibn Waqār it is reasonable to compare his tables to those of Ibn Kammād in addition to those of al-Battānī, although there are no significant differences between the planetary equation tables of Ibn al-Kammād and al-Battānī. As we will see, there are noticeable affinities of Ibn Waqār's tables with the Tables of Barcelona (c. 1381), which also depended heavily on Ibn al-Kammād. These tables, compiled by Jacob Corsuno, a Jew from Seville who became a member of the court of Pere el Cerimoniós, King of Aragon and Catalonia (1319-1387), are extant in Hebrew, Latin, and Catalan versions (Millás 1962; Chabás 1996; Chabás and Goldstein 2015).

We restrict our attention to Ibn Waqār's tables for the planetary equations (for his tables for finding true syzygy, see Chabás and Goldstein 2015, pp. 44-46; for his table for the lunar equation, see Chabás and Goldstein 2019, p. 342). The equations for all planets are treated in the same way: there are two main tables for the equations of center and anomaly, with two more tables for interpolation. In all cases the maximum equations of center and anomaly are the same as in Ibn al-Kammād and al-Battānī, with a small discrepancy for the equation of center of Mars: ±11;25° (Ibn Waqār and al-Battānī), and ±11;24° (Ibn al-Kammād and the Toledan Tables): see Chabás and Goldstein 2012, p. 77. Here we only present excerpts of the tables for Jupiter. The argument in Table 1 (40a-b) is the mean longitude and the argument in Table 2 is the true anomaly (44b). Table 3 (50b and 51a) is a multiplication table where one argument is the minutes of proportion found in Table 1, to be used for interpolation in Table 2. The structure of Tables 3 and 4 (51b) are described below. The headings are all in Arabic written in Hebrew script.

The entries in Table 1 agree very closely with those in the Tables of Barcelona (Millás 1962, pp. 204-205, Chabás 1996, pp. 496-499): see Table 1A.

TABLE 1 (Ibn Waqār):
Subtract the equated [longitude] from the mean [longitude]
of the Sun; the remainder is the equated anomaly [1].

Table for equating the mean [longitude] of Jupiter: enter it with the first mean [motion] and you will find the equated [longitude] (excerpt)

Degrees	0s	Min. of the equation	...	3s	Min. of the equation	...	5s	Min. of the equation	...
1	0s 2;53°	115		3s 5;52°	35		5s 1;53°	0	
2	0s 3;58	115		3s 6;50	34		5s 2;47	0	
3	0s 5; 4	114		3s 7;48	33		5s 3;42	0	
4	0s 6; 9	114		3s 8;46	32		5s 4;36	0	
5	0s 7;14	113		3s 9;44	31		5s 5;32	0	
...									
10	0s 12;39	111		3s 14;30	26		5s 10;36	0	
...									
20	0s 23;25	106		3s 23;58	17		5s 19;13	0	
...									
30	1s 4; 5	97		4s 3;20	12		5s 28;22	2	

[1] Written above the table, which is not numbered in the manuscript. The same statement, which is true for all outer planets, appears in chapter 6 of the canons to the Tables of Barcelona (Millás 1962, Heb.: p. 110:25-26; Catalan: p. 131:6-8; Latin: p. 149:18-19).

Comment

The argument for Table 1 is the mean longitude, $\bar{\lambda} = \lambda_A + \bar{\kappa}$, where λ_A is the longitude of Jupiter's apogee and $\bar{\kappa}$ is its mean center. The entry λ' is the longitude corrected for the equation of center (here called the equated longitude), $\lambda' = \lambda_A + \kappa$, where $\kappa = \bar{\kappa} - c_3$. The second entry for each argument is called «minutes of equation» and it is the usual minutes of proportion plus 60 ($c_4 + 60$) ranging from 0 to 120, instead of the standard entries for the minutes of proportion c_4 which range from –60 to +60. In this way subtractions are avoided. The minutes of proportion serve as one of the arguments in Tables 3a and 3b.

TABLE 1A: Ibn Waqār and the Tables of Barcelona

Ibn Waqār			Tables of Barcelona	
Arg.	equ. long.	c_4 + 60	equ. long.	c_4
1	0s 2;53°	115′	0s 2;54°	53′ [+ 60 = 113]
2	0s 3;58	115	0s 3;59	53′ [+ 60 = 113]
3	0s 5; 4	114	0s 5; 5	53′ [+ 60 = 113]
4	0s 6; 9	114	0s 6;10	54* [+ 60 = 114]
5	0s 7;14	113	0s 7;15	54* [+ 60 = 114]
...				
30	1s 4; 6	97	1s 4; 6	37 [+ 60 = 97]
...				
120	4s 3;20	12	4s 3;20	–51 [+ 60 = 9]
...				
180	5s 28;22	2	5s 28;17	–58 [+ 60 = 2]

*Vatican, Biblioteca Apostolica, MS heb. 356, 55b, reads 53.

In Table 1 the extreme value for c_3 is 5;15° at 65° – 72°, whereas in the Tables of Barcelona the extreme value for c_3 is 5;16° at 65° – 72° (Chabás 1996, p. 499). Jupiter's apogee is not given explicitly but, from the table, we can determine that its longitude is 5s 11° (= 161°), for the entry with this argument is 5s 11° (that is, c_3 = 0;0°), whereas in the Tables of Barcelona the apogee is specified as 5s 11;3° in the heading of the corresponding table. A comparison of the entries in Table 1 with those in the corresponding table in the zij of Ibn al-Kammād, shows that there is a shift of 161° (see Table 1B). The reason is that for Ibn al-Kammād the argument is $\bar{\kappa}$, whereas for Ibn Waqār (and the Tables of Barcelona) the argument is the mean longitude $\bar{\lambda}$ which already includes the apogee of the planet: see Table 1B, where the argument under Ibn Waqār is equal to 161° plus the argument under Ibn al-Kammād, and the entry under Ibn Waqār c_3 is the difference between the entry in Table 1 and the argument. The entries under c_4 are the minutes of proportion. We thus see that the table for the equated longitude both in the Tables of Barcelona and in the tables of Ibn Waqār derive from the table for the equation of center in the zij of Ibn al-Kammād, with a horizontal displacement of 161°, equal to the longitude of Jupiter's apogee, and similarly for the other planets. It is also noteworthy that the planetary apogees that can be derived from Ibn Waqār's tables are truncations of the values for the planetary apogees in the Tables of Barcelona: Saturn: 8s 1;0° (Barcelona: 8s 1;23°); Jupiter 5s 11;0° (Barcelona: 5s 11; 3°), Mars 4s 2;0° (Barcelona: 4s 2;23°); Venus: 2s 19;0° (Barcelona: 2s 19;25°); and Mercury 6s 21;0° (Barcelona: 6s 21;3°): see Chabás 1996, p. 496.

