A New Manuscript of Abū Bakr al-Ḥaṣṣār's *Kitāb al-bayān*

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The library of the University of Pennsylvania, Philadelphia, PA, has published in internet a provisional catalogue of about 400 manuscripts of scientific interest in Latin, Arabic and other languages from the private collection of Lawrence J. Schoenberg, Longboat Key, Florida. In 2001 I had the opportunity to inspect a number of astronomical[1] and mathematical manuscripts of the collection in Philadelphia – where a selected number of them had been transferred for an exhibition – and in Mr. Schoenberg's residence.

Here I give a short description of MS LJS 293 which caught my special attention, because in the internet catalogue its author, Abū Bakr, was declared to be the so far unidentified *Alabuchri*, author of a geometrical treatise known only in a Latin translation by Gerard of Cremona[2]. The contents of the Arabic manuscript, however, proves to be purely arithmetical and is, therefore, different from the Latin treatise (see below)[3].

[1] MS LJS 268 is an Arabic manuscript of the *Almagest*, copied in 1381 AD in Saragossa. See the description by P. Kunitzsch, "A Hitherto Unknown Arabic Manuscript of the *Almagest* ", in *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*, 14 (2001), 31-37.

[2] *Liber mensurationum ... Ababuchri*. Cf. F. Sezgin, *Geschichte des arabischen Schrifttums* V, Leiden 1974, 389-391; R. Lemay, article "Gerard of Cremona", in *Dictionary of Scientific Biography* XV [Suppl. I], New York 1978, 173ff., esp. p. 188 (no. 82). The treatise has been edited by H.L.L. Busard, "Le 'Liber mensurationum' d'Abû Bekr", in *Journal des Savants*, 1968, 65-124.

[3] Cf. also J.P. Hogendijk, "A Medieval Arabic Treatise on Mensuration by Qāḍī Abū Bakr", in *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*, 6 (1990),

MS LJS 293 comprises 87 folios (the fly-leaf is not counted). There are no folio numbers in the manuscript itself. I here follow the pagination added to the pages of the text in the library's scanned reproduction[4]. On 2r there appear the title and the author's name (written by the same hand which wrote the whole manuscript): *Kitāb al-bayān wa-l-tadhkār* [this word is partly obliterated, but the reading is clear, because the title is repeated in the author's preface on 2v] *fī ṣanᶜat ᶜamal al-ghubār, ta'līf al-shaykh al-ajall Abū Bakr ibn Muḥammad ibn ᶜAyyāsh al-Ḥaṣṣār*. Under the title there is an owner's name, ᶜAlī ibn Thaᶜlab al-Sāᶜātī al-Baghdādī. Deeper down on the page there is the notice, in Persian ductus: *min kutub Muḥammad ibn Muḥammad ibn al-Ḥasan al-Ṭūsī ghafara Allāhu lahu wa-li-wālidayhi* ("Belonging to the books of M. b. M. b. al-Ḥ. al-Ṭūsī"). It is of some historical interest to know that the manuscript once belonged to the library of that famous Muslim scholar[5]. At the bottom of the page a more recent Eastern Arabic hand[6], which has also added several more notes in the course of the text, has put down, for identification, in an upper line the Western Arabic forms of the nine numerals and, in a lower line, their Eastern Arabic correspondences.

The work begins on 2v with the author's preface and a list of the headings of ten chapters forming Part I (both the chapters and the Part are similarly called *bāb*). The text itself begins on 3v.

The manuscript is written in clear *naskhī* script, mostly undotted. On 87r there is the copyist's colophon. The copy is dated Ṣafar 590 = January-February 1194; it was written in the famous Niẓāmīya *madrasa* in Baghdad by Muḥammad ibn ᶜAbdallāh ibn al-M-ḥ-l al-Baghdādī al-Ḥāsib.

130-150, where comparison is made between this work and the Latin *Liber mensurationum*.

[4] The manuscript had been scanned and can now be read in internet on the homepage of the library of the University of Pennsylvania, Philadelphia, PA.

[5] The script of this owner's note is different both from the script of a collation notice allegedly in the hand of al-Ṭūsī, reproduced in M.T.M. Rizvi, *Aḥwāl wa-āthār-i Khwāja Naṣīr al-Dīn Ṭūsī*, Tehran 1354 sh, p. 166 [I am grateful to Prof. G. Saliba, New York, for providing me with a xerox of this notice from the book] and from the script of al-Ṭūsī's Persian translation of al-Ṣūfī's *Book on the Constellations*, contained in MS Istanbul, Aya Sofya 2595, dated 647 AH / 1250 AD, which is sometimes assumed to be al-Ṭūsī's autograph (the text of the Istanbul MS has been published, in facsimile, in Tehran, 1348 sh / 1969 AD). Which of the three could really be in al-Ṭūsī's hand remains uncertain

[6] This is evident from the word *qalam* which is written with two dots over the *q*, in the Eastern Arabic style.

