Aspects of Arabic Influence on Astronomical Tables in Medieval Europe

José Chabás

Abstract

Transmission in Medieval Europe of astronomical tables and their accompanying texts, originally written in Arabic, has a complex history and it involves various mechanisms, which include translation, revision, adaptation, appropriation, copying, and commentaries. This paper reviews such features and presents examples of them.

Keywords:

Transmission, astronomical tables, translation, appropiation, Arabic Influence

According to one school of thought on translation studies, "Western Europe owes its civilization to translators".[1] This view may seem to be an overestimation of the process of translation, but there is general agreement that translation plays a critical role in the transmission of scientific knowledge, and that scientific translated texts provide their readers the material enabling them to develop further knowledge. This general statement also applies in the case of the transmission of Arabic astronomy to those territories in Europe where Latin and vernacular languages were used, which we may call "Latin Europe", for simplicity. Several very active translation movements developed in medieval Western Europe especially, but not only, in the Iberian Peninsula, and made Arabic texts accessible in Latin and other languages, thus giving other European astronomers the essential tools on which to build their own works.

[1] L. G. Kelly 1979. *The True Interpreter: A History of Translation Theory and Practice in the West*, Oxford: Basil Blackwell, p. 1.

The transmission of astronomical tables composed in Arabic follows the same pattern as astronomy in general. However, because of the instrumental character of tabulated material and, consequently, the need to adjust them to new users, such transmission was diverse. It also involved a few mechanisms other than strict translation activity, including copying, commentaries, revision, adaptation, and appropriation, or any combination of them.

A fine example of the complexity in transmission of astronomical tables from East to West is that of the *Zīj al-Sindhind*. This set of tables was composed by al-Khwārizmī (fl. 830), working in Baghdad, on the basis of texts of Persian and Indian origin as well as material in the Ptolemaic tradition.[2] The original Arabic version, no longer extant, used the Persian calendar and the era of Yazdegird III (epoch: June 16, 632). The *zīj* reached al-Andalus shortly after its completion and in the late 10th century Maslama b. Aḥmad al-Majrīṭī of Madrid (d. 1007), active in Córdoba, revised it.[3] Al-Majrīṭī modified some of the tables to adapt them to the Arabic lunar year of 354;22 days and the Hijra era (epoch: July 15, 622), instead of the Persian year of 365 days, and to the Hijra as epoch, and included, among others, tables for the geographical coordinates of Córdoba. The original version by Maslama in Arabic is now lost, but it is preserved in two Latin translations. One is an adaptation made by Pedro Alfonso, a Jew from Aragon who converted to Christianity in the early 12th century and was previously called Moshe Sefardí.[4] While in England, Pedro Alfonso translated into Latin al-Khwārizmī /Maslama's tables and adapted them to the Julian calendar, with October 1, 1116 as epoch. Globally the reworking of the tables was not considered satisfactory because, for example, it maintained the arrangement of the mean motions of the planets in intervals of 30 collected years (= 10,631 days), which is an appropriate grouping for Arabic years, but not for Julian years of 365 1/4 days, since 30 is not a multiple of 4. This and other computational or structural problems probably led to a reconsideration of Pedro

[2] D. Pingree 1996. "Indian Astronomy in Medieval Spain", in J. Casulleras and J. Samsó (eds.). *From Baghdad to Barcelona: Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*. Barcelona, pp. 39–48. For Islamic *zīj*es, see "Astronomical Handbooks and Tables from the Islamic World (750–1900): An Interim Report", by D. A. King and J. Samsó, with a contribution by B. R. Goldstein, *Suhayl*, 2 (2001), 9–105.

[3] J. Samsó [1992], 2011. *Las Ciencias de los Antiguos en al-Andalus*, Madrid, 2nd ed. (Almería, 2011), pp. 84–93 and 468–470.

[4] J. Casulleras 1996. "Las Tablas Astronómicas de Pedro Alfonso", in M. J. Lacarra (ed.). *Estudios sobre Pedro Alfonso*. Huesca: Instituto de Estudios Altoaragoneses, pp. 349–366. For the text and the tables, and an analysis of them, see O. Neugebauer 1962. *The Astronomical Tables of al-Khwārizmī*. Copenhagen. See also J. M. Millás 1943. "La aportación astronómica de Pedro Alfonso", *Sefarad*, 3:65–105.

Alfonso's approach, and only a few years later, also in England, Adelard of Bath (fl. 1116–1149), made a new translation of al-Khwārizmī /Maslama's tables.[5] The text in Latin is also preserved in a very limited number of manuscripts and it consists of the translation, in the strict sense, of the tables (headings and entries) and the canons explaining their use. Adelard of Bath maintained the Arabic calendar originally found in the document he was translating, thus undoing Pedro Alfonso's not very successful work of adaptation.

Moreover, the tables of al-Khwārizmī /Maslama were also transmitted through commentaries. That of Ibn al-Muthannā (10th century), a scholar otherwise unknown, is now lost in its original in Arabic, but it has reached us in both Hebrew and Latin versions. One of the two extant Hebrew versions is anonymous and the other was due to Abraham ibn Ezra (d. 1167), from Tudela, Navarra, whereas Hugo of Santalla (*fl.* 1119– 1151), working in Tarazona, Aragon, was responsible for the Latin version.[6]

Another mode of transmission of al-Khwārizmī /Maslama's tables is the compilation of sets of tables totally or partially based on them, or on other Indian procedures. These works define the so-called Indian tradition in the Iberian Peninsula, where it was firmly rooted, and of which we shall only present three examples. In al-Andalus, Ibn Mu^{c}ādh of Jaén (d. 1093) is the author of a *zīj*, the *Tabule Jahen*, which has not survived, and only a Latin translation of its canons made by Gerard of Cremona (1114–1187), working in Toledo, is preserved.[7] At about the same time, and not far from Jaén, Ibn al-Kammād (c. 1100) compiled in Córdoba two *zīj*es as well as an abridged combination of them, called *al-Muqtabis*. Only one chapter of its canons is extant in the original Arabic, but the tables are preserved in two versions: one in Latin by John of Dumpno (1260), working in Sicily, and the other in Hebrew by the Spanish astronomer Solomon

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[5] For the text and the tables, see H. Suter. 1914. *Die astronomischen Tafeln des Muḥammad ibn Mūsā al-Khwārizmī.* Copenhagen. For a review of the tables, see Neugebauer 1962 (ref. 4). For the various origins of the tables, see B. van Dalen 1996. "Al-Khwārizmī's Astronomical Tables Revisited: Analysis of the Equation of Time", in Casulleras and Samsó (ref. 2), pp. 195–252. See also Mercier, R. P. 1987. "Astronomical Tables in the Twelfth Century", in Ch. Burnett (ed.). *Adelard of Bath. An English Scientist and Arabist of the Early Twelfth Century*. London: Warburg Institute, pp. 87–118.