TABLE 1B: Ibn al-Kammād and Ibn Waqār

Ibn al-Kammād			Ibn Waqār			
Arg.	c_3	c_4	Arg.	c_3	$c_4 + 60$	
1	–0; 6°	–60	162	0; 6°	0	[– 60 = –60]
2	–0;11	–60	163	0;11	0	[– 60 = –60]
3	–0;16	–60	164	0;16	0	[– 60 = –60]
...						
10	–0;53	–60	171	0;53	0	[– 60 = –60]
20	–1;43	–57	181	1;43	2	[– 60 = –58]
30	–2;32	–53	191	2;33	6	[– 60 = –54]

We now turn to Table 2 which represents the equation of anomaly. The argument is the true anomaly α, that is, $\alpha = \bar{\alpha} + c_3$, where $\bar{\alpha}$ is the mean anomaly and c_3 is derived from Table 1. The entries, however, are not standard, for they display the equation of anomaly at greatest distance headed 0′, and at least distance headed 120′: see Table 2. There are no entries for mean distance.

TABLE 2 (Ibn Waqār): Table of the equation [of anomaly] of Jupiter: enter its length with the equated [i.e., true] anomaly, and its width with the minutes of proportion (excerpt).

additive										
	0s		1s		2s		...	5s		
Degrees of anomaly	0′	120′	0′	120′	0′	120′		0′	120′	
1	0;10°	0;12°	4;40°	5; 2°	8;24°	9; 6°		6; 2°	6;45°	29
2	0;19	0;21	4;49	5;12	8;29	9;13		5;52	6;33	28
3	0;28	0;30	4;57	5;20	8;35	9;19		5;41	6;20	27
4	0;38	0;40	5; 6	5;29	8;41	9;25		5;29	6; 8	26

5	0;47	0;47*	5;14	5;39	8;46	9;32		5;16	5;53	25
...										
10	1;34	1;40	5;56	6;23	9;13	10; 1		4;20	4;51	20
...										
20	3; 4	3;19	7;11	7;47	9;55	10;50		2;13	2;30	10
...										
30	4;32	4;53	8;17	8;59	10;24	11;22		0; 0	0; 0	0
	11s		10s		9s		...	6s		Degrees of anomaly
subtractive										

* Sic; read: 0;49 or 0;50?

Comment

The maximum entry is 11;35° for arguments 3s 10°–12° and 120 minutes. The maximum entry in Ibn al-Kammād's zij is 11;3° (c_6) at arguments 99°–102°, with an additive amount at least distance of 0;32° (c_7); hence the sum is 11;35°, in agreement with Ibn Waqār for least distance. The corresponding entry for 0 minutes of proportion (greatest distance) is 10;34° [= Ibn al-Kammād: c_6(100°) – c_5(100°)], and 11;3° – 0;29° = 10;34°]. For distances other than greatest and least, interpolation is required, using Table 3. For the other planets the intervals in the argument for the minutes of proportion in Table 2 are: for Saturn (same as Jupiter); for Mars the entries for each degree of argument are for multiples of 12′ from 0′ to 120′ (that is, there should be 11 columns for each degree rather than just 2, as was the case for Saturn and Jupiter; however, there are 12 columns because, instead of a column for 60′, there are columns for 59′ and 61′, for which we have no explanation); For Venus the entries for each degree of argument are for multiples of 40′ from 0′ to 120′; and for Mercury the entries for each degree of argument are for multiples of 24′ from 0′ to 120′. There are more columns of entries in the tables for these planets because the range of the equation of anomaly at least and greatest distances is much greater than it is for Saturn and Jupiter. The tables of interpolation for the minutes of proportion then take into account the different intervals for each of the planets.

The planetary equation tables of John Vimond (Paris, fl. 1320) include a table similar to Table 2, but the intervals for the argument are 6° rather than 1°. There are many differences both in presentation and underlying parameters between Vimond's tables and those of Ibn Waqār, but the feature in common is the tabulation of the equation of anomaly at greatest distance without displaying the correction of anomaly at mean distance: see Chabás and Goldstein 2004, pp. 248-254; reprinted in Chabás and Goldstein 2015, pp. 264-269.

There are two subtables for Table 3 for interpolation in Table 2: Subtable 3a (50b), with suitable changes in the headings, is to be used for Saturn, Jupiter, Mars, and Mercury, and a similar subtable (3b) is displayed for Venus (51a). Note that these minutes of proportion range from 0′ to 120′, rather than the usual 0′ to 60′: see Table 3.

TABLE 3 (Ibn Waqār): Minutes of proportion.

Subtable 3a: Table for ... the minutes of the equation of Saturn, Jupiter, Mercury and Mars (excerpt).

Saturn/Jupiter	10′	20′	30′	40′	...	120′
Mercury	2	4	6	8	...	24
Mars	1	2	3	4	...	12
...						
0; 1	0; 0°	0; 0°	0; 0°	0; 0°		0; 1
0; 2	0; 0	0; 0	0; 1	0; 1		0; 2
0; 4	0; 1	0; 1	0; 1	0; 1		0; 4
0; 8	0; 1	0; 1	0; 2	0; 3		0; 8
0;12	0; 1	0; 2	0; 3	0; 4		0;12
...						
0;32	0; 3	0; 5	0; 8	0;11		0;32
0;36	0; 3	0; 6	0; 9	0;12		0;36
...						
0;48	0; 4	0; 8	0;12	0;16		0;48
...						
1;36	0; 8	0;16	0;24	0;32		1;36

Subtable 3b: Table for the minutes of the equation of Venus (excerpt).

Venus	1′	5′	9′	13′	17′	...	37′	39′
0; 1	0; 0°	0; 0°	0; 0°	0; 0°	0; 0°		0; 1°	0; 1°
0; 2	0; 0	0; 0	0; 0	0; 1	0; 1		0; 2	0; 2
0; 4	0; 0	0; 1	0; 1	0; 1	0; 2		0; 4	0; 4
0; 8	0; 0	0; 1	0; 2	0; 3	0; 3		0; 7	0; 8
0;12	0; 0	0; 2	0; 3	0; 4	0; 5		0;11	0;12
...								