The numerals (in ch. 2, fol. 5r, and through the whole manuscript) are Eastern Arabic numerals. On 5r the annotator has again added the corresponding Western Arabic forms.

The *Kitāb al-bayān* and its author, Abū Bakr al-Ḥaṣṣār, are known since more than a century[7]. Up to now, four manuscripts of the text had been located: Gotha, Pertsch 1489 (see below); Rabat, B.G. Q 917; al-Zāwiya al--Ḥamzawīya (Morocco); and Damascus, Ẓāhirīya *ᶜāmm* 9760[8]. The Gotha manuscript is in clear *naskhī*, written in 836 AH / 1432 AD. Contrary to the Schoenberg manuscript, where all numerals have been converted into Eastern Arabic numerals, the Gotha manuscript has retained the numerals in their Western Arabic form. The Damascus manuscript is dated 1003 AH / 1594-95 AD and consists of 116 folios; more details are not known[9]. The Rabat manuscript is in Maghrebi script and, of course, has Western Arabic numerals; it is not dated (judging from the script, one would ascribe it to a period around or after 1500 AD)[10]. As it seems, it contains only the ten chapters of Part I[11]. About the manuscript in the Zāwiya al-Ḥamzawīya we have no further information.

The Schoenberg manuscript is at present the fifth known representative of the text and, as it seems, the oldest. In 1901, H. Suter has given a detailed description of the book's contents based on the Gotha manuscript[12]. The work has also been translated into Hebrew, in 1271, by Moses ben Tibbon[13]. It is now also known that al-Ḥaṣṣār has edited two

[7] For the history of its identification and the present state of its knowledge, see M. Aballagh – A. Djebbar, "Découverte d'un écrit mathématique d'al-Ḥaṣṣār (XIIe S.): Le livre I du Kāmil", in *Historia Mathematica* 14 (1987), 147-158; D. Lamrabet, *Introduction à l'histoire des mathématiques maghrebines*, 1994, 56-60 (no. 330).

[8] Lamrabet (note 7), 57 note 23.

[9] See *Al-Fihris al-ᶜāmm li-makhṭūṭāt Dār al-Kutub al-Ẓāhirīya*, Damascus 1407/1987, p. 485 (no. 9760).

[10] The published catalogue of manuscripts in the Bibliothèque Générale in Rabat has not yet reached the collection "Q". Dr. Sa'id Lamrabeti of the manuscript department of the library was kind enough to send me information on the manuscript and copies of the first eight and the last three pages of the text (letter of June 12, 2001). The manuscript was once in the possession of Aḥmad ibn Muḥammad ibn Nāṣir al-Darᶜī (1057-1129 AH / 1647-1717 AD).

[11] Cf. Lamrabet (note 7), 57-59.

[12] H. Suter, "Das Rechenbuch des Abû Zakarîjâ el-Ḥaṣṣâr", in *Bibliotheca Mathematica*, 3. Folge, 2. Band, 1901, 12-40.

[13] Cf. Suter (note 12), 12; Aballagh – Djebbar (note 7), 148; Lamrabet (note 7), 57.

versions of his *Arithmetic*, a shorter one, the *Kitāb al-bayān*, perhaps for the use of students, and an extensive version, the *Kitāb al-kāmil* [14]. The lifetime of al-Ḥaṣṣār has so far been roughly ascribed to the 12th century AD. The Schoenberg manuscript, copied in 1194 AD in Baghdad, now gives a firm *terminus ante quem* for his activity. The *Kitāb al-bayān* must have been written, in the Maghreb, sufficiently earlier so that it could reach Baghdad, in the Arabic East, and be copied there in 1194 AD.

The contents of MS LJS 293 can be easily compared to, and identified with, the detailed description of MS Gotha given by Suter.

The Preface and the ten chapters (*bāb*) of Part I (*al-bāb al-awwal*) correspond to Suter, p. 13-23 (3v-25r). Then follows a section on the multiplication of fractions, in 72 *bāb*s, 25r-66r = Suter, p. 23-28. Hereafter follow more, unnumbered, chapters (*bāb*) on the treatment of fractions, up to 87r (Suter, p. 23-35). Here ends MS LJS 293. In the colophon it is said: *tamma al-juz' al-awwal* [now called *juz'*] *min al-bayān ... yalīhi fī l-thānī bāb qismat al-kasr*, "Finished is the first Part [*juz'*] of the *Bayān*; there follows in the second [sc. Part] the chapter [*bāb*] on the division of the fraction" (sic). But this remaining part of the work (about a quarter of what is in MS Gotha – in Suter reaching until p. 39) is absent from MS LJS 293, which ends here.