[6] For the Hebrew versions, see B. R. Goldstein 1967. *Ibn al-Muthannā's Commentary on the Astronomical Tables of al-Khwārizmī.* New Haven. For the Latin version, see E. Millás Vendrell 1963. *El comentario de Ibn al-Muṯannā a las Tablas Astronómicas de al-Jwārizmī: Estudio y edición crítica del texto latino, en la versión de Hugo Sanctallensis*. Madrid–Barcelona.

[7] See Samsó 2011 (ref. 3), pp. 152–166 and 484–487. See also H. Hermelink 1964. "Tabulae Jahen", *Archive for History of Exact Sciences*, 2:108–112; and Casulleras, J. 2010. *La astrología de los matemáticos. La matemática aplicada a la astrología a través de la obra de Ibn Mu^{c}āḏ de Jaén*. Barcelona, espec. 44–47.

Franco (14th century).[8] In the Christian part of the Peninsula, other tables contributed to the diffusion of Indian procedures and parameters, as well as to the survival of this tradition: the Tables of Barcelona were compiled in about 1381 by Jacob Corsuno of Seville at the behest of the king of Aragon, Pere el Cerimoniós (1319–1387). They are extant in Catalan, Hebrew, and Latin.[9] In brief, for about four centuries the tabular tradition inaugurated in al-Andalus by the *zīj* of al-Khwārizmī was progressively transmitted to other places in Europe to become one of the components of western astronomy. Figure 1 gives an example of a table in this tradition.

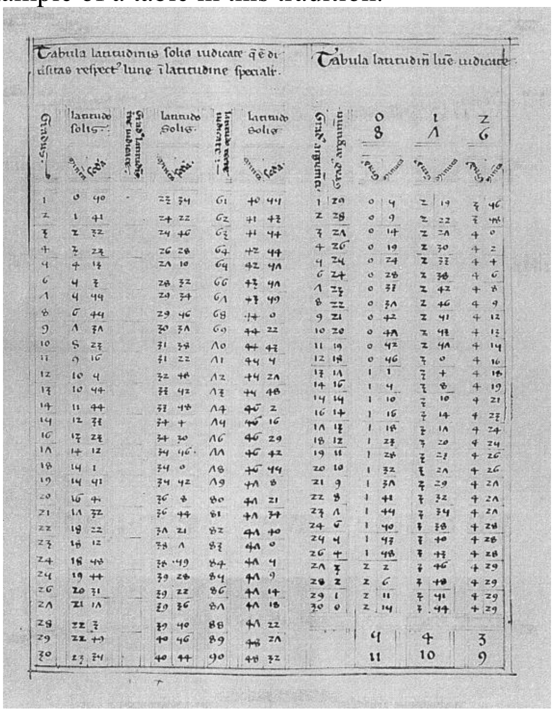

Figure 1: Table for the lunar latitude in the *zīj* of Ibn al-Kammād (Madrid, Biblioteca Nacional, MS 10023, f. 53r). The maximum value is 4;30º and the entries can be computed by means of the modern formula β = 4;30 · sin ω, where β is the lunar latitude and ω is the argument of latitude.

[8] J. Chabás and B. R. Goldstein 1994. “Andalusian Astronomy: *al-Zīj al-Muqtabis* of Ibn al-Kammād”, *Archive for History of Exact Sciences*, 48:1–41; B. R. Goldstein 2011, “Solomon Franco on the Zero Point for Trepidation”, *Suhayl*, 10:77–83.

[9] J. M. Millás 1962. *Las Tablas Astronómicas del Rey Don Pedro el Ceremonioso*. Madrid–Barcelona; Chabás, J. 1996. “Astronomía andalusí en Cataluña: Las Tablas de Barcelona”, in Casulleras and Samsó (ref. 2), pp. 477–525.

In the Iberian Peninsula the Indian tradition coexisted with the Greek tradition, based on the works of Ptolemy and represented by the *Zīj al-Ṣābi'* of al-Battānī (d. 929).[10] This *zīj* produced in Raqqa, Syria, made its way to al-Andalus by the end of the 10th century, for it was known to Maslama and his students at Córdoba.[11] A substantial difference between the two traditions is that Ptolemy, and consequently al-Battānī, uses tropical coordinates for the positions of the planets, that is, he introduces a motion in longitude of the fixed stars (now called precession), whereas the *zīj* of al-Khwārizmī /Maslama uses sidereal coordinates, that is, it allows for an oscillating motion of the vernal point (now called trepidation).

Two medieval Latin translations of the *zīj* of al-Battānī are known, both done in Northeast Spain at the time of the translation movement during the 12th century: one by Robert of Ketton (fl. 1141-1157), an English archdeacon in Pamplona, Navarra, and the other by Plato of Tivoli (fl. 1132–1146), an Italian translator working in Barcelona. Also in this town, the *zīj* of al-Battānī was adapted in Hebrew to the Jewish calendar by Abraham bar Ḥiyya in 1136, who can rightly be considered as the founder of Hebrew scientific culture and language.[12] In Toledo, an unnamed member of the group of scientific collaborators of Alfonso X, king of Castile and León (reigned: 1252–1284), translated al-Battānī's *zīj* from Arabic into Castilian, and thus opened the gate to its diffusion in a vernacular language.[13] Figure 2 offers an example of a table in this tradition.

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[10] C. A. Nallino 1903–1907. *Al-Battānī sive Albatenii Opus Astronomicum*, 2 vols. Milan. See also B. van Dalen F. S. Pedersen. 2008. "Re-editing the tables in the *Ṣābi' Zīj* by al-Battānī", in J. Dauben *et al.* (eds.), *Mathematics Celestial and Terrestrial: Festschrift für Menso Folkerts zum 65. Geburtstag*. Halle (Saale), pp. 405–428.