1;16	0; 2	0;10	0;18	0;25	0;33		1;11	1;15
1;20	0; 2	0;11	0;19	0;27	0;35		1;15	1;19

Comment

Subtables 3a and 3b are multiplication tables for interpolation in Table 2, where an entry is the product of both arguments divided by 120, e.g., in Subtable 3a, 0;16 = 0;48 · 40/120. The minutes of proportion are found in Table 1 (shown here for Jupiter) and in the corresponding tables for the other planets. The argument for the rows in Subtable 3a represents the difference for a given argument in Table 2 between the entries for 120′ and 0′ for Jupiter and Saturn, and the difference between adjacent entries for the other planets. There is a separate Subtable 3b for Venus because 40′/12 is not an integer: in this table the columns are at intervals of 4′ from 1′ to 37′, and then 39′.

To illustrate the computation of the true longitude λ according to Ibn Waqār's tables, let us consider for Jupiter $\bar{\lambda}$ = 3s 10° (= 100°) and α = 1s 20° (= 80°). In Table 1 the entries for argument 3s 10° are 3s 14;30° (=104;30°) for the equated longitude, and 26′ for the minutes of proportion. In Table 2 the entries for 1s 20° are 7;11° (0′) and 7;47° (120′), that is, the difference is 0;36°. Then in Subtable 3a (with interpolations), the amount to be added to 7;11° is 0;8° (= 26/120 · 0;36°), for a total of 7;19°. The true longitude l, based on Tables 1, 2, and 3a, is:

$$\lambda = \lambda' + c_6(\alpha) + [c_4(\varkappa) + 60]/120 \cdot \Delta$$

where

$$\Delta = [c_6(\alpha) + c_7(\alpha)] - [c_6(\alpha) - c_5(\alpha)] = c_7(\alpha) + c_5(\alpha).$$

Hence

$$\lambda = 104;30^\circ + 7;11^\circ + 0;8^\circ = 111;49^\circ.$$

This demonstrates that using these tables is easier than using the standard tables.

Although Tables 1, 2, and 3 are sufficient for calculating a planetary position, Ibn Waqār included Table 4 for interpolation in Table 2 for finding the true anomaly. The arguments in Table 2 are integer degrees; Table 4 is an interpolation scheme where the arguments for the rows are the differences between successive entries in the same column in Table 2, and the headings for the columns are the minutes that exceed an integer number of degrees of the argument in Table 2. Again, this is a multiplication table where an entry is the product of the headings for the column and the row (see Table 4).

Table 4 (Ibn Waqār): Interpolation (excerpt)

Argument	5′	10′	15′	…	50′	55′
0; 7[a]	0; 1°	0; 1°	0; 2°		0; 6°	0; 6°
0;14	0; 1	0; 2	0; 4		0;12	0;13
0;21	0; 2	0; 4	0; 5		0;18	0;19
0;28	0; 2	0; 5	0; 9[b]		0;23	0;20[c]
0;35	0; 3	0; 6	0; 8		0;29	0;30[d]
…						
0;56	0; 5	0;10	0;14		0;47	0;51
1; 3	0; 5	0;11	0;16		0;53	0;58
1;10	0; 6	0;12	0;17		0;58	1; 4
…						
3;23	0;16	0;34	0;51		2;49	3; 6
3;30	0;18	0;35	0;53		2;55	3;13

a. The alphabetical numeral in the MS looks like «50» but perhaps 7 was intended (in Arabic characters «7» and «50» are easily confused). This is evidence of an archetype in Arabic script.

b. Read: 0;7.

c. Read: 0;26.

d. Read: 32.

3. The «Persian Tables»

Solomon ben Elijah (late fourteenth century) lived in Saloniki (modern Thessalonika, Greece) and compiled a set of tables together with canons in Hebrew, based on a Greek version of the *Īlkhānī* zij, composed in Persian, by Naṣīr al-Dīn al-Ṭūsī (d. 1274), the director of the astronomical observatory in Maragha (Sayili 1960, pp.187-223; Mozaffari 2018-2019, p. 151). Solomon had the nickname *sharvit* *ha-zahav* (the golden sceptre: cf. Esther 4:11), probably a pun on the Greek name Chrysococces («with golden seeds»; cf. Tihon 2017, p. 323, n. 3; Tihon and Mercier 1998, p. 259), for George Chrysococces (c. 1347) is a prime candidate as author of the *Persian Syntaxis* in Greek that underlies Solomon's text, uniquely preserved in Paris, BnF, MS heb. 1042 [henceforth MS P] 34a-108a.

In the canons to Solomon's tables there is no mention of a Greek author or of a Greek text — or of a Persian author or a specific Persian text. However, as we shall see, internal evidence from the tables provides support for these claims. The canons begin by asserting that the astronomical tables produced by the Persians are easier to use than those in Ptolemy's *Almagest* (MS P 34a). As noted by Mercier (Tihon and Mercier 1998, p. 260), Solomon's Hebrew canons of the *Persian Tables* are different from the Greek canons of the *Persian Syntaxis*. The date for Solomon's canons can be established from references to two eclipses in his canons: a lunar eclipse in the middle of Elul 5134 A.M. (MS P 47a-48a), corresponding to August 22, 1374, and a solar eclipse at the beginning of Elul 5135 A.M. (MS P 48b), corresponding to July 29, 1375. Interestingly, there was a solar eclipse on this date, but it was below the horizon and not visible in Thessalonika. The canons inform us that the radices were computed by the original author in the city of Tivini, whose longitude is 72°, in the province of Khazaria (MS P 36a). As noted by Pingree, the tables on which George Chrysococces commented were translated from Persian (or Farsi) by Gregory Chioniades (fl. 1300) for a place whose longitude is 72° and whose latitude is 38° (Pingree 1964, p. 144; cf. Bardi 2018, p. 247); these coordinates correspond to the city of Dvin or Dabīl in Armenia (Kennedy and Kennedy 1987, p. 97; see also p. 338). However, according to Pingree, this place name might be a mistake for Tabriz — whose medieval coordinates are 82° (long.) and 38° (lat.) — for Dvin had been destroyed by the Mongols in 1236 (Pingree 1998, p. 104). Moreover, Chioniades is known to have traveled to Tabriz (Mercier 1984, p. 36). The relation of the Greek texts to their Persian antecedents has been the subject of several studies: see, e.g., Mercier 1984, Tihon 1990, and Bardi 2018. Of primary interest in the context of Solomon's *Persian Tables* is the claim in the previous literature on Byzantine astronomy that all the tables for the Sun, the Moon, and the planets in the *Persian Syntaxis* were drawn with few changes from the *Īlkhānī* zij. As we shall demonstrate, the same is true for the *Persian Tables* in Hebrew.