Abū Bakr al-Ḥaṣṣār seems to be one of the earliest Western Arabic mathematical authors of whom a work has survived. Unfortunately, the manuscript of his *Kitāb al-bayān* cannot really contribute to our knowledge of the development of the written forms of the Hindu-Arabic numerals in the Western Arabic style. As described above, the oldest manuscript, copied in Baghdad in 1194 AD, has transformed all the numerals into the common Eastern Arabic forms. The other known manuscripts were copied in later centuries; and it must be assumed that their copyists used forms of the numerals current in the periods of their activity[15].

"Hindu Reckoning", using the nine numerals in decimal position and a symbol for an "empty position" (i.e., void of any of the nine numerals), the zero, made its way from the Arabic East into the West at latest in the tenth century. In 955-56 AD Abū Sahl Dunas ibn Tamīm in Kairouan says in his

[14] See especially, Aballagh – Djebbar (note 7).

[15] D. Lamrabet informed me lately during the Congress in Marrakesh (cf. note 21) that in the meantime another manuscript of the *Kitāb al-bayān* has been found: it is in Rabat, Mu'assasat ʿAllāl al-Fāsī. It can be ascribed to the 8th c. AH / 14th c. AD and is, therefore, also one of the more recent copies..

commentary to the *Sefer Yeṣira*: "Les Indiens ont imaginé neuf signes pour marquer les unités. J'ai parlé suffisamment de cela dans un livre que j'ai composé sur le calcul indien connu sous le nom de حساب الغبار, c'est-à dire calcul du *gobar* ou calcul de poussière"[16]. Testimonies in Europe, in Spain, where the nine numerals and the new system of calculation were taken over from the Arabs in al-Andalus, begin in 976 AD[17] and continue in a rich tradition in Latin manuscripts on the abacus, until a new influx came in the great translation movement in the twelfth century.

In contrast to the Latin world, where we have a well documented continuous tradition of writing the nine numerals and zero from 976 on, there is an astonishing lack of evidence on the (Western) Arabic side for the first three and a half centuries of the Maghrebi and Andalusian acquaintance with the Hindu-Arabic numerals. All the known manuscripts of Western Arabic mathematical texts were copied in more recent periods. The oldest document showing Western Arabic numerals that has come to my knowledge is ms. Florence, Or. 152, in Maghrebi script, dated 1265-66 AD[18]. Here, in texts on the construction of some technical devices (machines for lifting water, etc.), both Eastern and Western Arabic numerals are inserted in the current text, in place of words[19]; the reason and purpose of the use of these numerals in the context still remains unexplained.

So, while there exists sufficient evidence for written Eastern Arabic forms of the numerals from the tenth century on (perhaps even one

[16] See J. Reinaud, in an *Addition* to his *Mémoire sur l'Inde*, in *Mémoires de l'Institut Impérial de France, Académie des Inscriptions et Belles-Lettres* 18 (1855), p. 565.

[17] To this year is dated the famous "Codex Vigilanus" (now in the Library of the Escorial) containing Isidor of Sevilla's *Etymologiae*; in an inserted passage it praises the Indian system of writing numbers and shows the forms of the nine numerals which are clearly of the Western Arabic type. (For an illustration, see, e.g., B.L. van der Waerden – M. Folkerts, *Written Numerals*, The Open University Press, Walton Hall, Milton Keynes, GB, 1976, p. 54; J. Vernet – J. Samsó (eds.), *El Legado Científico Andalusí*, Madrid, 1992, p. 24) A cognate manuscript is the "Codex Emilianus", dated 992 AD, also in the Escorial.

[18] I am grateful to Dr. S. Brentjes, Berlin, for bringing this manuscript to my attention. See Figs. 3a/b in P. Kunitzsch, The Transmission of Hindu-Arabic Numerals Reconsidered, to appear in a volume on Perspectives on Science in Medieval Islam, ed. by J. Hogendijk and A.I. Sabra, Cambridge, MA.

[19] There appear Eastern forms of 2 and 3; a clear Western 8; 1, 4 and 9 which can be as well Eastern as Western; and a figure which could be the Eastern 6 or the Western 5.

example from 260 AH / 873-74 AD in a papyrus from Egypt)[20], there remains a deplorable lack of written forms of Western Arabic numerals from the period before ca. 1400 AD. It is to be hoped that further research in the rich treasure of Maghrebi manuscripts will produce such evidence in the future.[21]

[20] For details, see Kunitzsch (note 18).

[21] A version of this paper was presented at the "7e Colloque Maghrébin sur l'Histoire des Mathématiques Arabes", May 30 – June 1, 2002, in Marrakesh.