[11] See Samsó 2011 (ref. 3), pp. 92–93.

[12] J. M. Millás 1959. *La obra Séfer Ḥeshbón mahlekot ha-kokabim de R. Abraham bar Ḥiyya ha-Bargeloni: edición crítica, con traducción, introducción y notas.* Barcelona; T. Lévy 1997. "Abraham bar Ḥiyya (Savasorda)" in H. Selin, *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures*, Dordrecht-Boston-London, pp. 151–152. For an analysis of the tables, see R. Mercier 2014. "Astronomical Tables of Abraham bar Ḥiyya", S. Stern and Ch. Burnett (eds.). *Time, Astronomy, and Calendars in the Jewish Tradition*, Leiden-Boston, pp. 155–207.

[13] For an edition of this text in Castilian, see G. Bossong 1978. *Los Cánones de Albateni.* Tübingen.

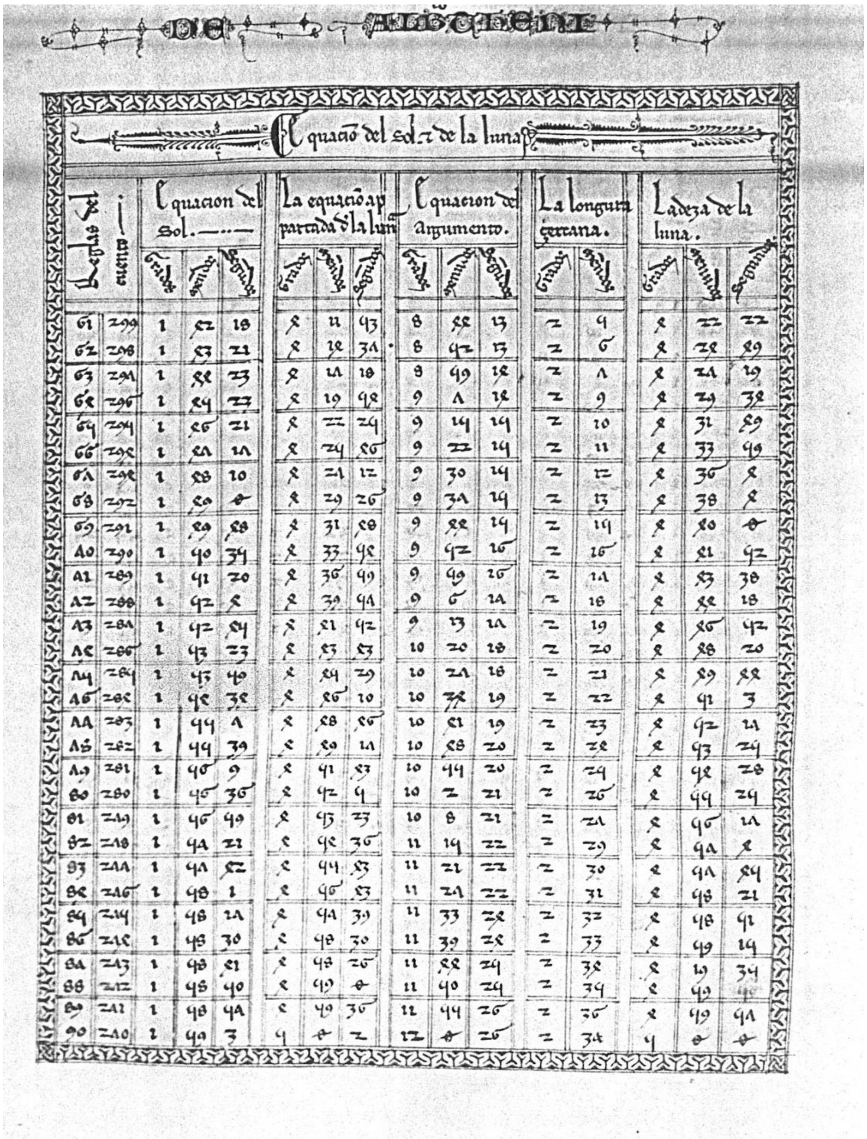

Figure 2: Table for the lunar latitude in the *zīj* of al-Battānī (Paris, Bibliothèque de l'Arsenal, MS 8322, f. 53r). The maximum value is i = 5;0º, the inclination of the lunar orb, and the entries can be computed by means of the modern formula $\beta = \arcsin(\sin i \cdot \sin \omega)$.

The Arabic *zīj*es associated with both the Indian and the Greek traditions were translated and adapted into Latin and other languages in the Iberian Peninsula, and were then disseminated throughout Western Europe. For the most part, this did not happen in a direct way, but through another *zīj* in which the two traditions met, the Toledan Tables. This *zīj* was compiled towards the end of the 11th century by Ṣāᶜid al-Andalusī (d. 1070) and his group of scholars working in Toledo, which included the noted astronomer, Ibn al-Zarqālluh (Azarquiel). The Toledan Tables were based on a small number of new observations and reflect both of these traditions, but they mainly follow the Ptolemaic framework, even though they use sidereal coordinates.[14] The Toledan Tables were originally composed in Arabic and are arranged for the Arabic calendar and the Hijra era. The original version in Arabic has not been found, whereas the number of copies of the Latin version is counted by the hundreds. Besides Latin, the Toledan Tables were translated into Greek towards 1340, and into Castilian at an unknown date, but of the latter only fragments of the tables for the planetary equations remain.[15]

In the 12th century the Toledan Tables were adapted to the Julian calendar on several occasions, and the mean motion tables were recast for different epochs and the meridians of various localities other than Toledo: Marseilles, Toulouse, Novara, Hereford, among others.[16] In all these cases Latin was maintained as the

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[14] F. S. Pedersen 2002. *The Toledan Tables: A review of the manuscripts and the textual versions with an edition*. Copenhagen; Richter-Bernburg, L. 1987. "Ṣāᶜid, the Toledan Tables, and Andalusī Science", D. A. King and G. Saliba (eds.). *From Deferent to Equant: A volume of studies in the of history science in the ancient and medieval Near East in honor of E. S. Kennedy. Annals of the New York Academy of Sciences*, 500, pp. 373–401; G. J. Toomer 1968. "A Survey of the Toledan Tables", *Osiris*, 15:5–174.