Apparently, there were contacts between Byzantine scholars and their counterparts in the Jewish community, although the circumstances are unknown (Tihon 2017, p. 333). The main evidence for the transmission of astronomical tables from Hebrew to Greek consists of Greek translations of treatises by Immanuel ben Jacob

Bonfils of Tarrascon (ca. 1350), Jacob ben David Bonjorn, called *ha-poʿel* (the table maker; late fourteenth century, Perpignan), and Isaac Ibn al-Ḥadib (d. ca. 1430 in Sicily): Tihon and Mercier 1998, pp. 255-261; Tihon 2017, 324-332. In all three cases the tables are restricted to the Sun and the Moon with nothing concerning the five planets. There are also notes in Hebrew script in a Greek astronomical manuscript (Tihon 2017, pp. 346-347), suggesting that at one time the manuscript was owned by a Jew. However, only one set of tables is known to have been transmitted from Byzantine Greek into Hebrew, namely, the *Persian Syntaxis* adapted in the *Persian Tables*, and they include planetary tables. It is unclear if the Hebrew tables depend on the version of the *Persian Syntaxis* by George Chrysococces or the version by Theodore Meliteniotes (fl. 1350): Tihon 1996, p. 246-248; Tihon 2017, p. 323; Bardi 2018, pp.252-255.

There is little information on Solomon ben Elijah apart from what can be gleaned from his canons: see Steinschneider 1893, p. 537; Steinschneider 1964, pp. 178-179. The catalogue of Hebrew manuscripts in the Vatican, Biblioteca Apostolica [henceforth BAV], indicates that MS heb. 393, 3a-8a, contains the first chapter of the canons (but no tables) to the *Persian Tables*: Richler et al. 2008, pp. 339-340. No other copy of Solomon's *Persian Tables* is extant.

The manuscripts consulted for the tables are (online versions checked Sept. 12, 2019):

- MS P. Paris, BnF, MS heb. 1042 (The *Persian Tables*). https://gallica.bnf.fr/ark:/12148/btv1b10549506z/f90.image.r=manuscrits%20hébreux%201042
- MS V. Vatican, BAV, MS gr. 210 (The *Persian Syntaxis*). https://digi.vatlib.it/view/MSS_Vat.gr.210
- MS B. Berlin, Staatsbibliothek, MS Sprenger 1853 (The *Īlkhānī* zij). https://digital.staatsbibliothek-berlin.de/werkansicht/?PPN=PPN635599538&PHYSID=PHYS_0005

The transcriptions of the tables are based on MS P and, in general, variants are not noted. The folio numbers in MSS V and B are given for comparison, mainly of the headings.

The *Persian Tables* are the earliest example of displaced planetary tables in Hebrew. The classic account of such planetary tables is given in Kennedy 1977, where the vertical and horizontal shifts are described. Kennedy demonstrated the equivalence of displaced tables with standard tables, that is, displaced tables yield the same results as the corresponding standard tables. Since 1977, many articles have appeared that discuss displaced tables in various sets of tables, including the *Īlkhānī* zij: see, e.g., Mercier 1989, van Brummelen 1998, Dalen 2004, and Mozzafari 2018-2019; cf. Chabás and Goldstein 2013 for discussion of a set of displaced tables in Latin. The underlying parameters for the planetary equations in the *Persian Tables* are the same as those in al-Battānī, with the exception of the maximum equation of anomaly for Mars: 42;12° (*Persian Tables* and the *Īlkhānī* zij); 41;9° (*Almagest* and al-Battānī's zij): MS P 68b; cf. Mozaffari 2018-2019, p. 240; Tihon 1981, p. 80; Chabás and Goldstein 2015, p. 159.

The basic rules are that $\overline{K} = \bar{\kappa} + d_1$, and $\bar{A} = \bar{\alpha} - d_2$, where $\overline{K}$ is the mean center in the displaced table, $\bar{\kappa}$ is the mean center in the standard table, $\bar{A}$ is the mean anomaly in the displaced table, $\bar{\alpha}$ is the mean anomaly in the standard table, d_1 is the vertical displacement in the displaced table for the equation of center and d_2 is the horizontal displacement in the displaced table for the equation of center. There is another vertical displacement in the table for the equation of anomaly d_3 such that $d_3 = d_1 - d_2$.

We are aware of one set of displaced tables in Latin, called «The Tables for the Seven Planets»: Chabás and Goldstein 2013; reprinted in Chabás and Goldstein 2015, ch. 5. This anonymous set is only preserved in Paris, MS lat. 10262, 2r-46v, and it has more than the three displacements in the *Persian Tables*.

In the *Persian Tables*, the rules are the same for all five planets (MS P 58b); Jupiter serves as our example, where $d_1 = 18°$, $d_2 = 6°$, and $d_3 = 12°$. For an excerpt of the table for the equation of center for Jupiter in the *Persian Tables*, see Table 5. The rule is that

$$C_3(K) = c_3(\kappa + d_1) + d_2 \qquad [7]$$

where K is the displaced true center, $C_3(K)$ is an entry in the displaced table, κ is the standard true center, $c_3(\kappa)$ is an entry in the standard table, d_1 is 18°, and d_2 is 6°. Subscripts to «c» refer to column numbers in al-Battānī's zij. For example, $C_3(30°) = c_3(30° + 18°) + 6°$. With the value in al-Battānī's zij (Nallino 1903-1907, 2:115),

$$C_3(30°) = -3;47° + 6° = 2;13°,$$

which is the entry in the *Persian Tables*. The maximum, found at 8s 6°–12° (246°–252°), is 11;15°, and the minimum, found at 2s 10°–18° (70°–78°), is 0;45°. The mean of the maximum and minimum entries is 6;0°: 6;0° – 0;45° = 5;15°, and 11;15° – 6;0° = 5;15°, which is the parameter used by Ptolemy and al-Battānī. This shows that there is a vertical displacement of 6°. Indeed, in the corresponding table in al-Battānī's zij (Nallino 1903-1907, 2:114-125), the extrema at 92°–97° and 263°–268° are ±5;15°.

Table 5 (*Persian Tables*):
Jupiter. First correction (MS P 64b; MS B 54a; MS V 88r)

	Table for Jupiter's [first] correction [i.e., the equation of center]; it is taken with the mean [center], to be added to the mean [center] and to be subtracted from the [mean] anomaly [1]										
[2]	0s	1s	2s	3s	4s	5s	6s	7s	8s	9s	10s	11s
0°	4;27°	2;13°	0;54°	0;56°	2;22°	4;51°	7;42°	10; 1°	11;12°	10;56°*	9;24°	7; 3°
1	4;22	2; 9	0;53						11;13		9;20	6;38
2	4;17	2; 5	0;52						11;13		9;16*	6;53
...												
6									11;15			
...												
10	3;38	1;39	0;45	1;15	3; 7	5;48	8;34	10;35	11;15	10;33	8;46	6;11
...												
12									11;15			
...												
18			0;45									
...												
20	2;52	1;12	0;46	1;44	3;57	6;47	9;21	10;57	11;10	10; 2	7;53	5;18
...												
29	2;16	0;55	0;54						10;58	9;58	7; 8	4;32

[1] The heading in Hebrew includes the phrase *ʿim ha-emṣaʿ huʾ nitpas* («with the mean [center] it is taken»), an unusual expression that reflects the heading in Greek which, in turn, is a literal translation of the Persian in the *Īlkhānī* zij: «Table of the first equation of Jupiter. [The entry] is taken with the [mean] center...». The sense is that the mean center is the argument for the entries in this table.

[2] MS P: «Degrees of half the motion»; *Īlkhānī* zij: «degrees (*ajzā*ʾ) of center». The Hebrew does not make sense: it seems to result from a misunderstanding of the Greek, «degrees of mean motion or center», translating «mean» as «half».

* 9s 0°. MS P: 11;16; read (with MS B): 10;56.

*10s 2°. MS P: 9;26; read (with MS B): 9;16.

For an excerpt of the equation of anomaly for Jupiter in the *Persian Tables*, see Table 6. The rule is that

$$C_6(A) = c_6(\alpha) + d_3, \qquad [8]$$

where A is the displaced corrected anomaly, α is the standard corrected anomaly, and A = α, as shown by Kennedy (1977, p. 15: note that his Γ′ corresponds to A, and his γ′ corresponds to α). Moreover, $C_6(A)$ is an entry in the displaced table, and d_3 = 12° is the vertical displacement. For example,

$$C_6(30°) = c_6(30°) + 12°,$$

where $c_6(30°)$ = 4;42° is the entry in col. 6 for the equation of anomaly for Jupiter in al-Battānī's zij (Nallino 1903-1907, 2:114). Then

$$C_6(30°) = c_6(30°) + 12° = 4;42° + 12° = 16;42°,$$

which is the entry in the *Persian Tables*. The minimum entry, found at 8s 16°–21° (256°–261°), is 0;57°, and the maximum entry, found at 3s 9°–13° (99°–103°), is 23;3°. The mean of the maximum and minimum entries is 12°: 12° – 0;57° = 11;3°, and 23;3° – 12° = 11;3°. Note that all entries in Table 6 are positive. This table corresponds to col. 6 in the table for Jupiter's equations in al-Battānī's zij, where the extrema at 99°–102° and 258°–261° are ±11;3°. There is no horizontal displacement.

TABLE 6 (*Persian Tables*):
Jupiter. Second correction (MS P 65a; MS B 54b-55a; MS V 88v)

	Table for Jupiter's second correction, [the entry] is taken with the corrected anomaly, and it is to be kept [1]							
Degrees of corr. anomaly	0s	1s	2s	3s	...	8s	...	11s
0	12; 0°	16;42°	20;38°	22;51°		1;37°		7;18°
1	12;10	16;51	20;44	22;53		1;33		7;27
2	12;20	17; 0	20;50	22;55		1;29		7;36
...								
9				23; 3				
...								

13				23; 3				
...								
16						0;57		
...								
21						0;57		
29	16;33	20;31	22;49	22;27		1; 7		11;50

[1] The heading in Hebrew corresponds to the headings in Greek and Persian.

Since the equation of anomaly is computed for the center of the planet's epicycle at mean distance from the observer, there are columns for adding to, or subtracting from, the relevant entry when the epicycle is not at mean distance. In al-Battānī's tables, col. 5 is for greatest distance (entry to be subtracted), and col. 7 is for least distance (entry to be added). Col. 4, where the argument is the true center, is for interpolation between least and mean distance and between mean and greatest distance.

In the displaced tables, the following equations hold (which are analogous to Equations [5] and [6]):

$$L = L_o + K + C_6(A) + C_4(K) \cdot C_5(A), \text{ if } C_4 \leq 0 \qquad [9]$$

or

$$L = L_o + K + C_6(A) + C_4(K) \cdot C_7(A), \text{ if } C_4 \geq 0, \qquad [10]$$

where L_o is the apogee in the displaced mean motion tables, and C_5, C_6, C_7, and C_8 in the entries in the displaced tables correspond to those in c_4, c_5, c_6, and c_7 in the standard tables. In equations [5] and [6], $c_6(\alpha)$ is positive for α from 0° to 180° and negative for α from 180° to 360°. When $c_6(\alpha)$ is positive, $c_4(\varkappa) \cdot c_5(\alpha)$ is negative, and when $c_6(\alpha)$ is negative, $c_4(\varkappa) \cdot c_5(\alpha)$ is positive. That is, the epicycle is farther from the observer than it is at mean distance, and this decreases the absolute value of the equation of anomaly at mean distance. By contrast, when $c_6(\alpha)$ is positive, $c_4(\varkappa) \cdot c_7(\alpha)$ is positive, and when $c_6(\alpha)$ is negative, $c_4(\varkappa) \cdot c_7(\alpha)$ is negative. That is, the epicycle is closer to the observer than it is at mean distance, and this increases the absolute value of the equation of anomaly over its value at mean distance. In the *Persian Tables* these rules are modified because $C_6(A)$ is always positive. So, instead of referring to C_6, the rules are stated in terms of A, distinguishing when A (= α) is between 0° and 180° from when it is between 180° and 360°.