[15] For Greek, see D. Pingree 1976. "The Byzantine Version of the Toledan Tables: the Work of George Lapithes?", *Dumbarton Oaks Papers*, 30:87–132. For Spanish, see J. Chabás 2012. "The Toledan Tables in Castilian: Excerpts of the Planetary Equations", *Suhayl* ,11:179–188.

[16] For these adaptations see Mercier 1987 (ref. 5) and Pedersen 2002 (ref. 14). More specifically, for the Tables of Marseilles, see Paris, Bibliothèque nationale de France, MS lat. 14704; and M.-T. d'Alverny, C. Burnett, and E. Poulle 2009. *Raymond de Marseille. Opera Omnia* (vol. I). Paris; for the Tables of Toulouse, see Paris, Bibliothèque nationale de France, MS lat. 16658; and E. Poulle 1994. "Un témoin de l'astronomie latine du XIIIe siècle: les tables de Toulouse", in *Comprendre et maîtriser la nature au moyen âge*: *Mélanges d'histoire des sciences offerts à Guy Beaujouan*, Geneva and Paris, pp. 55–81. The Tables of Hereford and those for Novara have not yet been the object of specific studies, but Pedersen 2002 (ref. 14) gives abundant information on them: for the Tables of Hereford, see Madrid, Biblioteca Nacional, MS 10016, and Mercier 1987 (ref. 5), p. 108; for the Tables of Novara, Fritz S. Pedersen has kindly consulted for the present work two manuscripts: Dublin, Trinity College, MS D.430 (444) (mean motion tables, etc.) and Paris, Bibliothèque nationale de France, MS lat. 16655 (syzygy tables only). There is also a copy of the tables of Campanus of Novara in Arabic written in Hebrew characters: B. R. Goldstein 1979. "The

language. There was also a translation into French paired with an adaptation to the Julian calendar and the meridian of Paris.[17] Table 1 displays some relevant features of such tables.

Table 1: Direct adaptations of the Toledan Tables

	Marseilles	Toulouse	Novara	Hereford	Paris
Difference in longitude from Toledo	1;6h (16;30º E)	0;48h (12º E)	1;16h–1;19h	–0;18h (4;30º W)	Not specified
Epoch	Incarna-tion	Incarna-tion	Incarna-tion	1092	1139
Interval for the mean motion (Sun)	28 years (10,227d)	24 years (8,766d)	28 years (10,227d)	28 years (10,227d)	28 years (10,227d)
Beginning of the year	January 1 (midnight)	March 1 (noon)	March 1 (noon)	January 1 (midnight)	March 1 (noon)

Most of the tables derived from the Toledan Tables make use of cycles of 28 Julian years to group the mean motions of the Sun and the planets in collected years, while intervals of 30 Arabic years are characteristic of most Arabic *zīj*es.[18] We also note that the beginning of the year, and even that of the day, was not yet firmly established among astronomers in the 12th century. In any case, the Toledan Tables, mainly through the Latin versions, were progressively disseminated from the Iberian Peninsula, and were assimilated by practitioners in the rest of Europe to become the standard computational tool for mathematical astronomy for more than two centuries. Figure 3 displays an example of a table in this tradition.

Survival of Arabic Astronomy in Hebrew", *Journal for the History of Arabic Science*, 3:31–39, espec. pp. 34–35, reprinted in B. R. Goldstein 1985. *Theory and Observation in Ancient and Medieval Astronomy*, Variorum (Essay XXI).

[17] J.-P. Boudet and M. Husson 2012. "The Earliest Astronomical Tables in French (c. 1271)", *Journal for the History of Astronomy*, 43:287–298.

[18] In the 12th and 13th centuries, other tables, such as the Tables for Pisa and those for London, and the Tables for Mechelen, used cycles of 20 Julian years (= 7,305 days). This is the same interval as in the *zīj* of Battānī; see Nallino 1903–1907 (ref. 10), 2: 73. For Pisa and London, see Mercier 1987 (ref. 5), pp. 108–110; for Mechelen, see E. Poulle 1964. "Astrologie et tables astronomiques au XIIIe siècle: Robert Le Febvre et les Tables de Malines", *Bulletin philologique et historique du Comité des travaux historiques et scientifiques*, reprinted in E. Poulle 1996. *Astronomie planétaire au Moyen Âge latin*, Variorum (Essay VII).

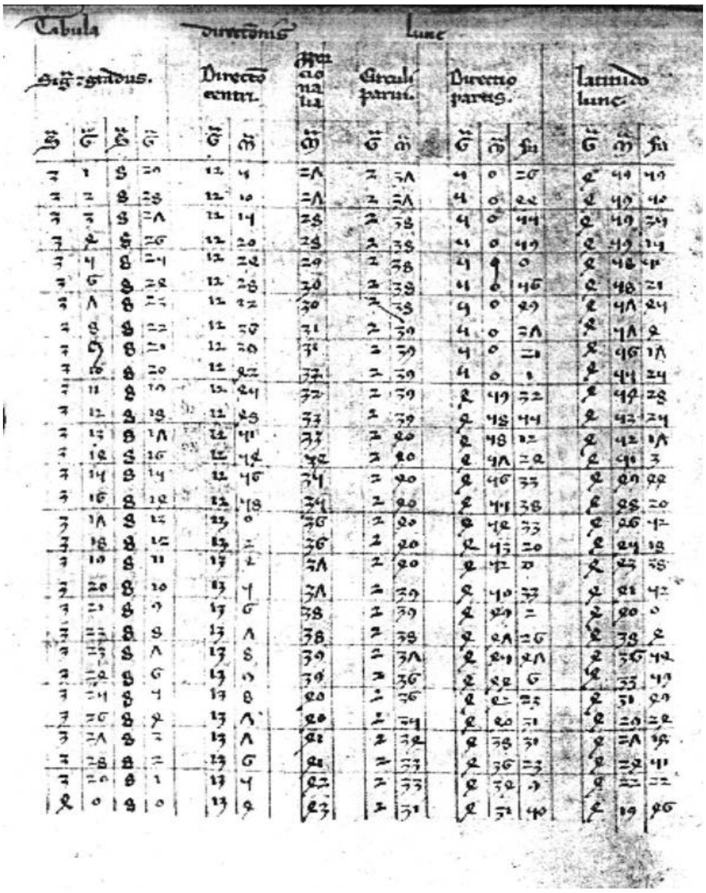

Figure 3: Table for the lunar equations in the Toledan Tables (Naples, Biblioteca Nazionale, MS VIII.C.49, f. 26v. Only one of the six pages of the complete table is displayed, corresponding to arguments 91º–120º and their complements 240º–269º, in the first two columns. The third column (*directio centri*) is for the equation of center, with a maximum of 13;9º at 115º. The sixth column (*directio partis*) is for the equation of anomaly, with a maximum of 5;1,0º at 95º.