Table 7a displays an excerpt of the tables for C_4 and C_5, where the entries in subtable 1 are values for C_4 and the entries in subtable 2 are values for C_5. The entries in C_4 are displaced by 12° (= d_3) with respect to entries in the corresponding column in al-Battānī's zij (Nallino 1903-1907, 2:116): e.g., in MS P, $C_4(270°)$ = 11′; in al-Battānī's zij, $c_4(282°)$ = 11′. The entries for C_5 correspond to those for c_5 in al-Battānī's zij, and they are not displaced. In MS P the maximum is 30′ at 3s 16°–28° (= 106°–118°); al-Battānī has a maximum of 30′ at 107°–119°. Despite the heading in MSS P and V, C_5 is for greatest distance.

Table 7a (*Persian Tables*):
Jupiter. Interpolation, Part 1 (MS P 65b; MS B 55b; MS V 89r)

[Subtable 1: minutes of proportion]							[Subtable 2: subtractive difference]							
	First lowering (*šiflut*) of Jupiter [1]							First minutes (*medublal ri'šon*). Least distance (sic) [2] The argument is the corrected anomaly. [The entry] is to be multiplied by [the minutes of proportion for] least distance and, if the product comes within these signs, it is to be subtracted from the second correction [i.e., the equation of anomaly].						
								subtract						
Degrees [of corrected center] [3]	8s	9s	...	11s	...	2s	Degrees of corrected anomaly	0s	1s	...	3s	4s	5s	[4]
0	0′	11′		57′		19′	0	0′	10′		27′	29′	21′	29
1	0	12		58		18	1	0	10		28	29	21	28
2	0	13		58		17	2	1	11		28	29	20	27
...							...							
8				60			...							
...							...							
...							16	5	15		30	26	10	13
17						1	...							
18						0	...							

...							...							
20	0	32					...							
21	1	33					...							
...							...							
...							28	9	19		30	22	2	1
29	9	40		60		0	29	10	20		29	22	1	0
								11s	10s		8s	7s	6s	
								add						
								But if [the entry] comes within one of these signs, [the product] is to be added to the second correction [i.e., the equation of anomaly]».						

[1] Subtable 1. This subtable corresponds to col. 4 for Jupiter in the tables for the equations of Jupiter in al-Battānī's zij. The heading in the *Īlkhānī* zij is: «Minutes of anomaly at greatest distance».

[2] Subtable 2. This subtable corresponds to col. 5 (greatest distance) for Jupiter in the tables for the equations of Jupiter in al-Battānī's zij. The heading in the *Īlkhānī* zij: «Difference to be subtracted according to the equated anomaly (*khāṣṣa muʿaddala*)». Despite the heading in both the Hebrew and the Greek versions, this subtable is for greatest distance, not least distance.

[3] *Īlkhānī* zij: «Degrees of corrected center (*ajzāʾ markaz muʿaddal*)».

[4] The entries is this column should all be increased by 1 to agree with the corresponding entries in the *Īlkhānī* zij, that is, symmetry requires that an entry for an argument such as 1s 1° should be the same as that for 10s 29° (not 28°) so that the sum of the arguments is 12s 0°. The Hebrew and Greek versions share the shift in the entries in this column.

Table 7b displays an excerpt of the tables for C_4 and C_7, where the entries in subtable 1 are values for C_4 and the entries in subtable 2 are values for C_7. Again, the entries for C_4 are displaced by 12° with respect to the entries in the corresponding column in al-Battānī's zij (Nallino 1903-1907, 2:116): e.g., in MS P, $C_4(90°) = 13'$; in al-Battānī's zij, $c_4(102°) = 13'$. The entries in Table 7b, subtable 2, correspond to col. 7 in al-Battānī's zij for the equation of Jupiter, «least distance» (Nallino 1903-1907, 2: 114-119). The maximum entry in MS P is 33′ for arguments 3s 21°–29° (= 111°–119°); in al-Battānī's zij, the maximum entry in col. 7 is 33′ for arguments from 111° to 120°. There is no displacement. Although this term, C_7, is labeled «least distance» by al-Battānī and it is additive, the instruction given in the *Persian Tables* is correct, for when A is between 6s 0° and 11s 30° (180° and 360°), $C_4 \cdot C_7$ is negative. This term has the effect of increasing the absolute value of the equation of anomaly. The terminology of the text reflects the difficulty of dealing with negative quantities in the Middle Ages.

Table 7b (*Persian Tables*):
Jupiter. Interpolation, Part 2 (MS P 65b; MS B 55b; MS V 89v)

[Subtable 1: minutes of proportion]							[Subtable 2: additive difference]						
Second lowering of Jupiter; the argument is the corrected mean [center] [1]							Second minutes (*medublal šeni*) for Jupiter's least distance; the argument is the corrected anomaly. If it belongs to these signs, [the product] is to be subtracted [2]						
								subtract					
[3]	2s	3s	…	5s	…	8s	[4]	6s	7s	8s	…	11s	[5]
0	0′	13′		57′		19′	0	0′	23′	33′		11′	29
1	0	14		57		18	1	1	23	33		11	28
2	0	15		57		17	2	2	24	33		10	27
…													
8							8			33			21
9										32			20
10				60									
16	0												
17	1												
…													
19						1	19	15	31	32		4	10
20						0	20	16	31	32		3	9
…													
27				60									
28	11	39		59									
29	12	40		59		0	29	22	32	31		0	0
								5s	4s	3s	…	0s	
								add					
								But if it belongs to one of these signs, [the product] is to be added to the second correction [i.e., the equation of anomaly].					

[1] Subtable 1. Heading in the *Īlkhānī* zij: «Minutes of anomaly at least distance».

[2] Subtable 2. Heading in the *Īlkhānī* zij: «Difference to be subtracted according to the equated anomaly».

[3] MS P: blank; *Īlkhānī* zij: «Degrees of corrected center (*ajzā' markaz mu'addal*)».

[4] MS P: blank; *Īlkhānī* zij: «Degrees of corrected anomaly (*ajzā' khāṣṣa mu'addala*)».

[5]. The entries in this column should all be increased by 1 to agree with the corresponding entries in the *Īlkhānī* zij.

The radices for Jupiter in MS P 62b for 720 AY, i.e., 1350 (where AY refers to years of the Yazdigird era whose epoch is June 16, 632 [JDN 1952063]), are consistent with those in the *Īlkhānī* zij, where the radices are for 601 AY, i.e., 1232, and with those in the *Persian Syntaxis*, where the radices are for 710 AY, i.e., 1340: MS V 84r-85v; cf. Mercier 1984, p. 55. Moreover, the entries in the equation tables in MS P agree with those in the *Īlkhānī* zij (for Jupiter, see MS B 54a-55b) and in the *Persian Syntaxis* (for Jupiter, see MS V 88r-89r).

The terminology in the headings in MS P is unusual, and reflects the terminology in Greek and Persian. For example, in the *Persian Tables* there is an unusual term in Hebrew for anomaly, *segula*, which was intended to translate the Greek *idion* and, in turn, the Arabic *khāṣṣa*, whose meaning in a non-technical context is «special», «characteristic», or «peculiar». The Hebrew term *šiflut* means «lowering», but in an astronomical context it refers to perigee or the lowest altitude of a star (Sela 2007, p. 388; Rodríguez Arribas 2016, p. 102). In Table 7 of the *Persian Tables šiflut* translates the Greek term *tapeinôma*, which also means «lowering». However, the relevant sense of «lowering» here is not evident. The Hebrew term *medublal* in the headings to Table 7 means «thin» or «sparse», and it is not a technical term in medieval Hebrew astronomy elsewhere; it is probably intended to translate the Greek *leptos*, which generally means «fine» or «thin» but in an astronomical context it means «minute». For an example of Persian phraseology underlying the Hebrew, see Table 5, n. 1 (cf. Neugebauer 1960, p. 25, col. 2).

4. The Hebrew Version of the «Šāmil» Zij

Vatican, BAV, MS heb. 381, 80a-154a, contains a unique copy of an anonymous set of tables (with no canons) for epoch 600 AY [= 1231] that was adapted from the anonymous *Šāmil* Zij. Richler (2008, pp. 324-325) dated this Hebrew manuscript to the mid-fifteenth century, and noted that the same manuscript has tables for Cyprus as well as a version of the Parisian Alfonsine Tables (see also Goldstein, 2019). The *Šāmil* zij is not mentioned in this Hebrew text, but the evidence for this identification is strong. Tihon (2008, p. 808) indicated that Cyprus was a center of astronomical activity in Greek in the late Middle Ages. Nevertheless, there is no indication that the author of these tables depended on a Greek version; rather, it seems that he translated these tables directly from an Arabic source. Although the *Šāmil* Zij is anonymous, it has long been argued that the author was Athīr al-Dīn al-Abharī (fl. 1240), based on similarities between his *Athīrī* Zij and the *Šāmil* Zij (see, e.g., Kennedy 1956, pp. 129 and 135; Dalen 2004, pp. 833-835; for a list of copies of this zij, see Mozaffari 2019, p. 547).

A characteristic feature of the *Šāmil* Zij is that an unnamed place with geographical longitude 84° appears in the headings of some tables, and this is also the case in the Hebrew version (Dalen 1989, p. 107; see, e.g., Vatican, BAV, MS heb. 381, 105b, and Paris, BnF, MS ar. 2529, 36a; cf. MS ar. 2528, 28a). Another characteristic of this zij is that the argument is given at intervals of 0;12° rather than the usual 1°. The equations are presented in three tables for each planet: we display excerpts of the tables for Jupiter only. Table 8 is for the equation of center, c_3, following al-Battānī's numbering, and Table 9 is for the equation of anomaly, c_6. Table 10 has two subtables: 10a for negative values of c_4 (near apogee, i.e., between mean distance and greatest distance) and 10b for positive values (near perigee, i.e., between mean distance and least distance). For the tables in the *Šāmil* Zij we refer to Paris, BnF, MS ar. 2528: for a list of other copies, see Dalen 1989, p. 142, n. 49. In this zij, the argument is the same for all columns in a given table, and the rules for finding the true longitude of a planet from the longitude of its apogee, its mean center, and its mean anomaly at a given time are the same as those that apply to the tables in al-Battānī's zij.

Table 8 (*Šāmil* Zij): First correction for Jupiter (excerpt).
Vatican, MS heb. 381, 119a-b (Paris, BnF, MS ar. 2528, 41b-42a).

Table for the first correction of Jupiter [equation of center], to be subtracted from the center and added to the anomaly [1]

Arg.	0s Aries					…	3s Cancer					…	
	0′	12′	24′	36′	48′		0′	12′	24′	36′	48′		
0	0; 0°	0; 1°	0; 2°	0; 4°	0; 5°		5;15°	5;15°	5;15°	5;15°	5;15°		29
1	0; 6	0; 7	0; 8	0; 9	0;10		5;15	5;15	5;15	5;15	5;15		28
2	0;11	0;12	0;13	0;15	0;16		5;15	5;15	5;15	5;15	5;15		27
3	0;17	0;18	0;19	0;20	0;21		5;15	5;15	5;15	5;15	5;15		26

4	0;22	0;23	0;24	0;25	0;26		5;15	5;15	5;15	5;15	5;15		25
5	0;27	0;28	0;29	0;30	0;31		5;15	5;15	5;15	5;15	5;15		24
...													
10	0;53	0;54	0;55	0;56	0;57		5;13	5;13	5;13	5;13	5;13		19
...													
20	1;43	1;44	1;45	1;46	1;47		5; 1	5; 1	5; 0	5; 0	4;59		9
...													
29	2;27	2;28	2;29	2;29	2;30		4;43	4;42	4;41	4;41	4;41		0
	60′	48′	36′	24′	12′		60′	48′	36′	24′	12′		
	11s Pisces					...	8s Sagittarius					...	Arg. [2]

[1] Ar. «First equation for Jupiter, to be subtracted from the center and added to the anomaly».

[2] The entries in this column (both in Hebrew and in Arabic) should be increased by 1 so that the sum of the entries in the first column and the last column always equals 30.

Comment

The argument is the mean center; the equations for integer degrees are the same as those in al-Battānī's zij, e.g., $c_3(20°) = 1;43°$ (= al-Battānī); there is no displacement. In particular, the maximum value is 5;15°, as in the zij of al-Battānī.