About two centuries after the compilation of the Toledan Tables, another set, also compiled in Toledo, gradually superseded them and had an even greater impact on European astronomy. The Castilian Alfonsine Tables were composed in Toledo no later than 1272 by Isaac ben Sid and Judah ben Moses ha-Cohen, who were in the service of king Alfonso X of Castile and León. The original tables are not extant, but the canons explaining their use survive in a unique manuscript in Castilian.[19] In the canons the authors state, in the first person, that the tables were based on new observations, "to replace and correct the positions and motions of some planets that Azarquiel had computed for Toledo".[20] We are also told that the tables were arranged for the era of Alfonso (explicitly given as January 1, 1252), and used the Julian calendar, with years beginning in January, days beginning at noon, and a leap day added every four years at the end of February. The canons describe, or refer to, tables of the same type found in other *zīj*es and addressing the same astronomical problems: chronology; mean motions (with collected years grouped in intervals of 20 years); equations (with a correction for the Moon differing from the standard Ptolemaic model but reminiscent of a lively tradition represented in previous *zīj*es in al-Andalus and the Maghrib); trigonometry and spherical astronomy (solar declination, right and oblique ascensions, sines and chords, meridian altitude of the Sun, length of daylight, shadow); latitudes; stations and retrogradations; visibility of the Moon, the planets, and the stars; velocity; domification, equation of time; syzygies; parallax; eclipses; trepidation; and projection of the rays. The Castilian Alfonsine Tables follow in many ways the pattern of a standard Arabic *zīj*, in particular in their organizational principle and the list of topics addressed, and thus can be considered as belonging to a long tradition of Arabic astronomy in al-Andalus as well as a case of transfer of knowledge to the rest of Europe.

Indeed, the Alfonsine Tables of Toledo reached Paris, where they were recast by a group of competent astronomers in the early 14th century. The mode of transmission from Castile, the process by which the Parisian astronomers modified these tables, and the role played by each of these scholars, are not yet entirely clear. But we know that, without challenging either the models or the parameters used by their predecessors in the Iberian Peninsula, the Parisian astronomers put a great deal of ingenuity into compiling tables that could facilitate the astronomers' task. From Paris, the Parisian Alfonsine Tables spread throughout Europe and progressively displaced the Toledan Tables, to become the preferred instrument for computational astronomy. For about two centuries nearly all European astronomers adhered to the Alfonsine Tables, and almost no changes in the models and the parameters occurred, preserving the Ptolemaic core transmitted through Arabic astronomy. Nevertheless, a number of the tables were adapted and presented in different formats to make them more user-friendly. All this mass of tables developed within the framework of the

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[19] J. Chabás and B. R. Goldstein 2003. *The Alfonsine Tables of Toledo.* Archimedes: New Studies in the History and Philosophy of Science and Technology, 8. Dordrecht and Boston.
[20] See Chabás and Goldstein 2003 (ref. 19), p. 20.

Alfonsine Tables fills in hundreds of manuscripts and, together with the printed editions (beginning in 1483), comprise what is now called the Alfonsine corpus.

The legacy of Arabic astronomy offers examples of another mechanism involved in the transmission of tables: the assimilation of certain elements characterizing them, particularly parameters. In the case of the lunar equations, the Greek tradition developed by Arabic astronomers used Ptolemy's second lunar model, with a movable deferent, which requires an equation of center and an equation of anomaly.[21] The maximum values of the two lunar equations used by Ptolemy are 13;9º and 5;1º, respectively, and these are the values found in the *zīj* of al-Battānī and the Toledan Tables (see Figure 3), to name only a pair of sets of tables in this tradition.[22] In contrast, the Indian tradition that flourished in al-Andalus employed a simple epicyclic model, where there is no need for an equation of center. In the *zīj* of al-Khwārizmī /Maslama, the maximum value for the equation of center is 4;56º, a parameter of Indian origin which is also found in the lunar tables in Andalusian-Maghribi astronomy: Ibn Muᶜādh, Ibn Isḥāq (Tunis, ca. 1193–1222), Ibn al-Bannā'al-Marrākushī (1256–1321), and Ibn al Raqqām (Granada, d. 1315).[23] This same value reappears in Paris in the early 14th century. Now, Jean Vimond was an astronomer working at Paris around 1320 who compiled a set of tables, uniquely preserved in Latin in a manuscript in Paris.[24] In his table for the lunar equation there is no column for the equation of center, and we find 4;56º for the maximum value of the equation of anomaly. Unfortunately, the brief canons do not mention the origin of Vimond's table. The Parisian Alfonsine Tables appropriated this parameter, and presented lunar tables for the equation of center (maximum of 13;9º) and the equation of anomaly (maximum of 4;56º). Apparently, Vimond had set the standard for the tables generated in Paris and their derivatives, and all manuscripts in the Alfonsine corpus have 4;56º as the maximum value of the equation of anomaly,

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[21] For lunar and planetary models, the equations of the Sun, the Moon, and the planets, and other topics such as Alfonsine precession, see, e.g., J. Chabás and B. R. Goldstein 2012. *A Survey of European Astronomical Tables in the Late Middle Ages*, Leiden-Boston.

[22] Note that according to W. D. Stahlman 1959. *The Astronomical Tables of Codex Vaticanus Graecus 1291*. Brown University, Ph. D. dissertation (unpublished: ProQuest, UMI, AAT 6205761), the maximum value of the equation of center in Ptolemy's *Handy Tables* is given as 13;8º (see p. 252).

[23] See B. R. Goldstein 1996. "Lunar velocity in the Middle Ages: A comparative study", in Casulleras and Samsó (ref. 2), pp. 181–194.

[24] J. Chabás and B. R. Goldstein 2004. "Early Alfonsine Astronomy in Paris: The Tables of John Vimond (1320)", *Suhayl,* 4:207–294.

to the point that almost all astronomers from the 14th to the 16th centuries, including Copernicus, used it.[25]

Displaced tables provide another example of the assimilation in Western Europe of Arabic techniques in table-making. The term “displaced” applied to tables was introduced by E. S. Kennedy in 1977 to translate the Arabic *waḍʿī* for tables derived from *aṣlī* (original) tables.[26] Displaced tables are intended to facilitate computations and avoid subtractions, by using functions which are always positive, according to the following principle. If $y = f(x)$ is the function underlying a table, or a column in a table, which takes on negative values in part of its domain, adding a sufficiently large vertical displacement, k_v, to all its entries ensures that all resulting entries are positive. The new function, $y = f(x) + k_v$ is said to be vertically displaced. Similarly, when a horizontal displacement, k_h, is applied to the variable, the new function, $y = f(x + k_h)$ is said to be horizontally displaced. Of course, both types of displacements can happen simultaneously. Displaced tables improved computational methods with respect to standard tables, for they made unnecessary the inevitable long and complicated rules found in so many texts explaining their use, at a time when negative numbers were not available, and decreased the possibility of user error. Although we find no displaced tables in any of the sets of tables mentioned so far, astronomers working in Arabic beginning in the 9th century had felt the need to address subtractive corrections and introduced displaced tables to ease the task of computers.

The *zīj* of Ḥabash al-Ḥāsib (*ca.* 850), known in two versions, makes use of displaced tables to compute lunar positions.[27] The tables for the equations of the Moon are the same in both versions and are based on Ptolemy’s second model, but use elongation, rather than double elongation, as the independent variable for the equation of center and the minutes of proportion. To counterbalance this, in

[25] See *De revolutionibus* IV.11. For Copernicus, see also N. M. Swerdlow and O. Neugebauer 1984. *Mathematical Astronomy in Copernicus’s De Revolutionibus*, New York-Berlin, 1: 41–48 (on the influence of Arabic astronomy) and 225–230 (on the lunar tables).

[26] E. S. Kennedy1977. “The Astronomical Tables of Ibn al-Aᶜlam”, *Journal for the History of Arabic Science*, 1:13–23, espec. p. 16.

[27] E. S. Kennedy1956. *A Survey of Islamic Astronomical Tables*. Transactions of the American Philosophical Society, 46.2. Philadelphia, pp. 126–127, 151–154. For the Istanbul copy, see Debarnot, M.-T. 1987. “The *Zīj* of Ḥabash al-Ḥāsib: A Survey of MS Istanbul Yeni Cami 784/2”, in D. King and G. Saliba (ref. 14), pp. 35–69. For an explanation of the displaced tables, see H. Salam and E. S. Kennedy 1967. “Solar and Lunar Tables in Early Islamic Astronomy”, *Journal of the American Oriental Society* 87:493–497. Reprinted in King and M. H. Kennedy (eds.) 1983. *Studies in the Islamic Exact Sciences by E. S. Kennedy, Colleagues and Former Students*. Beirut. pp. 108–113.

the *zīj* of Ḥabash the period of both functions is halved from 360º to 180º. Furthermore, to have all corrections additive, two simultaneous displacements are introduced in the equation of center: a vertical displacement of 13;8º and a horizontal displacement of 5º. In order to obtain the same final results as with Ptolemy's method, some adjustments have to be introduced to the equation of anomaly (a change of sign and an upwards displacement of 5º). This tradition was kept alive, for it is found in a set of tables for year 1172 as epoch and using Persian years,[28] as well as in the *zīj* compiled ca. 1285 by al-Baghdādī, where Ḥabash's displaced tables are used together with double-argument tables for the combined lunar equation.[29] Later, also in the East, we find examples of the use of displaced tables for the equations for the Moon, as well as for the equation of the Sun. In the *Zīj-i Sulṭānī* of Ulugh Begh (1393–1449), as transmitted by the *Zīj al-Sharīf* (17th century), the solar equation and the equation of center of the Moon are displaced vertically, and for the computation of the lunar anomaly a vertical displacement is also introduced.[30] Similarly, in the *Zīj Durr al-muntakhab* by Cyriacus (ca. 1480), the equation of anomaly is affected by a vertical and a horizontal displacement.[31]

Displaced tables were also used by Arabic astronomers in the computation of the planetary positions. The lost *zīj* of Ibn al-Aᶜlam (d. 985), only known though references in later astronomical works, contains several examples of displaced tables, as reported in the Persian *Zīj-i Ashrafī* (ca. 1310) by Sayf-i Munajjim.[32] According to it, the equation of center of each planet is given two displacements (see Table 2): one horizontal and one vertical, which in all cases amounts to the nearest integer degree higher than the respective maximum value of the equation of center used. This ensures that the new equation of center is always positive. In contrast, the equation of anomaly of each planet is only affected by a vertical

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[28] O. Neugebauer 1960. "Studies in Byzantine Astronomical Terminology", *Transactions of the American Philosophical Society* 50, part 2, pp. 1–45, espec. pp. 24–25.

[29] C. Jensen 1971. "The Lunar Theory of al-Baghdādī", *Archive for History of Exact Sciences* 8:321–328.

[30] J. Samsó 2003. "On the Lunar Tables in Sanjaq Dār's *Zīj al-Sharīf*", in J. P. Hogendijk and A. I. Sabra (eds.) 2003. *The Entreprise of Science in Islam: New Prespectives*. Cambridge-London, pp. 285–305.

[31] G. Saliba 1976. "The Double-Argument Lunar Tables of Cyriacus", *Journal for the History of Astronomy* 7:41–46.