TABLE 9 (*Šāmil* Zij): Second correction for Jupiter (excerpt).
Vatican, MS heb. 381, 120a-b (Paris, BnF, MS ar. 2528, 42b-43a)

The second correction of Jupiter [equation of anomaly], to be corrected and then added to the [corrected] center [1]

Arg.	0s Aries [2]					...	3s Cancer					...	
	0′	12′	24′	36′	48′		0′	12′	24′	36′	48′		
0	0; 0°	0; 2°	0; 4°	0; 6°	0; 8°		10;51°	10;51°	10;52°	10;52°	10;53°		29
1	0;10	0;12	0;14	0;16	0;18		10;53	10;53	10;54	10;54	10;55		28
2	0;20	0;22	0;24	0;25	0;27		10;55	10;55	10;56	10;56	10;57		27
3	0;29	0;31	0;33	0;35	0;37		10;57	10;57	10;58	10;58	10;59		26
4	0;39	0;41	0;43	0;45	0;47		10;59	10;59	10;59	11; 0	11; 0		25
5	0;49	0;51	0;53	0;55	0;57		11; 0	11; 0	11; 0	11; 1	11; 1		24
...													
10	1;37	1;39	1;41	1;43	1;45		11; 3	11; 3	11; 3	11; 3	11; 3		19
...													

20	3;11	3;13	3;15	3;16	3;18		10;55	10;55	10;54	10;54	10;53		9
...													
29	4;33	4;35	4;37	4;38	4;40		10;27	10;27	10;25	10;25	10;24		0
	60′	48′	36′	24′	12′		60′	48′	36′	24′	12′		
	11s Pisces					...	8s Sagittarius					...	Arg. [3]

[1] The Hebrew is a literal translation of the Arabic.

[2] The names of the zodiacal signs are omitted in Paris, BnF, MS ar. 2528.

[3] The entries in this column (both in Hebrew and in Arabic) should be increased by 1 so that the sum of the entries in the first column and the last column always equals 30.

Comment

The argument is the true anomaly; the entries for integral degrees generally agree with those in the corresponding table in al-Battānī's zij, but in some cases there is a difference of 1′ or 2′, e.g., $c_6(20°) = 3;11°$ (= al-Battānī). In particular, the maximum value is 11;3°, as in the zij of al-Battānī.

Table 10a (*Šāmil* Zij): Third correction for Jupiter, subtable 1 (excerpt.)
Vatican, MS heb. 381, 121a (Paris, BnF, MS ar. 2528, 43b)

	[I]						[II]					
	Corrected center [1]						Corrected anomaly [1]					
	Minutes of proportion [2]						Difference at greatest distance, to be corrected and subtracted from the second correction [2]					
	9s	10s	...	1s	2s		0s	1s	...	4s	5s	
0	0	29		52	29		0	11		29	21	30
1	0	30		52	28		0	11		29	21	29
2	0	31		51	27		0	11		29	20	28
3	1	32		51	26		1	11		29	19	27
4	2	33		50	25		1	11		29	19	26
5	3	34		50	24		1	11		29	18	25
...												
10	8	39		47	19		3	13		28	15	20
...												

20	18	47		41	9		7	16		25	8	10
...												
29	27	52		30	0		10	19		22	1	1
							11s	10s	...	7s	6s	

[1] Missing in Paris, BnF, MS ar. 2528.
[2] The headings in Hebrew agree with the corresponding headings in Arabic.

Comments

(1) The entries in the Hebrew version agree with those in the Arabic version, although in some cases there is a difference of 1′.

(2) The argument for the minutes of proportion is the true center near apogee, and the argument for the difference at greatest distance is the true anomaly. As indicated in the heading, the product of the relevant entries in [I] and [II], $c_4 \cdot c_5$, is to be subtracted from the equation of anomaly, c_6.

(3) The entries are all in minutes. The entries in [I] agree with those in al-Battānī's zij for Jupiter, col. 4, with differences of up to 2′, whereas the entries in [II] agree with those in al-Battānī's zij for Jupiter, col. 5, with differences of 1′ in some cases.

Table 10b (*Šāmil* Zij): Third correction for Jupiter, subtable 2 (excerpt)
Vatican, BAV, MS heb. 381, 121b (Paris, BnF, MS ar. 2528, 44a)

	[I]							[II]			
	Difference at least distance for Jupiter, to be corrected and added to the second correction [1]							Minutes of proportion [1]			
	0s	1s	...	4s	5s			3s	4s	...	7s	8s
0	0	11		34	22	30		1	30		52	30
1	0	11		33	21	29		2	31		52	29
2	1	12		33	21	28		3	32		52	28
3	1	12		33	20	27		4	33		51	27
4	1	12		33	19	26		5	34		51	26
5	2	13		33	19	25		6	35		50	25
...												
10	4	14		31	16	20		11	39		47	22
...												

20	8	18		27	9	10		21	46		39	12
...												
29	11	21		22	2	1		29	52		31	2
	11s	10s		7s	6s							

[1] The headings in Hebrew agree with the corresponding headings in Arabic.

Comments

(1) The entries in the Hebrew version agree with those in the Arabic version, although in some cases there is a difference of 1′.
(2) In Vatican, BAV, MS heb. 381, the order of [I] and [II] in Table 10b is inverted from their order in Table 10a.
(3) The argument for the minutes of proportion is the true center near perigee, and the argument for the difference at least distance is the true anomaly. As indicated in the heading, the product of the relevant entries in [I] and [II], $c_7 \cdot c_4$, is to be added to the equation of anomaly, c_6.
(4) The entries are all in minutes. The entries in [I] agree with those in al-Battānī's zij for Jupiter, col. 7, with differences of 1′ in some cases. The entries in [II] agree with those in al-Battānī's zij for Jupiter, col. 4, with differences of up to 2′.

Tables 10a and 10b are analogous to Tables 7a and 7b in the *Persian Tables*, that is, the minutes of proportion are separated into two subtables, one where the argument is near apogee and the other where the argument is near perigee.

Conclusion

The three sets of tables for planetary equations described here are of interest because they illustrate the ingeniousness of medieval astronomers in offering new presentations of tables without changing the model or the underlying parameters. In general, the goal was to ease the burden of the user who wished to compute planetary positions. In a survey of astronomical tables in Hebrew and Latin in the late Middle Ages (Chabás and Goldstein 2012), the impact of Islamic astronomy was shown to be pervasive. However, no references were found to zijes composed in the eastern Islamic lands after 1100. Indeed, there is still no evidence for the inclusion of such material in Latin astronomical tables before

Copernicus, but here we find tables in Hebrew that derive from two zijes, the *Īlkhānī* zij and the *Šamil* zij. This is a clear sign that zijes were still highly regarded in the Jewish community in the fourteenth and fifteenth centuries despite the availability of astronomical tables in Hebrew that provide much the same information.

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