[32] See Kennedy 1977 (ref. 25), and Mercier, R. 1989. "The parameters of the Zīj of Ibn al-Aᶜlam", *Archives Internationales d'Histoire des Sciences* 39:22–50. See also van Dalen, B. 2004. "The *Zīj-i Nāṣirī* by Maḥmūd ibn ᶜUmar" , in Ch. Burnett *et al.* (eds.) 2004. *Studies in the History of the Exact Sciences in Honour of David Pingree*. Leiden-Boston, pp. 841–842, where the assumption is made that Ibn al-Aᶜlam did not use displaced equations.

displacement (Saturn, 7º; Jupiter, 12º; Mars, 47º; Venus, 48º; and Mercury 26º) and, very cleverly, it is set to be the difference between the horizontal displacement applied to the equation of center and its vertical displacement (e.g., for Jupiter, 12º = 18º – 6º), to counterbalance the effects of the previously applied displacements. Moreover, the interpolation function (the minutes of proportion) also requires a horizontal displacement, which in each case has the same amount as that affecting the equation of anomaly (e.g. 12º for Jupiter). As demonstrated by Kennedy, when these conditions are fulfilled, the computed planetary longitudes with the new tables are the same as those obtained with the old ones.[33] The *Jāmiᶜ Zīj* by Kūshyār ibn Labbān (ca. 1000) also introduces displaced tables for the planets, as Van Brummelen demonstrated in the case of Mars, where the same values as those attributed to Ibn al-Aᶜlam are used.[34]

According to Samsó and E. Millás, the *Minhāj* of Ibn al-Bannā'contains the first documented case of the use of displaced tables in Western Islam: the solar equation is affected by two displacements, vertical and horizontal, both amounting to 4º; for the Moon, the equation of center is given a vertical displacement of 13;9º, corresponding to the standard maximum of this correction; and in the case of the planets, the equation of center of each planet has only a vertical displacement (see Table 2).[35]

Table 2: Displacements in the equation of center of the planets in various sets of tables

	Zīj-i Ashrafī[36]		Ibn al-Bannā'	Tables of the Seven Planets	
	k_v	k_h	k_v	k_v	k_h
Saturn	7º	14º	6º	7º	14º
Jupiter	6º	18º	6º	6º	18º

33 Kennedy 1977 (ref. 26), p. 15–16.

34 G. Van Brummelen 1998. "Mathematical Methods in the Tables of Planetary Motion in Kūshyār ibn Labbān's *Jāmiᶜ Zīj*", *Historia Mathematica* 25:265–280.

35 The case of the Sun is addressed in J. Samsó and E. Millás 1994. "Ibn al-Bannā', Ibn Isḥāq and Ibn al-Zarqālluh's solar theory", in J. Samsó *Islamic Astronomy and Medieval Spain,* Variorum (Essay X); for the Moon and the planets, see J. Samsó and E. Millás 1998. "The computation of planetary longitudes in the *zīj* of Ibn al-Bannā'", *Arabic Sciences and Philosophy* 8: 259–286.

36 According to Mercier 1989 (ref. 32, p. 26), the corresponding values for the vertical and horizontal displacements of Saturn in the case of Ibn al-Aᶜlam are 6º and 13º, respectively, because the maximum value of the equation of center for this planet is 5;48º, which differs from the 6;31º found in Ptolemy's *Almagest* and *Handy Tables*. It should be noted that 5;48º is also the parameter for Saturn used by Ibn al-Bannā'in his *Minhāj*, according to Samsó and E. Millás 1998 (ref. 35, p. 273); a clue worth pursuing.

Mars	12º	59º	12º	12º	61º
Venus	2º	50º	4º	3º	51º
Mercury	4º	30º	4º	4º	28º

These examples suffice to show that the enhanced computational methods represented by displaced tables were maintained as a lively tradition among astronomers both in Eastern Islam, and in al-Andalus and the Maghrib. We also know that this tradition reached Byzantium. Shortly after 1347 Georges Chrysococces wrote a set of tables called *Persian Syntaxis*, based on a Persian *zīj*, the *Zīj-i Īlkhānī* (ca. 1270).[37] The tables of Chrysococces contain several displaced tables, and so does an anonymous set of tables for Cyprus with 1346 as epoch.[38] According to Tihon, the solar equation is vertically displaced in both sets, but only in the table of Chrysococces does it always have positive values; the equation of center of the Moon has the same vertical displacement in both sets, whereas the equation of anomaly of the Moon presents different vertical displacements, to have all entries positive; the equation of center of each planet is affected by two displacements, vertical and horizontal, and only Saturn and Jupiter have their equation of anomaly displaced.[39]

The first documented case of the use of displaced tables in Latin is provided by an anonymous set of tables, called the Tables for the Seven Planets, most likely of French origin and uniquely preserved in Paris, Bibliothèque nationale de France, MS 10262.[40] The epoch of the tables is 1340, that is, about five

[37] A. Tihon 1990. "Tables islamiques à Byzance". *Byzantion, Revue internationale des études byzantines* 60:401–425, reprinted in A. Tihon 1994. *Études d'astronomie byzantine*, Variorum (Essay VI). See also R. Mercier 1984. "The Greek 'Persian Syntaxis' and the *Zīj-i Īlkhānī*", *Archives Internacionales d'Histoire des Sciences* 34:35–60. The text of Chrysococces was translated into Hebrew by Shelomo ben Eliyahu of Saloniki (fl. 1374–86); see B. R. Goldstein 1979, "The Survival of Arabic stronomy in Hebrew", *Journal for the History of Arabic Science* 3(1979):31–39, reprinted in B. R. Goldstein 1985. *Theory and Observation in Ancient and Medieval Astronomy*, Variorum (Essay XXI).

[38] A. Tihon 1994. "Un traité astronomique chipriote du XIVe siècle", *Janus. Revue internationale de l'histoire des sciences, de la médecine, de la pharmacie et de la technique* 64 (1977):279–308; 66 (1979):49–81; 68 (1981):65–127, reprinted in Tihon 1994 (ref. 37), Essays VIIa, b, and c.

[39] For each planet, the vertical and horizontal displacements of the equation of center are, respectively, 7º and 14º (Saturn), 6º and 18º (Jupiter), 12º and 12º (Mars), 2º and 2º (Venus), and 4º and 4º (Mercury); for Saturn and Jupiter, the equation of anomaly is also displaced vertically by 7º and 12º, respectively: see Tihon 1994 (ref. 37), Essay VIIc, espec. p. 76 and 114–123. We note that, for the equation of center, the vertical displacements in all five cases and the horizontal displacements for Saturn and Jupiter are the same as those in the *Zīj-i Ashrafī*.

[40] J. Chabás and B. R. Goldstein 2013. "Displaced tables in Latin: the Tables for the Seven Planets for 1340", *Archive for History of Exact Sciences*, 67:1–42.

centuries after the first displaced tables in Arabic. The ambitious purpose of this set, not specified by its anonymous author, is to compute the positions of the Sun, the Moon, and the five planets, within the framework of the Parisian Alfonsine Tables, without appealing to any negative quantity. These tables require 40 displacements, truly a *tour de force*: the solar equation presents a vertical displacement equal to the longitude of the solar apogee; the eighth sphere is vertically displaced by 9º, the standard maximum of the equation of the eighth sphere in Alfonsine astronomy; the Moon is affected by three vertical displacements (13;9º for the equation of center, 2;40º for the increment, and 4;56º for the equation of anomaly); and each of the planets are assigned seven displacements (two for the equation of center – see Table 2 and Figure 4 –, two for the minutes of proportion, and three for the equation of anomaly – see Table 3). The absence of any instructions in the manuscript as well as unusual technical terms in the headings contribute to make it difficult to explain fully these tables, many of which are unprecedented. The underlying principles of the displacements in the Tables for the Seven Planets and even the amounts of several of them are already found in Arabic *zīj*es presented above. But the endeavor goes much beyond, and we do not know of any previous set of displaced tables, in any language, with such a large number of displacements while maintaining the same results as the original tables. Nevertheless, the computational method is the same as that used by Arabic astronomers beginning in the 9th century, and therefore the Tables of the Seven Planets belong to this computational tradition in astronomy.

Table 3: Displacements in the equation of anomaly of the planets in the Tables for the Seven Planets

	Minutes of proportion	Equation of anomaly[41]		
		kv_5	kv_6	kv_7
Saturn	7º	0;21º	6;13º	0;25º
Jupiter	12º	0;30º	11; 3º	0;33º
Mars	49º	5;38º	41;10º	8; 3º
Venus	48º	1;42º	45;59º	1;52º
Mercury	24º	3;12º	22; 2º	2; 1º

[41] While kv_6 is the vertical displacement applied to the equation of anomaly, kv_5 and kv_7 are those applied to the subtractive difference near apogee and to the additive difference near perigee, respectively.

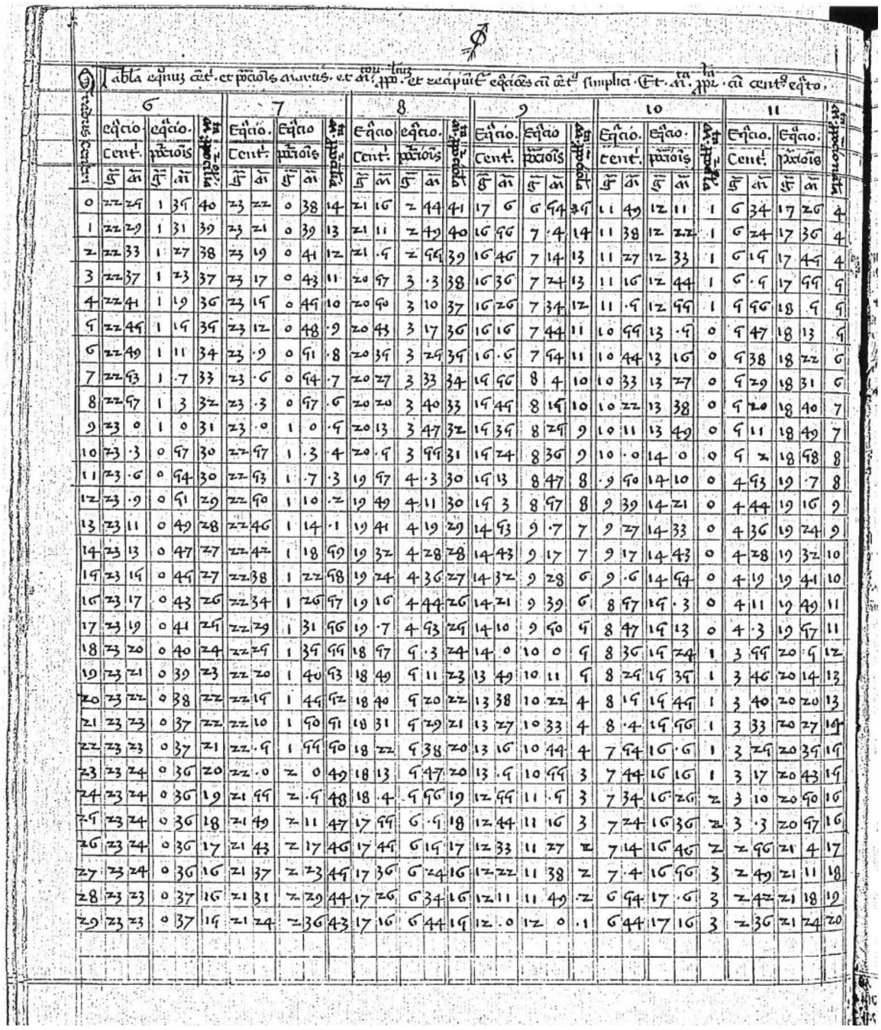

Figure 4: Table for the equation of center for Mars in the Tables of the Seven Planets (Paris, Bibliothèque nationale de France, MS 10262, f. 28v). Only the second page is displayed here. The maximum value under *equatio centri* is 23;24º (at 6s 23º–6s 27º of the argument). A vertical displacement of 12º has been applied to the standard Alfonsine Table, where the maximum value, 11;24º, is reached at 264º–268º. Thus, a horizontal displacement of 61º has also been applied (see Table 2).

There is no doubt that astronomy made in the medieval Western world is heir to Arabic astronomy, and the astronomical tables developed in Latin Europe are a natural continuation of Arabic *zīj*es. Moreover, a common feature in the late Middle Ages is that almost all astronomical tables had Arabic archetypes, usually available in the Iberian Peninsula. The overall picture of the transmission of Arabic astronomical tables to medieval Europe seems quite clear, but to fill the gap between Andalusian and Maghribi tables up the 13th century and those developed in Latin Europe beginning in the early 14th century, more research has to done on both sides.

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